

Algebraically Defined Graphs and Their Girth

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An algebraically defined graph $\Gamma_{\mathcal{R}}(f(x, y))$ is constructed using a specific ring \mathcal{R} and function $f(x, y)$. These graphs are bipartite with each partite set consisting of all coordinate pairs in \mathcal{R}^2 . We denote the vertices of the first partite set by (a_1, a_2) and of the second by $[x_1, x_2]$. Two vertices (a_1, a_2) and $[x_1, x_2]$ are adjacent when their coordinates satisfy the equation $a_2 + x_2 = f(a_1, x_1)$. The focus of our study is the girth, or length of a shortest cycle, of these graphs. In this talk, we will use incidence geometry to motivate our study of algebraically defined graphs. We will also discuss the effect that changing the ring \mathcal{R} and function $f(x, y)$ has on the girth of the algebraically defined graph $\Gamma_{\mathcal{R}}(f(x, y))$, with particular emphasis on the case $\mathcal{R} = \mathbb{R}$.

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