

Progress on a characterization of signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 3$.

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A signed graph is a pair (G, Σ) , where G is an undirected graph (we allow parallel edges but no loops) and $\Sigma \subseteq E(G)$. The edges in Σ are called odd, while the other edges are called even. If (G, Σ) is a signed graph with vertex-set $V = \{1, \dots, n\}$, $S(G, \Sigma)$ is the set of all real symmetric $n \times n$ matrices $A = [a_{i,j}]$ with $a_{i,j} > 0$ if i and j are adjacent and connected by only odd edges, $a_{i,j} < 0$ if i and j are adjacent and connected by only even edges, $a_{i,j} \in \mathbb{R}$ if i and j are adjacent and connected by both even and odd edges, $a_{i,j} = 0$ if i and j are not adjacent, and $a_{i,i} \in \mathbb{R}$ for all vertices i . The parameter $\nu(G, \Sigma)$ is defined as the largest nullity of any positive semidefinite matrix $A \in S(G, \Sigma)$ satisfying the Strong Arnold Hypothesis. This invariant is closed under taking minors. Arav, Hall, van der Holst, and Li gave a characterization of the class of signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 2$. A characterization of the class of signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 3$ is still open. In this talk, we present some classes of signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 3$.

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