Site percolation threshold bounds for the $4, 8^2$) Archimedean tiling

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Percolation models are infinite random graph models which are used to model phase transitions and critical phenomena. In the site percolation model, each vertex in an infinite graph G is retained independently with probability p and deleted otherwise. The percolation threshold is the probability $p_c(G)$ such that if $p > p_c(G)$ there is positive probability that the random subgraph induced by the retained vertices has an infinite connected component, while all components are finite if $p < p_c(G)$. There are very few lattice graphs for which the site percolation threshold is exactly known, and rigorous bounds for unsolved lattices are very imprecise. The substitution method for computing bounds for the more common class of bond percolation models must be substantially modified to apply to site models. The complications will be illustrated with an application to the $(4, 8^2)$ Archimedean lattice, which is a vertex-transitive tiling of the plane by squares ad regular octagons.

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