Combinatorics of lane merging: Limited Capacity Lanes

Shaun Sullivan*, Erik Insko, Katie Johnson, Florida Gulf Coast University

A two lane road approaches a stoplight. The left lane merges into the right just past the intersection. Vehicles approach the intersection one at a time, with some drivers always choosing the right lane, while others always choose the shorter lane, giving preference to the right lane when the lanes are equal. In previous work, we analyzed the expected number of cars in the left lane with unlimited lane capacity. Now we consider lanes with limited capacity so that there will generally be a gap in the left lane where cars can no longer access. Given an arrival sequence of vehicles, represented as a binary string where the zeros represent drivers always choosing the right lane, and the ones represent drivers choosing the shorter lane, we create lattice paths representing how the cars stack up in the lanes. We give closed formulas for the number of these 'merging' paths reaching the point (n, m) with exactly k zeros in the arrival sequence, and the expected gap size. Many connections to the ballot numbers also emerge.