Aggregate Sequences in Sage

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In the game "Triple Town" pieces of grass are placed such that three or more pieces of grass promote to a bush, three or more bushes promote to a tree, three or more trees promote to a red house, and so on until a castle is built. The game is played on a cartesian product of two paths of length six, though we generalize the game to any graph and define a sequence of ones to be promoted to 2, 3, etc according to the recursive promotion rules of the game. There are several parameters to optimize for any given graph. The first is to find the shortest sequence (respectively longest) sequence of ones to be placed on the vertices for the game to terminate, the second is to maximize the sum of the weights of the vertices (1 for grass in the game, 2 for a bush, 3 for a tree, etc) after the game terminates.

We define a greedy algorithm, implemented in Sage, which fixes an ordering of the vertices for each 1 to be placed, including recursive promotions. We create a certificate in Sage to prove the weights on vertices at the end of a game are valid. We also consider a cryptographic protocol based on reversing a compact form of the certificate to reconstruct the original moves of the game.

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