List homomorphism problem for signed graphs

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In the talk, we are interested in classifying the computational complexity of the list homomorphism problem for signed graphs. Signed graphs were introduced by Zaslavsky in the 1980s. A signed graph is an undirected graph $G$ in which each edge has a positive or negative sign. We denote such a graph as a pair $(G, \Sigma)$, where $\Sigma$ is the set of its negative edges. Loops and parallel edges are allowed in signed graphs. However, there is at most one positive and at most one negative edge with the same endpoints. The switching operation can be applied to any vertex, resulting in flipping the signs of the incident edges.

Two signed graphs $(G, \Sigma)$ and $(H, \Pi)$ are said to be homomorphic if there is a graph $(G, \Sigma')$, obtained from $(G, \Sigma)$ by a sequence switchings, such that there is a mapping from the vertex set of $(G, \Sigma')$ to the vertex set of $(H, \Pi)$ preserving both adjacencies of the vertices and colors of the edges. The list homomorphism problem for signed graphs can then be defined in the following way. Let $(H, \Pi)$ be a fixed signed graph. The List-S-Hom$(H, \Pi)$ problem has on its input a signed graph $(G, \Sigma)$ with lists $L(v) \subseteq V(H)$ for every $v \in V(G)$. We ask if there exists a homomorphism $f$ from $(G, \Sigma)$ to $(H, \Pi)$ such that $f(v) \in L(v)$ for every $v \in V(G)$.

We shall provide both polynomial and NP-complete cases of this problem, obtained as partial results in our effort towards solving the full dichotomy of the problem.

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