Multithreshold Graphs

Robert E. Jamison, Alan P. Sprague*, Clemson University, U. Alabama at Birmingham

Chvatal and Hammer defined a graph to be a threshold graph if every vertex $v$ has a real rank $r(v)$ such that two vertices $v$ and $w$ are adjacent precisely when $r(v) + r(w) \geq 0$. We extend this notion: we define a graph to be a $k$-threshold graph if every vertex $v$ has a real rank $r(v)$ and there exist $k$ real numbers called thresholds such that two vertices $v$ and $w$ are adjacent precisely when $r(v) + r(w)$ is greater than or equal to an odd number of thresholds. The 1-threshold graphs are precisely the threshold graphs of Chvatal and Hammer. The class of 2-threshold graphs is intermediate between the class of bipartite permutation graphs and the class of permutation graphs. We will report on graph classes such that few thresholds suffice, classes requiring many thresholds, and show an upper bound on the number of thresholds for all graphs on $n$ vertices.

Keywords: threshold graphs, permutation graphs