Graphs with few trivial characteristic ideals

Carlos A. Alfaro Montufar, Banco de México; Michael D. Barrus*, University of Rhode Island; John Sinkovic, Brigham Young University–Idaho; Ralihe Villagráñ, Center for Research and Advanced Studies of the National Polytechnic Institute, Mexico City

Many interesting properties of a graph $G$ can be related to properties of its adjacency matrix $A$ or Laplacian matrix $L$. For example, the sandpile group of $G$, also known as the critical group, is a graph invariant with strong ties to $L$. A number of recent, interesting questions focus on the structure of this critical group.

In this report on an ongoing project, we will see how a question about critical groups with few invariant factors equal to 1 can be approached using matrix minors related to the characteristic polynomial of $A$. It turns out that when we bound the number of trivial “characteristic ideals” in $A$, the corresponding graphs have characterizations in terms of forbidden induced subgraphs. We present complete forbidden subgraph characterizations for the first two of these classes and describe progress on a conjectured characterization of the third.

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