The Upper Transversal Number of Uniform Hypergraphs

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The upper transversal number of a hypergraph is the maximum size of a minimal transversal, also known as a vertex cover or hitting set. Minimal transversals of hypergraphs have been intensely studied in the contexts of computational complexity and data mining. We show that for \( k \geq 4 \), the upper transversal number of a \( k \)-uniform hypergraph of order \( n \) is bounded below by \( \sqrt[4]{n} \), for sufficiently large \( n \). This settles a conjecture of Henning and Yeo in all but finitely many cases for each fixed \( k \). We also construct a \( k \)-uniform hypergraph of order \( n \) such that the upper transversal number is at most \( 2k \sqrt[4]{n} \), thus showing that the lower bound is tight up to a linear factor in \( k \).

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