$(0,1)$-Matrices and Discrepancy

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Let m and n be positive integers, and let $R = (r_1, \ldots, r_m)$ and $S = (s_1, \ldots, s_n)$ be nonnegative integral vectors. Let $A(R,S)$ be the set of all $m \times n$ $(0,1)$-matrices with row sum vector R and column vector S. Let R and S be nonincreasing, and let $F(R)$ be the $m \times n$ $(0,1)$-matrix where for each i, the i^{th} row of $F(R,S)$ consists of r_i ones followed by $n - r_i$ zeros. Let $A \in A(R,S)$. The discrepancy of A, $disc(A)$, is the number of positions in which $F(R)$ has a one and A has a zero. In this paper we investigate the possible discrepancy of A^t versus the discrepancy of A. We show that if the discrepancy of A is ℓ then the discrepancy of the transpose of A is at least $\frac{\ell}{2}$ and at most 2ℓ. These bounds are tight.

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