List Colorings: Some Refinements of Hall’s Condition.


Let $C$ be an infinite set of symbols. A function $L$ is a list assignment to a graph $G$ if $L$ assigns to each vertex of $G$ a non-empty finite subset of $C$, called a list. A proper $L$-coloring of $G$ is an assignment of “colors” to the vertices of $G$, from their lists, so that adjacent vertices are colored with different colors. Interpreted as a theorem about proper list colorings of complete graphs, P. Hall’s theorem on systems of distinct representatives inspires a generalization, a necessary condition for proper colorings, known as Hall’s Condition (HC). We present some refinements of HC, and explore some conditions for proper $L$-colorings of the graph $G$.

Key Words: Hall’s Condition, List Coloring, Hall plus, Hall plus plus, HC plus, HC plus plus.