**C₄-face-magic torus labelings on** $C_{2m} \times C_{2n}$

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For a toroidal graph $G = (V,E)$ embedded in the torus, let $\mathcal{F}(G)$ denote the set of faces of $G$. Then, $G$ is called a $C_n$-face-magic torus graph if there exists a bijection $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ such that for any $F \in \mathcal{F}(G)$ with $F \cong C_n$, the sum of all the vertex labelings along $C_n$ is a constant $S$. Let $x_v = f(v)$ for all $v \in V(G)$. We call $\{x_v : v \in V(G)\}$ a $C_n$-face magic torus labeling on $G$. We investigate $C_4$-face-magic torus labelings on $C_{2m} \times C_{2n}$. We say that a $C_4$-face-magic torus labeling $\{x_{i,j}\}$ on $C_{2m} \times C_{2n}$ is antipodal balanced if $x_{i,j} + x_{i+m,j+n} = (1/2)S$ for all $(i,j) \in V(C_{2m} \times C_{2n})$. We show that there exists a $C_4$-face-magic antipodal balanced torus labeling on $C_{2m} \times C_{2n}$ for all positive integers $m$ and $n$. We investigate properties of these labelings on $C_{2m} \times C_{2n}$. We also determine all $C_4$-face-magic antipodal balanced torus labelings on $C_4 \times C_4$ up to symmetries on a torus.

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