On Graceful Spectra of Graphs


For integer $k \geq 1$, a $(p, q)$-graph $G = (V, E)$ is called a $k$-graceful graph if there exists an injective mapping $f : V(G) \rightarrow \{0, 1, \ldots , k + q - 1\}$ such that the induced map $f^* : E(G) \rightarrow \mathbb{N}$ defined by $f^*((u, v)) = |f(u) - f(v)|$ for all $(u, v) \in E(G)$ is an bijection to $\{k, k + 1, \ldots , k + q + 1\}$. The graceful spectra of $G$ is the set $\{k \mid G$ is a $k$-graceful graph$\}$ and denoted by $\text{GS}(G)$. In this paper, we investigate the set $\text{GS}(G)$ and exhibit infinite families of graphs $G$ with $\text{GS}(G) = \mathbb{N}$ but not admit $\alpha$-labeling.

Keywords: vertex labeling, edge labeling, $k$-graceful, graceful spectra, $\alpha$-labeling