On Clique Immersions in Line Graphs

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We prove that if $L(G)$ immerses $K_t$ then $L(mG)$ immerses $K_{mt}$, where $mG$ is the graph obtained from $G$ by replacing each edge in $G$ with a parallel edge of multiplicity $m$. This implies that when $G$ is a simple graph, $L(mG)$ satisfies a conjecture of Abu-Khzam and Langston. We also show that when $G$ is a line graph, $G$ has a $K_t$-immersion iff $G$ has a $K_t$-minor whenever $t \leq 4$, but this equivalence fails in both directions when $t = 5$. 