Functions on the Integers Modulo $n$

Dylan Cruz$^1$, Andrés Ramos$^1$, Gary Mullen$^2$, Ivelisse Rubio$^1$
1: University of Puerto Rico, Río Piedras, 2: Penn State University

In a recent paper, Ullman and Velleman studied which functions from an abelian group $G$ to itself can be expressed as a difference of two bijections from $G$ to itself. The results have applications to juggling and bus scheduling. If $G = \mathbb{Z}_n$ one has that $a : \mathbb{Z}_n \to \mathbb{Z}_n$ can be written as a difference of bijections if and only if $\sum_{i \in \mathbb{Z}_n} a(i) \equiv 0 \pmod{n}$. We present results on a more general question: When can a function from a group to itself be expressed as a difference of two functions with the same image as a multiset? We also study connections to juggling sequences, latin squares and cyclic group orthomorphisms.

Keywords: Functions on Groups, Image of Function as Multiset, Applications to Juggling Sequences