Ramsey number for a tree versus a small wheel

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For any graphs $G$ and $H$, we define the Ramsey number $R(G, H)$ as the smallest natural number $k$ such that for any graph $F$ of order $k$, either $F$ contains $G$ as a subgraph or its complement of $F$ contains $H$ as a subgraph. The problem of finding the Ramsey number of a tree $T_n$ on $n$ vertices versus a wheel $W_m$ on $m + 1$ vertices has been extensively investigated. However, the problem is far from completely solved. Y. Chen, Y. Zhang, and K. Zhang (2004) strongly conjectured that $R(T_n, W_m) = 2n - 1$ if the maximum degree of $T_n$ is small and $m$ is even. For a tree $T_n$ with large maximum degree and even $m$, the $R(T_n, W_m)$ is also unknown in general. In this paper, we shall determine the Ramsey number $R(T_n, W_8)$ for all trees $T_n$ of order $n$ with the maximum degree of $T_n$ is at least $n - 3$.

Keywords: Ramsey number, tree, wheel.