Are Complete Graphs the Only $\chi$-Robust-Critical Graphs?

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Given a proper $k$-coloring of a graph, a local recoloring of a vertex $v$ is a proper $k$-coloring in which the color of $v$ changes but the colors of vertices not in $N[v]$ remain the same. A proper $k$-coloring is robust if every vertex is locally recolorable. The robust-chromatic number, $\rho(G)$, is the smallest $k$ for which graph $G$ has a robust $k$-coloring. A graph $G$ is $\chi$-robust-critical if $\rho(G) = \chi(G)$, and for any vertex $v$, $\rho(G - v) = \rho(G) - 1$.

We conjecture that complete graphs are the only $\chi$-robust-critical graphs. We show that $K_4$ is the only $\chi$-robust-critical graph with chromatic number 4 and maximum degree less than or equal to 5.

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