Modifications to the Birkhoff Polytope and Complexity Class Boundaries

Stephen Gismondi*, University of Guelph

Assuming that P ≠ NP and Graph Isomorphism (GI) is not in P, this paper proposes an approach toward observing complexity class boundaries. A P problem is initially modelled as an IP feasibility problem whose relaxed feasible region is the Birkhoff polytope. An NP-complete problem and GI are also modelled as IP feasibility problems subject to relaxed feasible regions that are respectfully the intersection set of the Birkhoff polytope and 1) additional quadratic equations (the solution set is no longer a polytope), and, 2) linear homogeneous equations (the solution set remains a polytope, but not the Birkhoff polytope). These models are developed via incremental modifications to the Birkhoff polytope yielding new feasible regions or sets of feasible points. By noting changes in the complexity of algorithms that decide the existence of integer extrema of these new regions or sets, as these models are developed i.e. as incremental modifications to the Birkhoff polytope are implemented, these changes might be indicative of complexity class boundaries. Regarding the GI model as incrementally developed here, it’s easy to prove that not all integer extrema become infeasible by adding a few additional linear constraints (as required by the model) i.e. LP remains a viable solution technique and this seeming in-between P and GI problem is in P. But eventually this becomes unprovable, and more complex algorithms are needed. Similar but more complicated comments can be made re: quadratic constraints.

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