Hall t-chromatic spectra and weak Hall t-chromatic spectra of the Petersen Graph and wheels with odd numbers of spokes
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A color demand function on a graph $G$ is a function $\kappa : V(G) \to \mathbb{N}$. A proper $(t,\kappa)$-coloring of $G$ is a function $\phi$ assigning each vertex of $G$ a subset of $[t] = \{1, 2, \ldots, t\}$ so that for each $v \in V(G)$, $|\phi(v)| = \kappa(v)$ and for each $uv \in E(G)$, $\phi(u) \cap \phi(v) = \emptyset$. $\alpha(G)$ is the vertex independence number of $G$. $G$ and $\kappa$ satisfy Hall’s $t$-condition if and only if for each subgraph $H$ of $G$

$$ta(H) \geq \sum_{v(H)} \kappa(v)$$

It is clear that Hall’s $t$-condition is necessary for the existence of a proper $(t,\kappa)$-coloring of $G$. If it is sufficient (i.e. If $G$ is properly $(t,\kappa)$-colorable for every color demand $\kappa$ on $G$ such that $G$ and $\kappa$ satisfy Hall’s $t$-condition) then $G$ is Hall $t$-chromatic. If Hall’s $t$-condition with the equation $ta(G) = \sum_{v(G)} \kappa(v)$ suffice for the existence of a proper $(t,\kappa)$-coloring of $G$, then $G$ is weakly Hall $t$-chromatic.

We show that the Petersen graph is Hall 3-chromatic and determine the weak Hall $t$-chromaticity of wheels with odd numbers of spokes.

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