On constant sum partitions and applications to distance magic-type graphs

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Let $G$ be an additive abelian group of order $n$ and let $n = a_1 + a_2 + ... + a_p$ be a partition of $n$ where $1 \leq a_1 \leq a_2 \leq ... \leq a_p$. A constant sum partition (or $t$-sum partition) of $G$ is a pairwise disjoint union of subsets $A_1, A_2, ..., A_p$ such that $G = A_1 \cup A_2 \cup ... \cup A_p$, $|A_i| = a_i$, and $\sum_{a \in A_i} a = t$, for some fixed $t \in G$ and every $1 \leq i \leq p$.

In 2009, Kaplan, Lev, and Roditty proved that a 0-sum partition of the cyclic group $\mathbb{Z}_n$ exists for $n$ odd if and only if $a_2 \geq 2$. In this talk, we address the case when $n$ is even. In particular, we show that a $\frac{n}{2}$-sum partition of $\mathbb{Z}_n$ exists for $n$ even and $p$ odd if and only if $a_2 \geq 2$. Moreover, we provide applications to distance magic-type graphs including the classification of $\mathbb{Z}_n$-distance magic complete $p$-partite graphs for $p$ odd.

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