New Families of Cubic Surfaces in Characteristic Two

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Cubic surfaces with 27 lines are beautiful objects from classical geometry. Several infinite families are known, due to Fermat, Clebsch and Hilbert-Cohn-Vossen. Over finite fields, these families exist also, but there are more. One interesting example is a family due to Dickson and Hirschfeld, which exists whenever the field has a subfield $\mathbb{F}_4$. We present two new families of cubic surfaces with 27 lines. One exists whenever the field $\mathbb{F}_8$ is a subfield. The other requires $\mathbb{F}_{16}$ to be a subfield. These families are all nonisomorphic. For instance, over the field $\mathbb{F}_{64}$, both the Dickson-Hirschfeld family and the $\mathbb{F}_8$ family exist. We can tell them apart by looking at the number of Eckardt points. The Dickson-Hirschfeld surface has 45 Eckardt points, while the $\mathbb{F}_8$-surface and the $\mathbb{F}_{16}$-surface each have 13. Also, their automorphism groups are different. Many more surfaces exist which do not fall into any of these families.

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