Generalizing $p$-goodness to ordered graph Ramsey numbers

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Let $R(G, H)$ denote the usual two-color Ramsey number. A graph $G$ of order $n$ is said to be $p$-good if $R(G, K_p) = (n - 1)(p - 1) + 1$. In other words, $G$ is $p$-good iff the lower bound for $R(G, K_p)$ implied by the appropriate Turán graph is actually tight. It is known that the only connected graphs that are $p$-good for all $p$ are trees.

In this talk we consider ordered Ramsey numbers $r_{<}(T^<, K_p)$ of ordered trees vs complete graphs. Call an ordered tree $T^<$ on $n$ vertices order-$p$-good if $r_{<}(T^<, K_p) = (n - 1)(p - 1) + 1$. Every monotone path is order-$p$-good for all $p$, but not every ordered tree is order-$p$-good for all $p$. We attempt to characterize the class of ordered trees that are order-$p$-good for all $p$, and consider some reasonable-sounding guesses that turn out to be incorrect.

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