Beta-Packing Sets in Graphs

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A set $S \subseteq V$ is $\alpha$-dominating if for all $v \in V - S$, $|N(v) \cap S| \geq \alpha|N(v)|$. The $\alpha$-domination number of $G$ equals the minimum cardinality of an $\alpha$-dominating set $S$ in $G$. Since being introduced by Dunbar, et al. in 2000, $\alpha$-domination has been studied for various graphs and a variety of bounds have been developed. In this paper, we propose a new parameter derived by flipping the inequality in the definition of $\alpha$-domination. We say a set $S \subset V$ is a $\beta$-packing set of a graph $G$ if $S$ is a proper, maximal set having the property that for all vertices $v \in V - S$, $|N(v) \cap S| \leq \beta|N(v)|$ for some $0 < \beta \leq 1$. The $\beta$-packing number of $G$ ($\beta$-pack($G$)) equals the maximum cardinality of a $\beta$-packing set in $G$. The single greatest interest of studying $\beta$-packing sets, as with $\alpha$-dominating sets, is finding the value of $\beta$-pack($G$) for some graph. In this research, we set out to find $\beta$-pack($G$) for different types of graphs. We focused particular attention on $1/2$ $\beta$-packing sets; that is, where we set $\beta = 1/2$.

Keywords: $\alpha$-domination, $\beta$-packing, graph theory, graph parameters