Continuity, Differentiation and Quantum State Transfer in Discrete Structures

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The focus here is twofold: (1) to show how one may move continuously in continuous time on just the vertices of directed graphs (without moving along edges) and (2) an application to the transfer of a quantum state through a continuous path in a simple quantum network. The key idea is that directed graphs are pretopological spaces. Bourbaki (1949) introduced an approach to topology with filters as the basic notion. Subsequently, a simple hierarchy developed in which the class of topological spaces is included in the class of \textit{pretopological spaces} and is in turn included in the class \textit{convergence spaces}. Continuity and limit in convergence spaces are defined as in Bourbaki and agree with continuity and limit in topological spaces. Continuity of mappings among directed graphs is homomorphism. Digraph homomorphism can be coarse-grained corresponding to a coarsening of convergence structure for applications to data-partitioning, but may involve operations in functional-NP. We will give the definitions of these spaces, continuity, and differentiation, with straightforward examples including the transfer of a quantum state along the vertices of a continuous path in a simple quantum network where the path to be followed is described by an ODE.

Keywords: directed graph, pretopological space, convergence space, filter, quantum state