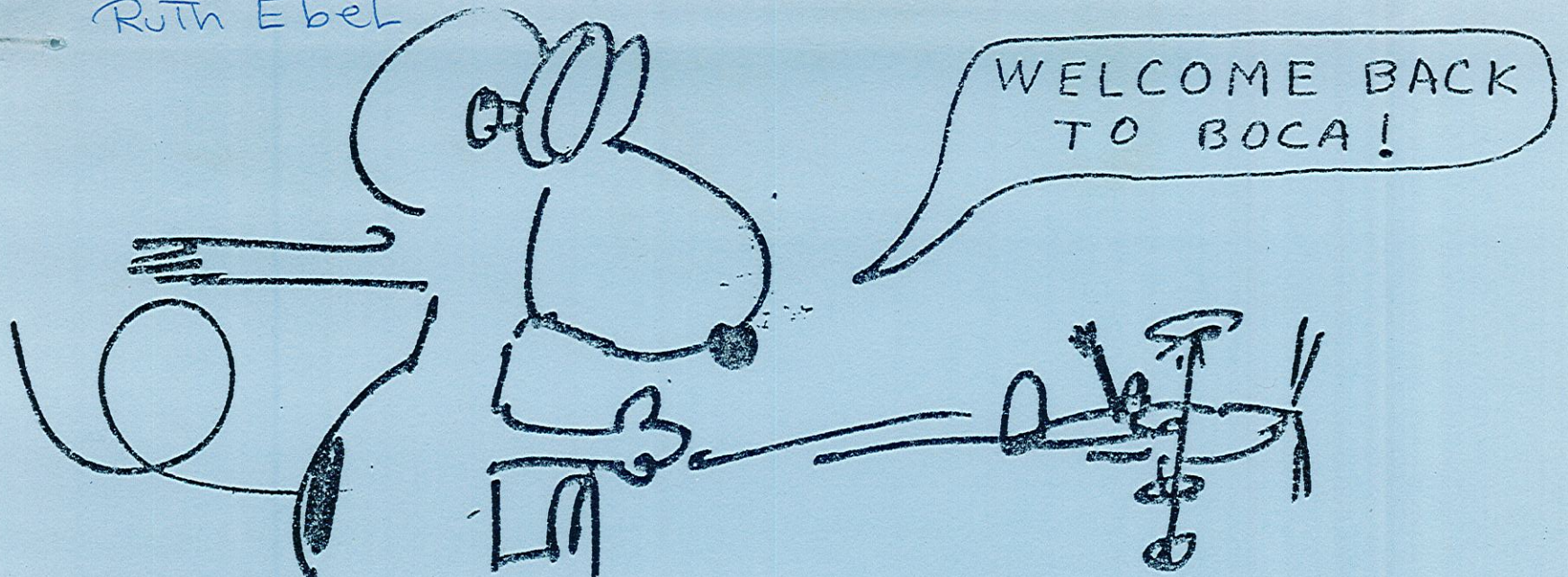
Ruth Ebel



PROGRAM and SCHEDULE

The Ninth Southeastern Conference on

COMBINATORICS GRAPH THEORY & COMPUTING

January 30-
February 2,
1978

Florida Atlantic University

GCR is the Gold Coast Room of the University Center.

Rooms 207 and 232 are entered from the second floor lounge of the University Center.

Coffee available in the other half of the Gold Coast Room.

CONFERENCE COCKTAIL PARTY in Casablanca Room of Howard Johnson's Ocean Resort Monday, January 30, 1978 at 6:00PM.

CONFERENCE BANQUET in Windsor Ballroom of the Howard Johnson's Ocean Resort Tuesday, January 31, 1978. Seating at 6:45PM, service at 7:00PM. Take advantage of "Happy Hour" prices in Billy Bowlegs Lounge, 5:00-6:30PM.

NOTE: Conference bus will leave University Center at 5:10PM Monday and Wednesday, 4:50PM Tuesday, and 1:00PM Thursday.

	MONDAY						TUESDAY						WEDNESDAY						THURSDAY	
	GCR		207		232		GCR		207		232		GCR		207		232		GCR	207
8:30	Registration (from 8AM)						Jackson	33	B. Reid	35			Fajtlowicz	66	Ballas	68			Freeman 96	Hadlock 98
9:00	Opening remarks Welcome by Pres. Creech & Dean Baker						Fleischner	34	Ismail	36			Cockayne	67	Killgrove	69			Mullin 97	Rogers et al 99
9:30	ULAM						BONDY						WILLIAMS						ROTA	
10:30	COFFEE																			
10:40	Marimont	1	Barkauskas	5	G. Berman	9	Levow	37	Robertson	41	Perry	45	Laskar	70	Catlin	74	Wagner	78	E	
11:00	Owens	2	Johnson	6	Wang	10	Kalbfleisch	38	Mohanty	42	T. Reid	46	Wallis	71	Dewdney	75	Rogers	79	R D O	
11:20	Ball	3	Eggleton	7	Devitt	11	Haggkvist	39	Faudree	43	S. Zaks	47	Nishizeki	72	Christopher	76	Cot	80	S	
11:40	Levinson	4	Hammer	8	Regener	12	Caccetta	40	Sheehan	44	Cook	48	Kotzig	73	Hell	77	Cummings	81	ROTA	
12:00	LUNCH																			
1:30	ROSENFELD	13					BONDY						WILLIAMS							
2:40	Carmony	14	Payne	21	Chinn	28	Schelp	49	Di Paola	55	Toan	61	Parsons	82	Mesner	89				
3:00	Bennett	15	J. Zaks	22	Slater	29	Ringeisen	50	D.G. Hoffman	56	Hansen	62	Simmons	83	Swanson	90				
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MONDAY, January 30, Professor S. Ulam will speak on "Some Combinatorial Studies Suggested by Biological Problems."

TUESDAY, January 31, Professor J. Adrian Bondy will speak on "Cycles in Graphs and Digraphs."

WEDNESDAY, February 1, Professor Hugh C. Williams will speak on "Recent Advances in the Use of Computers for Primality Testing."

THURSDAY, February 2, Professor Gian-Carlo Rota will speak on "The Umbral Calculus."

THURSDAY, February 2, Professor Paul Erdös will speak on "Current Problems in Combinatorics and Graph Theory."

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NOTE:
Schedule lists papers by presenters.

1 CONNECTIVITY, COMBINATORICS, AND A CONTINUANT

Rosalind B. Marimont National Institutes of Health

Determinants, principal minors, characteristic equations, and cofactors of matrices may sometimes be easily evaluated by use of the connectivity properties of their associated digraphs. If A is an n x n zero axial tridiagonal matrix, with superdiagonal a_i and subdiagonal b_i, and if $a_i b_i = p$, its characteristic equation may be written by inspection, using the fact that the number of ways of choosing j non-adjacent objects out of a row of n-1 objects is $\binom{n-j}{j}$. Writing C(n,j) for $\binom{n-j}{j}$, we have

$$F(n) = \sum_{j=0}^{INT(n/2)} C(n,j)(-p)^j(-\lambda)^{n-2j} \qquad (1)$$

From the theory of determinants, F(n) for this matrix is known to obey the recursion equation

$$F(n) = -\lambda F(n-1) - pF(n-2);\quad F(0) = 1;\quad F(1) = -\lambda \qquad (2)$$

By treating eq (2) as a difference equation, the sum for the series of eq (1) can be evaluated in closed form and its behavior studied. For example, if $\lambda^2 - 4p < 0$, F(n) will oscillate. The coefficients C(n,j) obey the relation C(n,j) = C(n-1,j) + C(n-2, j-1) and can therefore be generated recursively in a way similar to the generation of binomial coefficients by Pascal's triangle.

2 Enumeration of Weighted Binary Codes

Frank W. Owens - Ball State University

The transmission of an encoded message through a binary channel with noise is subject to error upon reception. There are various schemes (codes) for correcting and/or detecting errors upon decoding the message received.

The design of error-correcting and error-detecting codes requires a knowledge of weight sets. The number of ones required for the transmission of a given binary code word is called the weight of the code word, and the multiset of the weights of the various code words in a given binary code is called the weight set of the binary code. Two binary codes are said to be equivalent if they have the same weight sets. A weighted binary code is defined as an equivalence class of binary codes under this relation. This paper presents algorithms for the enumeration of weighted binary codes having n code words and of weighted binary codes having n code words with the longest code word having length m.

3 A COMPLETE LIST OF CF(2,5) CODES

A.H. Ball, University of Waterloo

Comma-free codes are synchronizable, that is, they do not require a distinct symbol of the alphabet to separate codewords. Recently, the isomorphism classes of comma-free codes with word length 3 over an arbitrary alphabet have been determined. We discuss this result and give a complete list of all binary comma-free codes with word length 5.

4 Topologies on Finite Sets

R. Bumby, R. Fisher, H. Levinson* and R. Silverman
Rutgers University and New Jersey Institute of Technology

The structure and uniqueness of maximal non-discrete topologies on arbitrary finite sets are investigated. All such topologies are constructed together with a chain of topologies, each properly maximal in the preceding topology of the chain. It is proved that the length of a maximum chain in the lattice of all topologies on a set containing n elements is $\frac{n^2+n}{2}$. An explicit construction is given which realizes such a chain.

5 A Constructive Characterization of Trees with Two Disjoint Minimum Dominating Sets

D. W. Bange and A. E. Barkauskas,* Univ. of Wisconsin
P. J. Slater, Sandia Laboratories

A set of vertices A in a graph G is a dominating set if every vertex not in A is adjacent to at least one vertex in A, and A is a minimum dominating set if there is no dominating set of smaller cardinality. The class of trees with two disjoint minimum dominating sets will be called D2. A constructive characterization of D2 trees is given.

6 THE STRUCTURE OF SIMPLE GRAPHS

R.H. Johnson, Sonoma State College

A graph is called simple if it is determined up to isomorphism by its degree sequence. We summarize attempts to characterize such graphs. Simple trees (called Giap graphs) are shown to be exactly those trees with diameter less than or equal to three. Simple disconnected and simple separable graphs are then determined. Two key geometric results are then derived: the radius and diameter of a simple connected graph is less than or equal to two and three respectively. Simple block graphs are characterized in part and directions of further research are indicated.

7 Generalized Graphic Sequences

R. B. Eggleton (University of Newcastle)* and D. A. Holton (University of Melbourne)

The notion of degree sequence of a graph is well-known. There are two rather different characterizations of integer sequences which arise as degree sequences of finite graphs, known as the Havel-Hakimi Theorem and the Erdős-Gallai Theorem. There are also extensions to multigraphs and directed graphs. In this paper we further extend these concepts, as follows. We introduce the notion of generalized graphic sequences, and the corresponding generalized graphs which realize these sequences. Then we focus on a class of generalized graphs with three possible kinds of incidence (which are represented by edges, loops and spines) and deal with some analogues of the Havel-Hakimi and Erdős-Gallai characterizations. Further, we introduce the notion of the graph of realizations of a generalized graphic sequence. The vertices of this graph are the various generalized graphs which realize the given sequence. Some structural results concerning this graph are obtained.

8 DEGREE SEQUENCES OF THRESHOLD GRAPHS

P.L. Hammer,* T. Ibaraki and B. Simeone, University of Waterloo

A threshold graph is a finite, loopless graph, without multiple edges, having the property that there exist real numbers w_i associated to the vertices v_i, and a threshold number t, such that for any i and j, v_i and v_j are adjacent iff $w_i + w_j > t$. It has been remarked in [1] that threshold graphs are completely determined by their degree sequences. A graph whose vertex set can be partitioned into two subsets, one inducing a complete subgraph, and the other an edgeless one, is called a split graph. Every threshold graph is split.

Consider a sequence $d_1 \geq \ldots \geq d_n$ of nonnegative integers, and let $m = \max\{i | d_i \geq i - 1\}$. It is easy to see that the first m Erdos - Gallai conditions for the sequence to be graphic imply the last n-m ones. It has been proved in [2] that a graphic sequence corresponds to a split graph iff the m-th Erdos - Gallai inequality holds with equality.

The main result of our paper characterizes threshold graphs as those graphs for which the first m Erdos - Gallai inequalities hold all with equality.

Further, given a graphic degree sequence, the "closest" degree sequence corresponding to a threshold graph is defined, and the set of all these sequences is shown to form a lattice with unique minimal and with unique maximal elements.

[1] V. Chvatal, P.L. Hammer, Aggregation of Inequalities in Integer Programming, Annals of Discrete Mathematics, 1, 1977, 145-162.

[2] P.L. Hammer, B. Simeone, The Splittance of a Graph, Department of Combinatorics and Optimization, University of Waterloo, Research Report CORR 77-39.

9 BOOLEAN MINIMIZATION USING FREE BOOLEAN ALGEBRAS

Gerald Berman*, University of Waterloo
A.W. Colijn, University of Calgary

Free Boolean algebras are used to construct the set of minimal representations of a Boolean function f from the set of its prime implicants. This is done with the aid of a matrix of Boolean functions associated with f. The minimum representations of f relative to any criterion can then be obtained immediately from the list of minimal representatives. The method is extended to handle incompletely specified functions.

A LINEAR ALGORITHM FOR COPYING BINARY TREES USING BOUNDED WORKSPACE

10

Chung C. Wang
Department of Computer Science
University of Kentucky
Lexington, Kentucky 40506

The problem of copying a binary tree is assumed to have the constraint that only a constant amount of workspace is available, besides that required to hold the original tree and the copy. Although the previous best known algorithms are linear, they have the restriction that a contiguous block of storage must be available for the copy. This paper presents a linear algorithm which does not have this restriction.

11 An Enumerative Approach to a Problem in Probability

J.S. Devitt University of Waterloo

Consider a computer which uses exactly one of m distinct resources for each job which is run. Assume that the resource used for a given job is independent of all other jobs and that the jobs are run in the order submitted. The resources need not all have the same probability of use, but assume that the probability of a given resource being used for a job is fixed for all jobs.

A model based on sequences over the finite alpabet {1,...,m} is introduced to describe the various job queues, classifying each job according to the resource it uses. Known results concerning the enumeration of such sequences according to the number of rises, levels and falls are then used to obtain the various moments of "The number of changes of resources required in a sequence of n jobs". The techniques involved consist primarily of algebraic operations on certain generating functions and demonstrate a useful approach to problems for which such enumerative information is available.

12 Programming the Penrose pattern.

Eric Regener, Concordia University, Montreal, Quebec

The Penrose pattern, discovered by Roger Penrose, tiles the plane with congruent copies of two four-sided tiles but has no translation-symmetry. Our program systematically fills in the area 'forced' by any given vertex of the pattern and then 'inflates' it, level by level, with heavy use of recursion. The vertices are regarded as polynomials with integer coefficients over the field generated by a complex tenth root of unity. This method defines each vertex uniquely as a quadruple of integers and thus avoids the computational inaccuracy associated with considering the vertices as coordinate pairs in the real plane.

An auxiliary program independent of the Penrose properties plots the portion of the pattern falling within a rectangular 'window' of any size and orientation. Using this program we can piece together plots larger than the standard paper size. Another program takes a general graph ready for plotting and produces a plot with the theoretical minimum of trails(i.e., half the number of vertices of odd degree if there are no isolated points), thus speeding up the activity of the plotter by reducing the number of pen-lifts necessary to draw the graph.

All programs except the last-named have been operational for more than a year, and one of our plots was included in Martin Gardner's article on the Penrose pattern in the January 1977 issue of Scientific American.

13 DISTANCE GEOMETRY AND THE CAPACITY OF GRAPHS

Moshe Rosenfeld, Simon Fraser University & Ben Gurion Univeristy

In 1956 C.E. Shannon introduced the notion of the Capacity of a Graph in his investigation of noisy channels in Information Theory. The problem of determining the capacity of various families of graphs or obtaining bounds for the capacity have attracted many authors, who employed diversified techniques such as linear programming, packing problems, Abelian groups in their attempts. The simplest family of graphs whose capacities have not been determined yet are the odd cycles. We present here a survey of various problems and techniques of realizing graphs as distance sets in Euclidean space E^n, or on the unit sphere S^n and their applications. One of the most striking applications was recently discovered by L. Lovász who shows, using these techniques, that the capacity of the pentagon is $\sqrt{5}$. We'll discuss Lovász' work, obtain new bounds for the capacities of odd cycles, other families of graphs and related problems.

14 Composition of T-Designs

Lowell A. Carmony, Southern Illinois University

We consider the problem of composing t-designs to obtain larger t-designs. It is shown that if D_1 and D_2 are t-designs on the same point set with consecutive block sizes ($k_2 = k_1 + 1$) and such that the sum of the replication numbers is the number of blocks of the first design ($b_1 = r_1 + r_2$), then the designs can be composed to yield a new t-design D_3 on one additional point with $b_3 = b_1 + b_2$, $r_3 = b_1$, $k_3 = k_2$, $\lambda_3 = \lambda_1 + \lambda_2$. Also a useful composition theorem for block designs is extended to t-designs and various families of 3- , 4- , and 5- designs are derived.

15 Some Remarks on 2-Designs $S_2(2,3,v)$.

F.E. Bennett* and N.S. Mendelsohn, University of Manitoba.

Let λ, t, k, v be positive integers such that $t \le k \le v$. A t-design $S_\lambda(t,k,v)$ is a pair (X,B), where X is a set of v elements and B is a collection of k-subsets of X, called blocks, such that every t-subset of X is contained in exactly λ blocks. An $S_\lambda(t,k,v)$ is called simple if it has no repeated blocks. A cyclic triple system of order v (CTS(v)) is an $S_2(2,3,v)(X,B)$ in which the blocks of B (triples) have been arranged cyclically, and such that every ordered pair of elements of X appears in exactly one cyclic triple. A CTS(v) is an example of a block design on the directed graph with vertices a,b,c and arcs (ab), (bc), (ca). A CTS(v) exists for every $v \equiv 0$ or $1 \pmod 3$, where $v \ge 3$ and $v \ne 6$. A 2-design $S_2(2,3,v)$ will be called non-orientable if its triples cannot be arranged cyclically so as to form a CTS(v). Recursive constructions are used to show that a non-orientable simple $S_2(2,3,v)$ exists for every $v \equiv 0$ or $1 \pmod 3$, where $v \ge 6$ and $v \ne 7$.

16 A few more squares, II

S. M. P. Wang and Richard M. Wilson*
The Ohio State University

It is proved that there exist three mutually orthogonal Latin squares of order n for every integer $n > 14$. Difference methods and the computer were used to find several examples which recursive constructions had failed to supply. These examples (e.g., $n = 18, 22, 26$) consist of a self-orthogonal square together with a symmetric orthogonal mate.

17 PERPENDICULAR ARRAYS

Paul J. Schellenberg, John van Rees, University of Waterloo
Scott A. Vanstone*, St. Jeromes' College, University of Waterloo

A perpendicular array $PA[k,n]$ is a k by $\binom{n}{2}$ array defined on a set of n symbols such that any two rows contain all unordered pairs exactly once. We give direct and recursive constructions for such arrays. From these arrays we also construct an $OA[6,15]$.

18 THE CONSTRUCTION OF BIBDS USING NONABELIAN GROUPS

W. H. Mills
Institute for Defense Analyses

Balanced incomplete block designs with $\lambda = 1$ are particularly difficult to construct in cases where neither v nor $v - 1$ is divisible by k. We show how such a BIBD can sometimes be constructed using difference family methods based on a nonabelian group of order v or $v - 1$. The known BIBD with $v = 111$, $k = 6$, $\lambda = 1$ was constructed this way. Using one of the nonabelian groups of order 135 we obtain a BIBD with $v = 136$, $b = 612$, $r = 27$, $k = 6$, and $\lambda = 1$.

19 BALANCED RECTANGLE DESIGNS

F. K. Hwang, Bell Labs

A balanced rectangle design (BRD) is an arrangement of v symbols in an m×n rectangle such that either the rows and columns together form a balanced block design, or they form two separate balanced block designs. A BRD is said to be complete (semicomplete) if every cell (nonempty cell) contains a fixed number of symbols. Examples of complete BRD are Latin squares, Youden squares, Agrawal Youden squares, generalized Youden squares and lattice squares. Examples of semicomplete BRD are Room squares, t-tuple Room squares and Howell designs. An example of BRD which is not semicomplete is the generalized Room squares studied by Schellenberg and Taylor.

The present paper reviews the above designs as well as some new BRD designs. The concept of BRD is also extended to higher dimensions.

20 MULTIPLYING CERTAIN HOWELL DESIGNS BY 3

B. A. Anderson* and K. B. Gross
Arizona State University

In order to settle the existence question for types of Howell Designs of odd side, it would be helpful to be able to prove the following Conjecture. If $p \ge 7$ is a prime and all $H^*(p,2n)$ exist except possibly $H^*(p,2p-2)$, then all $H^*(3p,2n)$ exist except possibly $H^*(3p,6p-2)$. We obtain some results in this direction for primes $p = 6t + 1$, t odd. For such primes, we show how to construct a strong (partial) starter on $Z_{3p} \setminus Z_3$. Often this can be done in such a way that the starter can be used to build designs of all types $H^*(3p,2n)$, $3p + 3 \le 2n \le 6p - 2t$.

Generalized Quadrangles of Order Sixteen

21

Stanley E. Payne*, Miami University, Oxford, Ohio and
Ray Kilgrove, California State University at Los Angeles, and St. Lawrence University, Canton, New York

Any generalized quadrangle S of order s that has an incident point-line pair $((\infty), [\infty])$ with desarguesian projective planes based at both (∞) and $[\infty]$ must be coordinatized by a pair (α, β) of permutations of $GF(2^e)$, $s = 2^e$. S is a translation generalized quadrangle with base point (∞) if and only if α is an additive map, with a dual result holding for β. If α and β are both additive, then both are automorphisms for which $\alpha\beta^{-1}$ is an automorphism of maximal order e. The interest here is in the case in which α is additive but not an automorphism, so β is not additive. It has been shown that no such example exists with $e < 4$, and it is the purpose of this note to indicate that an exhaustive computer search has failed to produce an example in the case $s = 16$.

Few Hamiltonian Results.

22

Joseph Zaks, Univ. of Haifa, Israel, and Ohio State Univ., Columbus, Ohio.

I. Following B.Grünbaum and H.Walther (JCT'73) and G.Ewald (J.Geometry'76), let $\rho(\mathcal{G})$ denote the shortness coefficient of a family $\mathcal{G}$ of graphs, and let $G(q;r)$ denote the family of all the polytopal graphs having maximum vertex (face) degree $\leq r$ ($\leq q$). Refuting conjectures of Grünbaum, Walther and Ewald, we prove: Theorem 1: $G(7;3)$ has a non-Hamiltonian member having 54 vertices. Theorem 2: $\rho(G(7;3)) \leq 57/58$. Theorem 3: $\rho\{G \mid G \in G(8;3), p_k(G) \neq 0 \rightarrow k=5;8\} \leq 44/45$.
II. Theorem 4: $\forall n \geq 3$ and $\forall k$, $\exists$ an n-valent n-connected (or n-valent 3-connected and planar, $n \leq 5$) graph which is the union of all of its Hamiltonian circuits, yet is not the union of any k of them. Let $a(n) = \limsup a(G)/V(G)$, taken over all the n-connected n-valent graphs G, $a(G)$ = #of those edges belonging to every Hamiltonian circuit of G; define $b(n)$ similarily for those edges on no Hamiltonian circuit; let the planar case be denoted by $*$. Theorem 5: $a(3)=a^*(3)=1$, $b(3)=b^*(3)=\frac{1}{2}$, $a^*(4)=a^*(5)=b^*(4)=b^*(5)=0$, and $\forall n \geq 3$, $a(n) \geq 1/(4n-2)$ and $b(n) \geq (n-1/8)$.
Corollary: $\forall n \geq 3$, $\exists$ an n-valent n-connected graph admitting no pairs of edge-disjoint Hamiltonian circuits.
III. C.Kiram Babu, a student in my class at O.S.U., refutes A.Gyárfas and J.Lehel's conjecture (see C.L.Liu and S.Zaks, 8th S.E. Conf.) by showing that $K_{n,n+1}$ is the union of $P_2, P_4, \ldots, P_{2n}$, (where P_m is the path of length m), for all even n, $n \geq 4$.

Hamiltonian Cycles in Cayley Color Graphs of Semi-Direct Products

23

J.B. Klerlein* & A.C. Starling
Western Carolina University

In this paper we consider the Cayley color graph, using the standard presentation, of the semi-direct product of cyclic groups. We investigate the problem of determining when these graphs are hamiltonian. Certain necessary conditions and certain sufficient conditions for a hamiltonian cycle to exist are obtained. In the process we generalize results obtained by Trotter and Erdos for the cartesian product of directed cycles to be hamiltonian.

On Maximal Triangulations, Optimal Drawings and Hamilton Cycles

24

Selim G. Akl, McGill University

A triangulation of n points in the plane is defined as the plane graph obtained by joining the points by non-intersecting straight-line segments so that every region interior to their convex hull is a triangle. This structure finds applications in the finite element method and in the numerical interpolation of functions of two variables. Very little is known, however, about the geometrical properties of triangulations. This note studies a special type of triangulations. In particular, we give a method of placing n points in the plane and joining them by straight-line segments that yields a triangulation with the maximum possible number of edges. Triangulations of this type are shown to be hamiltonian and a conjecture relating them to crossing-number-optimal rectilinear drawings of complete graphs is made.

The spanning tree number of polygon triangulations

25

Kenneth A. Berman, University of Waterloo

An algorithm is given for obtaining the number of spanning trees of a polygon triangulation where there are no interior vertices. A rooted polygon triangulation T with no interior vertices can be decomposed into an ordered pair of rooted triangulations. By successive decompositions T can be reduced to a bracketed string of degenerate triangulations consisting of single edges. The number of spanning trees is then obtained directly from this encoding of T.

Combinatorial Properties of Helly-type Sets

26

John D. Baildon, Pennsylvania State University, Worthington Scranton Campus, and Ruth Silverman*, New Jersey Institute of Technology.

Suppose an ordered pair of sets (S,K) in a linear topological space is of Helly type $(n+1,n)$, i.e., for every $n+1$ distinct points in S there is a point in K which sees at least n of them via S. If S is closed, K compact, $n \geq 3$, and $n \geq 2r-1$, we show the nontrivial visibility sets in K are r-wise nondisjoint.

If (K,K) has Helly type $(n+1,n)$ for all integers $n>0$, we say K has Helly type ∞. We show that a compact set of Helly type ∞ differs from a starshaped set at most by an isolated point.

Planar examples are provided of sets of various Helly types.

Polarities of Elliptic Semi-Planes

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R. D. Baker, University of Delaware

An elliptic semi-plane is a finite symmetric partial plane which has a parallel relation on lines and on points. A subclass of elliptic semi-planes, called Baer semi-planes, arise as a projective plane minus a Baer subset. Given a projective plane of order n that has a polarity π with α absolute points the following results hold: 1) (Baer) $\alpha \geq n+1$; 2) (Baer) if $\alpha = n+1$ then π is called orthogonal and the absolute points are either collinear (if n is even) or form an oval (if n is odd); 3) (Seib) $\alpha \leq n^{3/2} + 1$; 4) (Seib) if $\alpha = n^{3/2} + 1$ then π is called unitary and the absolute points and non-absolute lines form a unital. This paper investigates generalizations of these results to polarities of elliptic semi-planes.

28 The Path Center of a Tree

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The path number of a point in a tree is the number of distinct paths which pass through that point.

The path center of a tree consists of all points having maximum path number.

The path number of a tree is the path number of the points in the path center.

Three formulas are given for calculating path numbers.

There exist trees with single point path centers and any path number except 2.

There exist trees with two point path centers, with the two points adjacent or arbitrarily far apart.

All points in the path center of a tree are collinear.

Structure of the k-Centrums in a Tree

29 Peter J. Slater - Sandia Laboratories, 5121

The concepts of the center and the centroid of a graph G can be generalized as follows. For $1 \le k \le |V(G)|$ and $u \in V(G)$ let $r_k(u) = \max\{\Sigma_{s \in S} d(u,s) : S \subseteq V(G), |S| = k\}$. The k-centrum of G, denoted $C(G;k)$, is defined to be the subset of vertices u in G for which $r_k(u)$ is a minimum. Thus $C(G;1)$ is the center of G, and $C(G;|V(G)|)$ is the centroid. A discussion of the structure of these sets for trees will conclude with a proof of the following.

Theorem. For any tree T there exists a tree T′ such that $\bigcup_{k=1}^{n} C(T';k) \approx T$ where $n = |V(T')|$.

Characterization of Maximal Planar Graphs 30

by Ilker Baybars, Carnegie-Mellon University, Pittsburgh, PA

We show that (1) the connectivity of a maximal planar graph (MPG) is 3 iff it contains a C_3 which is not a face; (2) every vertex v_i of a MPG is the core of exactly s_i+1 distinct wheels where s_i is the number of sets S_j such that $v_i \in S_j$ and S_j is the jth distinct separating set of triplet of adjacent vertices; and (3) a (p, 3p-6) graph $G = (V,E)$ is planar and 4-connected iff every $v_i \in V$ is the core of exactly one distinct wheel $W_{i,n} = (V_{i,n}, E_{i,n})$ in G such that $V_{i,n} = \{v_i\} \cup \{v_j \in V | (v_i, v_j) \in E\}$.

31 THE INDUCIBILITY OF GRAPHS AND HYPERGRAPHS

Martin Charles Golumbic, Courant Institute

We shall be concerned with the most number of ways a given hypergraph H can appear as an induced sub-hypergraph of a hypergraph of fixed larger size p and how this number behaves as p goes to infinity. This notion, called the (vertex) inducibility of H, was introduced for graphs in Pippenger and Golumbic [1975], and many of the theorems presented there carry over to hypergraphs.

32 On the Number of Graphical Partitions

Paul R. Stein, Los Alamos Scientific Laboratory

A graphical partition is one whose parts constitute the degree sequence of at least one simple linear graph (no loops or multiple lines). For graphs with p nonisolated points and q lines, the number of possible graphical partitions is denoted by $g_{p,q}$. In the present paper, enumeration formulae are derived for the simplest cases $q - p \le 6$, and a general method for obtaining further results is sketched. In addition, a table of $g_{p,q}$ is included which covers the cases $q \le 27$ for all possible values of p.

33 CYCLES IN BIPARTITE GRAPHS

Bill Jackson, University of Waterloo

We prove the following result. Let G be a bipartite graph with bipartition $V(G) = A \cup B$. Let each vertex of A have degree at least k and suppose that $|B| \le 2k - 2$. Then G contains a cycle of length 2ℓ, for all ℓ, $2 \le \ell \le \min(|A|, k)$. This solves a problem communicated to us by P. Erdős.

34 THE RECONSTRUCTION OF LINE-CRITICAL BLOCKS

Herbert Fleischner, Austrian Academy of Sciences & MIT

A line-critical block (l.c.b.) is a two-connected graph such that G-e is not two-connected for any e in E(G). In a DT-graph, every line is incident to a point of degree two. D(G) contains exactly those e of E(G) which are not incident to a point of degree two. Dirac and Plummer proved that for a l.c.b., G-e is always a block-chain and that D(G) induces an acyclic subgraph of G. There exists some f in D(G) (if G is not a DT-graph) such that G-f has an endblock which is a DT-subgraph of G. Denote the set of these f's by $D_1(G)$. In terms of point-deletion, a two-connected G is a l.c.b. iff G-v has exactly deg v endblocks whenever degv>2. These and some other facts are the basis for reconstructing l,c.b.'s; it takes 12 steps (some require blood & sweat, but no tears). In some steps one determines $|D(G)|$, $|D_1(G)|$, and other parameters. In the main part of the proof one shows that if G contains certain subgraphs, then G is reconstructible. By induction on $|D(G)|$ one finally shows that such subgraphs always exist in l.c.b.'s.

35 SCORE SETS FOR TOURNAMENTS

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The score set of a tournament (an irreflexive, asymmetric, complete digraph) is the set of scores (outdegrees) of the vertices of the tournament. Is every nonempty set of nonnegative integers the score set for some tournament? In case the given set has cardinality one, two, or three, or in case the given set forms an arithmetic or geometric progression, we prove that the set is a score set. Reference will be given to related work on graphs and digraphs.

36 Weighted Permutation Problems and Special Functions

Mourad E.H. Ismail, McMaster University

A number of illustrations are given of how to transform some combinatorial problems to equivalent problems involving special functions and vice versa. The main tool used is MacMahon's Master Theorem. For example, consider $n = n_1 + n_2 + n_3$ different objects given initially in three boxes, n_1 in the first, n_2 in the second and n_3 in the third. Rearrange these objects so that the number of objects in each box does not change. A result of Askey and Gasper is proved by showing that there are more rearrangements in which an even number of objects move to another box than there are rearrangements where an odd number of objects move to another box. Another interpretation of the same result can be formulated in terms of distances in Hamming schemes. Related results of Koornwinder will also be phrased in combinatorial language and will be proved combinatorially in some special cases.

37 On Uniquely Four-Colorable Planar Graphs

Roy B. Levow
Florida Atlantic University

We say that a graph is uniquely 4-colorable if all possible 4-colorings are equivalent under permutation of the colors. It is clear that uniquely 4-colorable planar graphs may be constructed recursively starting with K_3 and then taking a graph already constructed and adding a new vertex connected by an edge to each of the three vertices of one of the triangular faces of the graph. We say that a uniquely 4-colorable planar graph is "non-trivial" if it cannot be constructed in this way. We examine the question of whether such graphs exist.

A class of minimal graphs analogous to the irreducible graphs of the 4-color problem is described and a number of structure theorems are obtained for this special class.

38 ON A COLOURING PROBLEM

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Let $F(k,n)$ be the minimum number of k-subsets of an n-set which must be coloured in order that every $(k+1)$-subset contains a coloured k-subset. It is obvious that $F(1,n) = n-1$, and it is easily shown that $F(2,n) = \lceil (n-1)^2/4 \rceil$. A general construction is given which provides an upper bound for $F(3,n)$ and it is shown that the bound is exact for $n \leq 9$. Some results on the uniqueness of minimal colourings are also described.

This colouring problem is equivalent to a covering problem. If complements are taken of the blocks in a (k,n)-colouring, one obtains a family of $(n-k)$-subsets with the property that every $(n-k-1)$-subset is covered by (i.e. is contained in) at least one of them. Thus $F(k,n) = N(n-k,n)$, where $N(t,n)$ is the minimum number of t-subsets needed to cover all $(t-1)$-subsets of an n-set. Most of the published results on $N(t,n)$ are for small values of t.

39 On minimal digraphs with given girth

L. Caccetta
R. Haggkvist, University of Waterloo

Define $f(r,g)$ to be the minimum number of vertices in an r-regular digraph of girth g. The function $h(r,g)$ is defined to be the minimum number of vertices in a digraph of girth g in which every vertex has out degree at least r. The function $f(r,g)$ has been studied in the literature and it has been conjectured that $f(r,g) = r(g-1) + 1$. We conjecture that $f(r,g) = h(r,g)$, and prove this for the case $r = 2$. We also present some bounds for both functions.

Extremal graphs of diameter 3

40 L. Caccetta, University of Waterloo

This paper is concerned with graphs of order n and diameter at most 3 having the property that by deleting any s or fewer vertices (edges) the resulting subgraphs (partial graphs) have diameter at most λ. A graph satisfying the above constraints and having minimum number of edges is said to be extremal. A characterization of extremal graphs is presented for the case $s = 1$. For $s > 1$ a conjecture is stated.

41 ON MAXIMAL CHAIN OF CYCLIC-4-CONNECTED SUBGRAPHS

Neil Robertson, The Ohio State University

A graph G is cyclic-4-connected when its cyclamatic number is at least 4 and its edge set cannot be partitioned into two parts, each containing a circuit, without G having at least 4 vertices incident with edges of both parts. Let $L(4,2)$ be the set of cyclic-4-connected non-separable finite graphs. Graph $G \in L(4,2)$ is derived from a subgraph $H \in L(4,2)$ when a chain $H = H_0 \subset H_1 \subset \cdots \subset H_k = G$ exists such that for $1 \leq i \leq k$ we obtain $H_i \in L(4,2)$ by "properly" adjoining an arc or Y-graph to each H_{i-1}. A theorem is obtained whereby any $G \in L(4,2)$ either

1) belongs to 2 simple families of graphs,
2) is planar and is derived from a subdivided cube,
3) is a subgraph of $K_{4,5}$ and is derived from a cube,

or 4) is derived from a subdivided $K_{3,3}$.

This theorem has applications to the problem of embedding a graph in the topological projective plane.

42 Enumeration of higher dimensional paths within two boundaries.

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and
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The enumeration of lattice paths lying between two boundaries in two dimensional space has been done and the explicit expression is a determinant. By considering a natural extension of the boundaries in higher dimensional space, a generalized recurrence relation is established, the solution of which gives the number of paths in higher dimension not crossing the boundaries, in a determinant form.

43 RAMSEY-FINITE GRAPHS

S. A. Burr - A. T. & T. Long Lines, P. Erdos - Hungarian Academy of Sciences, R. J. Faudree and R. H. Schelp - Memphis State Univ.

For graphs (finite, undirected, without loops or multiple edges) F, G and H, write $F \rightarrow (G,H)$ to mean that if the edges of F are colored with two colors, say red and blue, then either the red subgraph of F contains a copy of G or the blue subgraph contains a copy of H. Let R(G,H) denote the class of all graphs F such that $F \rightarrow (G,H)$. A graph F in R(G,H) is minimal if no proper subgraph of F is in R(G,H). Denote the class of such minimal graphs by R'(G,H). The pair (G,H) is Ramsey-finite if R'(G,H) is a finite set. The following is proved.

THEOREM: For $G = nK_2$ $(n \geq 1)$ and H an arbitrary graph, the pair (G,H) is Ramsey-finite.

Also evidence will be given to support the following.

CONJECTURE: If G is a graph such that (G,H) is Ramsey-finite for all H, then $G = nK_2$.

44 Ramsey Numbers for Books

C.C. Rousseau and J. Sheehan,* Memphis State University

The graph $B_n = K_2 + \overline{K}_n$ is called the n-book. Solved Ramsey number problems involving books have used a striking variety of critical colorings. As examples, we cite our results concerning the diagonal Ramsey number $r(B_n)$ and the book-tree Ramsey number $r(B_m,T)$ for the special cases $T = P_n$ (path of length n) and $T = K_{1,n}$ (star of degree n). Theorem 1. If 4n+1 is a prime power, then $r(B_n) = 4n+2$. If 4n+1 cannot be expressed as the sum of two integer squares, then $r(B_n) \leq 4n+1$. In the first example of the latter, $r(B_5) = 21$. Theorem 2. Set $k = [(m-1)/n]$. Then $r(B_m,P_n) = \max\{(k+2)n + 1, m + 2[(m-1)/(k+1)]\}$ and $r(B_m,K_{1,n}) \leq \max\{(k+2)n + 1, [(k+3)((k+2)(m-1) + 2(k+1)n)/(k+1)(k+4)] + 1\}$. Furthermore, the result for $r(B_m,K_{1,n})$ holds with equality in case (i) $kn \leq m-1 \leq \{k + 2/(k+2)(k+3)\}n$ or (ii) $\{k + 2/(k+2)(k+3)\}n < m-1 < (k+1)n$ and there exists integers p and q such that $(k+1)(p-2)q = m-1$ and $(p+k+2)q = n$. Theorem 3. If q is a prime power then: (i) if q is even $q^2+q+2 \leq \gamma(C_4,B_{q^2-q+1}) \leq q^2+q+3$; (ii) if q is odd then $q^2+q+2 \leq \gamma(C_4,B_{q^2-q+1}) \leq q^2+q+4$.

45 Distance Between Partitions on a Finite Set

E. L. Perry, Baylor University

In this paper, a metric is defined on the collection of all partitions of a finite set S. It is then shown that under a very reasonable set of axioms, this is the only metric possible on this collection.

46 REDUCTION OF PREDICATE/FUNCTION GRAPHS

Thomas F. Reid and William R. Edwards
University of Southwestern Louisiana

Data flow graphs (DF) represent a viable approach to expressing algorithms which exhibit concurrency. In contrast to control flow graphs (flow charts), DF nodes represent operators on data and DF arcs represent paths for passage of data between nodes.

A species of DF, the predicate/function graph (PF), is defined in which the semantics of a node are expressed in a notation based upon a predicate/function pair. The predicate determines whether any action takes place and the function determines what action, if any, takes place.

A class of well-formed PF graphs will be described. The coalescing rules necessary to reduce a PF graph to a simpler, equivalent graph consisting of a single node for each graph output will be presented.

47 ON A GRAPH LABELLING PROBLEM

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In connection with the scheduling of independent tasks on a set of identical processors under certain resource constraints, the following graph labelling problem is of interest: given an undirected graph, find a labelling of its vertices with k-dimensional vectors over the interval [0,1], such that for any two vertices a and b we have: a and b are connected by an edge iff the sum of their labels does not exceed the k-dimensinal vector (1,1, ... ,1) . The class of graphs which can be 1-labelled is completely characterized, and properties of the classes of graphs, which can be k-labelled, are discussed.

48 CONTEXT-FREE GRAMMARS WITH GRAPH CONTROL

Curtis R. Cook, Oregon State University

Context-free grammars with graph control provide a general framework for the various types of context-free grammars with regulated rewriting. The vertices or edges of the directed graph are labeled with the productions of the grammar. The only strings in the language generated by the grammar are those whose derivations correspond to labeled paths in the graph. Inclusion relations among the various classes of context-free grammars with regulated rewriting such as programmed grammars, matrix grammars, periodically time-variant grammars, state grammars, unordered scattered context grammars, grammars with regular control, etc., are easily obtained using context-free grammars with graph control. Adding negative context to graph control we are able to generate parallel context-free languages and languages generated by TOL-systems.

49 THE EDGE DISJOINT PLACEMENT OF TWO COPIES OF A GRAPH

R. J. Faudree, C. C. Rousseau, R. H. Schelp* - Memphis State University and S. Schuster - Carleton College

Several conditions are known to be sufficient to place two graphs on n vertices edge disjointly in K_n. Here we consider the special problem of embedding a graph in its complement, i.e., placing two copies of a graph G edge disjointly in K_n where $n = |G|$. We prove the following theorems.

Theorem 1. Except for five infinite classes of graphs each (p,p) graph, $p \geq 10$, is embeddable in its complement.

Theorem 2. Let G be a graph with p vertices and at most $\frac{6p}{5} - 2$ edges. If G has no cycles of length three or four and is not a star, then G is embeddable in its complement.

Conjecture. Theorem 2 is true when the restriction on the number of edges is deleted.

50 ON THE STRONG GRAPH EMBEDDING CONJECTURE

Charles H. C. Little, Royal Melbourne Institute of Technology, and Richard D. Ringeisen*, Indiana University-Purdue University at Fort Wayne

At the 8th Southeastern Conference on Combinatorics, Graph Theory, and Computing, G. Haggard defined an operation called barring an edge of an embedded graph. He also stated the strong graph embedding conjecture: Every block has a disc embedding such that the boundary of every face is a cycle (a connected regular graph of degree two). In this paper, we further use the barring operation to help establish the following result. Theorem. If T is an orientable embedding of a connected bridgeless graph G with a monofacial edge (an edge which occurs in the boundary of exactly one face), then there exists an embedding of G with fewer monofacial edges than T.

A TOPOLOGICAL SPACE FOR WHICH GRAPH EMBEDDABILITY IS UNDECIDABLE

51 Stephane Foldes and Richard Steinberg*
University of Waterloo

We show the existence of a topological space T such that there is no algorithm to decide whether a finite graph is embeddable into T. In fact, T will be a path-connected subspace of the real plane.

52 Graph recognition and embeddability algorithms

Claude Benzaken and Stéphane Foldes*
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There exist very efficient algorithms to decide whether a graph is planar. How difficult it can be to decide whether a graph is embeddable into some specified subset S of the plane? It is shown that the degree of polynomial complexity of such embeddability problems can be prescribed in the following sense. To every class C of graphs one can associate a path-connected subset $p(C)$ of the plane, defined as a topological realization of an appropriate countable graph, in such a way that the problem of recognizing the class C is polynomially Turing-equivalent to the problem of testing embeddability into $p(C)$.

53 Sample Graphs for Isomorphism Testing

Rudolf Mathon, University of Toronto

An adjacency refinement technique is used as a classification scheme for simple undirected graphs. The concept of subgraph-regularity is derived and its relation is given to the complexity of certain graph-isomorphism algorithms. Sample graphs representing various classes of subgraph-regular graphs are given together with their automorphism groups and automorphism partitionings. Since these graphs may cause difficulties for graph isomorphism algorithms it is recommended to use them as sample graphs for isomorphism testing.

Lower Bounds for the Graph Isomorphism Problem

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The graph isomorphism decision problem (short: GIDP) is the following: Given any two graphs X^1 and X^2 decide whether they are isomorphic. Several algorithms (by Corneil & Gotlieb, Knödel, Levi, Sirovich, Unger, etc.) exist for this problem. Tinhofer has shown that these algorithms are all based on the same concept: First the orbits of the vertices of the input graphs under the automorphism group actions are determined; these orbits are invariant under graph isomorphism; thus in a second step only those bijective mappings between the sets of vertices of the input graphs have to be tested for isomorphism which preserve membership in corresponding orbits. In this paper it is shown that all algorithms based on this concept must take time exponential in the number of vertices of the input graphs. Thus completetly new ideas are required in order to solve the GIDP for general graphs in polynomial time and not only a refinement of existing algorithms.

55 Isomorphic Triple Systems with k Triples in Common

Jane W. DiPaola
Florida Atlantic University

For a fixed i let B_i be a collection of 3-subsets of a set S of cardinality v forming a Steiner triple system $STS(v)$. Two such systems (S,B_1) and (S,B_2) are said to intersect in k triples if $|B_1 \cap B_2| = k$. Let $J^*[v]$ denote the set of all integers k such that there exists a pair of isomorphic $STS(v)$ intersecting in k triples. $J^*[v]$ is unknown except for $v = 3,7,9$. In this paper it is shown that $0,1,2,3,4,5,6,8,11,12,13,16,17,18,19,20,22,26 \in J^*[13]$. It is known that $21,23,24,25 \notin J^*[13]$. Undecided are $7,9,14,15$. Previous work shows $0 \in J^*[v]$ whenever v is a power of a prime.

56 On Conjugate Invariant Subgroups of n-Quasigroups

By D. G. Hoffman Auburn University

We construct some n-quasigroups based on an arbitrary ring, and calculate their conjugate invariant subgroups. Direct products of some of these quasigroups prove our main result, that any semiregular group of permutations of n + 1 symbols is the conjugate invariant subgroup of some finite n-quasigroup. In particular, a simple construction shows that any finite group G is isomorphic to the conjugate invariant subgroup of a finite (|G| - 1)-quasigroup.

A Note on Partial S(s-1,s+1,v) Subsystems of a S(s,s+1,v)

57 K. T. Phelps, Georgia Institute of Technology

A Steiner systems S(t,k,v) is a collection q of k-element subsets of a v-set Q such that every t-element subset of Q occurs in exactly one of the k-element subsets in q. A partial S(t,k,v) has all of the above properties, except one and that one is: every t-element subset of Q occurs in at most one of the k-sets in q. Given (Q,q) an S(t,k,v) where Q and q are as defined above, we ask what is the largest sub-collection $q' \subseteq q$ such that (Q,q') is a partial S(t',k,v) where $1 \leq t' < t$. In this note we give a lower bound for |q'| when t = s, k = s + 1 and t' = s - 1. This gives us a decent lower bound when s = 3. However as s increases the bound becomes weaker.

58 SOME REMARKS ON RESOLVABLE QUADRUPLE SYSTEMS

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Auburn University

A Steiner quadruple system (or more simply, a quadruple system) is a pair (Q,q) where Q is a finite set and q is a collection of 4-element subsets of Q (called blocks) such that every 3-element subset of Q is a subset of exactly one block of q. The number |Q| is called the order of the quadruple system (Q,q) and in 1960 Hanani proved that a quadruple system of order n exists if and only if $n \equiv 2$ or 4(mod 6). If (Q,q) is a quadruple system and π is a collection of blocks of q which partition Q, then π is called a parallel class of blocks. A quadruple system whose blocks can be partitioned into parallel classes is said to be resolvable. An obvious necessary condition for the existence of a resolvable quadruple system of order n is that $n \equiv 4$ or 8(mod 12). Until recently the only known resolvable quadruple systems were all of order 2^m. The first two numbers $\equiv 4$ or 8(mod 12) which are not powers of 2 are 20 and 28. This paper gives a method of searching for resolvable quadruple systems which produces resolvable quadruple systems of orders 20 and 28.

59 AN EXTENDED BIBLIOGRAPHY AND SURVEY OF STEINER SYSTEMS

Jean Doyen, Université Libre de Bruxelles
Alexander Rosa, McMaster University

Given three integers t, k, v such that $2 \leq t < k < v$, a Steiner system S(t,k,v) is a v-set S together with a family B of k-subsets of S called blocks such that any t-subset of S is contained in exactly one block. We present a new bibliography on Steiner systems which extends a similar listing compiled five years ago; it contains about 650 titles (versus some 400 titles in the old bibliography). A survey of "small" Steiner systems is extended to comprise the available information on the existence and enumeration of Steiner systems S(t,k,v) for all sets of admissible parameters with $v \leq 28$.

60 TWO VARIABLE ORTHOGONAL DESIGNS OF ORDER 28

Peter Eades, University of Waterloo

Necessary and sufficient conditions for the existence of two variable orthogonal designs of order $n \equiv 4$ (mod 8) such that $n \leq 28$ are given. The result is proved by the tabulation of computer constructed orthogonal designs of order 28.

SOLUTION OF LINEAR EQUATIONS AND MASON THEOREM BY PATH FORMALISM 61

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Multiterm linear recursion equations with nonconstant coefficients has been solved recently by Adel F. Antippa by a formalism based on ordered partitions of interval. The purpose of this paper is to develop a form of solution of multiterm linear recursion equations with nonconstant coefficients based on path of a certain flow graph. The solution is obtained by defining a suitable homomorphism on the set of path of flow graph. A direct application of this path formalism is the demonstration of Mason's theorem. A second application is also presented by solving Fibonacci's three term recursion equation and the solution of one class of three term recursion equations with nonconstant coefficients is obtained.

ORBIT AND STABILIZER RESULTS RELATED TO BURNSIDE'S COUNTING THEOREM

62 Rodney T. Hansen, Montana State University

The concepts of a group acting on a set and sets defined as orbits and stabilizers are fundamental to the Burnside Counting Theorem. Extending orbits and stabilizers as given in L. W. Shapiro's Introduction to Abstract Algebra, we define for group G acting on set S and $\phi \neq H \subset S$,

$\mathrm{Stab}(H) = \{g \in G \mid g(H) = H\}$, and $\mathrm{Orb}(H) = \bigcup_{g \in G} g(H)$, where

$g(H) = \{g(h) \mid h \in H\}$. Various basic algebraic and set properties involving these sets and elementary set operations are given. Further, relationships of these sets to the lemmas upon which the Burnside theory is built are considered.

Frieze Patterns as Matrix Multiplication Tables

63 Duane M. Broline Auburn University

The frieze patterns studied by Conway, Coxeter, and others consist of an infinite strip of numbers arranged like bricks on a wall. The pattern is bordered by rows of zeroes and ones and for every diamond of the form

$$\begin{array}{ccc} & b & \\ a & & d \\ & c & \end{array}$$

the so-called unimodular rule holds, i.e. ad - bc = 1. For example,

```
... 0 0 0 0 0 0 0 0 0 0 0 0 0 ...
... 1 1 1 1 1 1 1 1 1 1 1 1 1 ...
... 1 2 2 1 3 1 2 2 1 3 1 2 ...
... 1 3 1 2 2 1 3 1 2 2 1 ...
... 1 1 1 1 1 1 1 1 1 1 1 1 ...
... 0 0 0 0 0 0 0 0 0 0 0 0 0 ...
```

We show that a frieze pattern can be viewed as an array of suitably oriented, overlapping 2 x 2 matrices in such a way that the unimodular rule corresponds to matrix multiplication. Also the constant rows on the bottom of the pattern correspond to a certain matrix being in the center of GL(2). By altering this "multiplication", it is possible to get patterns other than those generated by the unimodular rule. These new patterns share many of the properties of those previously studied.

On the Numbers of Solutions of Certain Equations in a Design

64 K. Nageswara Rao, North Dakota State University

$GF(p^n)$ denotes a fixed Galois field of order p^n, p being an odd prime and n an arbitrary positive integer. For any α and $0 \neq \mu$ of $GF(p^n)$, let

$$\tau(\alpha) = \alpha + \alpha^p + \ldots + \alpha^{p^{n-1}},$$

$$e(\alpha) = \exp\left(\frac{2\pi i \tau(\alpha)}{p}\right),$$

and

$$e_\mu(\alpha) = e(\mu\alpha).$$

E_μ stands for the set, called a block, of those elements λ of $GF(p^n)$ which are such that

$$\sum e_\mu(\beta\gamma) = p$$

$$\beta \in GF(p).$$

The object of this paper is to study the numbers of solutions of certain equations with solutions from different blocks using finite Fourier series and character sums.

Characterization of Outerplanar Graphs

65 by Ilker Baybars, Carnegie-Mellon University, Pittsburgh, PA

We show that a 2-connected graph $G = (V,E)$ is outerplanar iff (i) it contains a unique spanning cycle C_p, and (ii) $G - \{v_i, v_j\}$ is disconnected $\forall (v_i, v_j) \in E_c$ such that $G^p - E_c = C_p$ and, furthermore, we present an algorithm, OPTA-K2, for outerplanarity testing.

Results and Problems Concerning the Independence Ratio for Graphs

66 S. Fajtlowicz, University of Houston

Let G be a finite connected regular graph of degree p, $p \geq 3$, and let n be the number of elements of a maximum independent subset of G. The purpose of this paper is to prove some theorems and state some conjectures concerning the independence ratio $r = \frac{n}{|G|}$.

Theorem 1. If G contains no complete q-element subgraphs then $r \geq \frac{2}{p+q}$. Moreover, if $r = \frac{2}{p+q}$ then we have $3q - 2p \leq 5$.

Theorem 2. If G contains no complete p-element subgraphs then there are constants $c_p > \frac{1}{p}$ such that with two exceptions (found by Albertson, Bollobas and Tucker) we have that $r \geq c_p$.

The second theorem answers positively a question asked me by Professor Erdős.

Graph Theoretic Parameters concerning Domination Independence and Irredundance

67

by

B.Bollobás, Trinity College, Cambridge, U.K.
and E.J.Cockayne, University of Victoria, Victoria B.C., Canada.

Vertex x in a subset X of vertices of an undirected graph is redundant if its neighbourhood is contained in the union of closed neighbourhoods of vertices of X-{x}. In the context of a communications network, this means that any vertex who may receive communications from X may also be informed from X-{x}. The irredundance number ir(G) is the smallest cardinality of a maximal set of vertices having no redundancies. The domination number $\gamma(G)$ is the smallest cardinality of a dominating set of G and the independent domination number i(G) is the smallest cardinality of a maximal independent set of vertices of G.

The paper contains results which relate these parameters. For example we prove that for any graph G, $ir(G) > \gamma(G)/2$ and for any graph G, with p vertices, $i(G) \leq p - \gamma(G) + 1 - \{\frac{p-\gamma(G)}{\gamma(G)}\}$.

PACKING POLYTOPES AND FACET PRODUCING GRAPHS

68 EGON BALAS, Carnegie-Mellon University, Pittsburgh, Pa. 15213

The packing polytope P(G) associated with a simple graph G is the convex hull of the incidence vectors of all stable sets of vertices of G. A linear characterization of P(G) involves a description of its facets. We investigate several classes of facets of P(G) and their connections with certain subgraphs of G.

Combinatorial problems in the Foundations of Geometry

69 R. B. Killgrove, St. Lawrence University
D. I. Kiel, Cal. State University, Los Angeles
E. T. Parker, University of Illinois

Finite planes is one type of combinatorial problem in geometry. A little progress in this area is reported. The other problem discussed is the Sylvester-Gallai conjecture for planes. In particular, it is shown that the statement of the conjecture is not equivalent to the transversal axiom for ordered planes. The question was raised by Professor Crowe. The answer is given for affine ordered planes.

On Domination and Some Related Concepts in Graph Theory

70 Robert B. Allan and Renu Laskar,* Clemson University

Let $G = (V,E)$ be a finite, undirected, loopless graph with no multiple edges. Let $\lambda(G)$, $i(G)$, $d(G)$, and $ir(G)$ denote the domination, independent domination, domatic and irredundant numbers of a graph, respectively. The following results are discussed.

Proposition 1. If G is $K_{1,3}$-free then $\lambda(G) = i(G)$.

Proposition 2. $\lambda(G) \leq d(\overline{G})$, where $\overline{G}$ is the complement of G.

Proposition 3. $ir(G) \leq \lambda(G) \leq 2ir(G) - 1$, and these bounds are the best possible ones.

A SET OF PAIRWISE ORTHOGONAL FACTORIZATIONS

71

I.R. Beaman and W.D. Wallis*, University of Newcastle

There exists a set of four pairwise orthogonal one-factorizations of K_{10}, or equivalently a four-dimensional Room hypercube of side 9.

Lower bounds on the cardinality of the maximum matchings of graphs

72 Takao Nishizeki, Carnegie-Mellon University

Lower bounds on the cardinality of the maximum matchings of simple finite graphs are established. Let $M(G)$ denote a maximum matching of a graph G, p the number of vertices, $p(i)$ the number of vertices of degree i $(i=1,2)$, κ the vertex-connectivity, δ the minimum degree, γ the genus of a graph, $\lfloor x \rfloor$ the greatest integer $\leq x$, and $\lceil x \rceil$ the least integer $\geq x$. We express the lower bound for arbitrary connected graphs in terms of a linear polynomial of p, $p(1)$, $p(2)$ and γ as follows:

$$|M(G)| \geq \min \{ \lfloor p/2 \rfloor, \lceil (p - p^{(1)} - p^{(2)} - 4\gamma + 2)/3 \rceil \}.$$

We obtain more precise lower bounds for planar graphs, distinguishing several cases in accordance with δ and κ. Furthermore we show that all our lower bounds are best possible in the sense that neither the coefficients of p, $p(1)$, $p(2)$ and γ nor the constant terms can be improved.

Problems and Recent Results on 1-Factorizations of Cartesian Products of Graphs

73 Anton Kotzig, CRM-Université de Montréal

The following nearly trivial results are well known: (1) If there is a 1-factorization of a graph G, then G is regular; (2) A regular graph G is 1-factorizable if and only if every component of G is 1-factorizable; (3) A cartesian product of n graphs $H = G_1 \times G_2 \times \ldots \times G_n$ is connected (resp. is regular) if and only if every component G_i is connected (resp. is regular). Two recent results: (4) Let $\{G_1, G_2, \ldots, G_n\}$ be a set of regular graphs having at least one of the following properties: (*) for at least one $i \in \{1,2,\ldots,n\}$ G_i is 1-factorizable or (**) there exist i and j in $\{1,\ldots,n\}$ $(1 \leq i < j \leq n)$ such that G_i and also G_j contain at least one 1-factor, then H is 1-factorizable; (5) Let F be a cycle of length greater than three and let G be an arbitrary cubic graph. Then $H = F \times G$ is 1-factorizable. Consequences of the recent results and some open problems will be discussed.

Vertex Neighborhood Families of Graphs

74

Paul A. Catlin, Wayne State University, Detroit MI 48202

The neighborhood of a vertex v in a graph is the selection (counting multiplicities, in the case of multiple edges) of vertices adjacent to v. The vertex neighborhood family (VNF) of a graph is the family of neighborhoods of the vertices of the graph.

If the correspondence of vertices to their respective neighborhoods in the VNF is given, a graph is determined. However, even if this correspondence is not known, the VNF alone is usually enough to determine a graph uniquely up to isomorphism. Several examples are illustrated. The smallest counterexample involving two simple graphs is the 6-cycle and the pair of triangles.

We characterize those graphs that are not determined uniquely by their VNF's, and we give an algebraic formula, involving conjugate classes in an automorphism group, for the number of nonisomorphic graphs having a given VNF.

A POWER HIERARCHY OF GRAPH INVARIANTS

75 A.K. Dewdney, University of Western Ontario

A hierarchy of graph invariants is described. Classes within the hierarchy differ in their power to characterize graphs. The lowest class includes almost all of the "useful" graph invariants such as chromatic number, connectivity, girth, and so on. The highest class includes all the graph codes. From the point of view of this hierarchy, evidence is given that a set of invariants which is complete in Harary's sense is not likely to include any invariants of the sort commonly in use.

Each class in the hierarchy is given a purely structural definition and its power, the ratio (number of n-vertex graphs distinguished by an invariant in this class)/(total number of n-vertex graphs), is deduced from the definition.

MINIMAL GRAPHS WITH TETRAHEDRAL AUTOMORPHISM GROUP

76 Peter R. Christopher, Worcester Polytechnic Institute, Mass.

Graphs having automorphism group isomorphic to the tetrahedral group - also known as the alternating group A_4 - are examined. Such graphs with minimum number of 16 vertices and 30 edges are exhibited.

The general minimum edge problem is to determine $e(G,n)$ - the least number of edges required for a graph on n vertices to have automorpism group isomorphic to a given group G. The numbers $e(A_4,n)$ are determined for all admissable n.

GRAPHS WITH GIVEN GROUP, ORDER, AND SIZE

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L.V. Quintas, Pace University, New York, N.Y. 10038

77 We ask when there exists a graph X with the (abstract) automorphism group G, number of vertices (order) n, and number of edges (size) e. Clearly, it is necessary that n be at least as large as $n(G)$, the minimum order among graphs with the group G, and that e be between $e(G,n)$, and $E(G,n)$, the minimum and the maximum size among graphs with n vertices and group G.

We show that for any fixed G and all sufficiently large n, this necessary condition is also sufficient. We discuss examples of groups for which the condition is sufficient, respectively insufficient for small values of n, and pose related open problems.

78 Minimal Multiplicative Covers of an Integer

Carl G. Wagner, University of Tennessee

Let $|S| = n$. The number $m(n,k) = |\{(S_1,\ldots,S_k): \cup S_i = S$ and, $\forall t \in [1,k], \cup_{i\neq l} S_i \neq S\}|$, have been studied previously by Hearne and Wagner. The present paper treats three arrays, $\tilde{m}(n,k)$, $\bar{m}(n,k)$, and $\hat{m}(n,k)$, related to certain sequences of divisors of n. These arrays extend $m(n,k)$ in the sense of Carlitz, i.e., $\tilde{m}(p_1\ldots p_s,k) = \bar{m}(p_1\ldots p_s,k) = \hat{m}(p_1\ldots p_s,k) = m(s,k)$ for all sequences $(p_1,\ldots,p_s)$ of distinct primes.

79 Similarity Relations and Semi-Orders

D.G. Rogers, Mathematical Institute, Oxford, England

The two types of relation in the title have arisen independently in statistical work. As part of a combinatorial investigation, it is shown that they are, in a sense, converse notions and correspondences with lattice paths are established.

SOME COMBINATORIAL PROPERTIES OF GENERALIZED FIBONACCI NUMBERS

Norbert Cot, Institut De Programmation, Universite Paris VI, France

80 Generalized Fibonacci numbers appear in some fields of computer science, in particular in the theory of dynamic storage allocation schemes, as well as in coding theory. They are also related to some optimal binary trees.

Various relations involving these numbers have been recently presented in the literature. In this paper, we first give new interpretations of some known results and we then exhibit new results concerning in particular the convolution of generalized Fibonacci numbers.

ON THE CONSTRUCTION OF THUE SEQUENCES

81 L.J. Cummings

UNIVERSITY OF WATERLOO

It is often rediscovered fact that there exist arbitrarily long sequences on 3 or more symbols having no pair of adjacent identical segiments. There are, however, arbitrarily long sequences on 2 symbols without triples of adjacent identical segiments. Many related problems concerning forbidden patterns of segiments have been studied.

We describe all finite sequences on 3 symbols which cannot be extended indefinitely without the occurrence of a pair of adjacent identical segiments. This classification is used to justify an easily implemented algorithm which generates sequences without adjacent identical segiments.

82 The Search Number of a Connected Graph

T.D. Parsons

The Pennsylvania State University

The search number of a connected graph (finite simplicial 1-complex) G is the least number of pursuers, following a predetermined plan of continuous movement within G, required to intercept an arbitrary continuously moving evader in G whose initial position and movements are unknown. We present an algorithm for computing this strangely behaved topological invariant.

Almost all n-Dimensional Rectangular Lattices are Hamilton Laceable*

83 Gustavus J. Simmons, Sandia Laboratories

Numerous investigators have discovered that a Hamilton circuit exists in every n-dimensional rectangular lattice with an even number of vertices. Recently Steven Alpern has also shown that if the lattice has an odd number of vertices, a Hamilton path exists whose endpoints are both adjacent to a common vertex. Both of these conclusions, however, are simply special cases of the following more general result.

A bipartite graph G with vertex sets V_1 and V_2 is defined to be Hamilton laceable if

a) $|V_1| = |V_2|$ and for every $p \in V_1$ and $q \in V_2$

or else

b) $|V_1| = |V_2| + 1$ and for every $p, q \in V_1$, $p \neq q$,

there exists a Hamilton path in G with endpoints p and q.

The principal result of this paper is to prove that all n-dimensional rectangular lattices are Hamilton laceable except for the two-dimensional lattices

a) $2 \times m$, $m \neq 2$

and

b) $3 \times 2m$.

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"Greedy" Algorithms for some Lattice Polyhedra Problems

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84

Several "greedy" algorithms, which generalize results of Johnson and Edmonds, are presented to solve linear programming problems on certain "lattice polyhedra" defined by Hoffman and Schwartz (Proc. Keszthely Combinatorics Conference, 1975).

One of the problems is the following: We assume that we are given a partially ordered set L, two funtions on $L \times L \to L$, "$\vee$" and "$\wedge$", that satisfy $a \vee b = b \vee a \geq a, b$ and $a \wedge b = b \wedge a \leq a, b$, a finite set U, and a mapping $f: L \to 2^U - \emptyset$ which satisfies $f(a) \cup f(b) \supseteq f(a \vee b) \cup f(a \wedge b)$ and either $u \in f(a)$, $a \leq b \Rightarrow u \in f(b)$, or $u \in f(a)$, $a \geq b \Rightarrow u \in f(b)$. We also assume that a nonnegative function r is defined on L such that $r(a \vee b) + r(a \wedge b) \geq r(a) + r(b)$, that is, r is supermodular. Also, $d(u) \geq 0$ for all $u \in U$.

The following linear program and its dual are considered:

Primal $\min \sum_{u \in U} c(u)x(u)$, where $c(u) \geq 0 \ \forall u \in U$, subject to

$$\sum_{u \mid u \in f(a)} x(u) \geq r(a) \quad \forall a \in L, \text{ and } 0 \leq x(u) \leq d(u) \quad \forall u \in U.$$

Dual $\max \sum_{a \in L} r(a)y(a) - \sum_{u \in U} d(u)z(u)$, subject to

$$\sum_{a \mid u \in f(a)} y(a) - z(u) \leq c(u) \ \forall u \in U, \ y(a) \geq 0 \ \forall a \in L, \text{ and } z(u) \geq 0 \ \forall u \in U.$$

It has previously been proven by Hoffman and Schwartz that if c(u), d(u), and r(a) are integral, then there exists an integral optimal solution to the program. In this paper, we present an algorithm which finds an optimal solution, which is integral if c(u), d(u), and r(a) are integral.

85 TWO GREEDY HEURISTICS FOR THE WEIGHTED MATCHING PROBLEM

David Avis - McGill University

Recent heuristics for the travelling salesman problem indicate the need for determining a low cost perfect matching in a weighted complete graph K_{2n} in time $O(n^2)$. Two greedy heuristics are presented and analyzed for this problem. The first runs in $O(n^2)$ time and the expected weight of the matching obtained is given in terms of the moments of the order statistics of the edge weights. In particular, if the edge weights are chosen according to a uniform - (0,1) distribution, the expected weight of the matching found is $\frac{1}{2} \ln n$. Similar results are obtained with this edge weight distribution for a second more sophisticated heuristic that has a worst case running time of $O(n^2 \ln n)$ and an expected running time of $O(n^2)$. Worst case results are also presented for both heuristics and various improvements are discussed.

86 Broadcasting in Grid Graphs

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Stephen Hedetniemi
Department of Computer Science
University of Oregon

Broadcasting refers to the process of message dissemination in a communication network whereby a message, originated by one member of the network becomes known to all members. We model a communication network by a graph G=(V,E), consisting of a set V of vertices (members) and a set E of edges (lines). Messages are transmitted by phone calls over the edges of G, each from an informed vertex to an uninformed vertex. Each call requires one unit of time. In (local) broadcasting an informed vertex can only call an adjacent vertex.

In this paper we study optimal broadcasting in a variety of graphs in which there is an underlying grid (checkerboard) structure, i.e. a subgraph of the form $P_m \times P_n$, where P_m and P_n are paths of length m and n, respectively, and 'x' denotes "product". The results we present indicate that grid graphs are not optimal broadcasting structures. They are of practical interest, however. Graphs of this type are frequently used to describe the interconnection pattern of multiprocessor computers, such as Illiac IV.

87 A Census of Minimum Broadcast Graphs with N ≤ 11 Vertices

S. Mitchell,* U. of Louisville and S. Hedetniemi, U. of Oregon

Let a connected graph G = (V,E) represent a communication network, where the vertices V represent communication centres or nodes, and the edges E represent communication lines over which phone calls can be made. Consider the process of disseminating a piece of information from a vertex u to all other vertices by a series of phone calls over the edges of the network such that: i) each phone call requires one unit of time, ii) a vertex can participate in only one call per unit of time, and iii) a vertex can only call an adjacent vertex. This process is called broadcasting. The minimum time required to broadcast from vertex u is defined to be the broadcast time for u, b(u). The broadcast time of a connected graph G, b(G), is equal to the maximum broadcast time of any vertex in G. A graph G with n vertices for which $b(G) \quad \lceil \log n \rceil$ but for any proper spanning subgraph G' of G, $b(G') > b(G)$, is a minimal broadcast graph; G is a minimum broadcast graph (mbg) if it has a minimum number of edges among all minimal broadcast graphs with n vertices. In this paper all mbg's with fewer than 12 vertices are determined.

88 SWITCHING CONNECTIVITY IN GRAPHS

Marc J. Lipman* and Richard D. Ringeisen
Indiana University-Purdue University at Fort Wayne

J.H. Van Lint and J.J. Seidel introduced the operation of switching in a graph. By "switching a graph G at a set $S \subseteq VG$" we mean creating a new graph $G^*(S)$ on the vertex set VG by removing all edges joining S to VG - S, adding all edges joining S to VG - S which were not in EG, and leaving all other edges unchanged. A switching cutset of the connected graph G is any set $S \subseteq VG$ so that $G^*(S)$ is disconnected. The switching connectivity of G is the minimum cardinality among the switching cutsets of G. We show that switching connectivity is independent of both connectivity and edge connectivity. We characterize switching cutsets in a way which yields the following interesting Corollary: If S VG is a switching cutset of G, then either S is a cutset of G, VG - S is a cutset of G, or G = S + (VG - S).

89 ON THE DISTRIBUTION OF NONZERO ELEMENTS IN CERTAIN SPARSE MATRICES

Dale M. Mesner, Univ. of Nebraska - Lincoln

A 1976 theorem of Ganter and Teirlinck says that a (2t) x (2t) matrix with at most 2t nonzero elements can be partitioned into four t x t submatrices of which at most two contain nonzero elements. An easily proved generalization is that a (dt) x (et) matrix with at most (de-2)t nonzero elements can be partitioned into t x t submatrices (de in number) of which at most (de-2) contain nonzero elements. More generally, we may conjecture that a (dt) x (et) matrix with at most kt nonzero elements can be partitioned into t x t submatrices of which at most k contain nonzero elements. The author has (a) proofs of some other cases of this conjecture, (b) optimism about the truth of the full conjecture, and (c) pessimism about the existence of an easy general proof.

90 COMPLETE ORTHONORMAL SYSTEMS GENERATED BY MATRICES

R.B. Crittenden and L.G. Swanson*, Portland State University

Application of a modified Kronecker product to a class of "Hadamard-like" matrices generates sets of functions which are complete orthonormal systems. The Walsh functions and the systems generated by the characters of finite cyclic groups are special cases. The technique can be modified to generate generalized finite Fourier expansions.

Enumeration of Many Body Diagrams Using the Permutation Group

91 Edwin Ihrig, McMaster University

Many body graphs that occur in green function expansions are seen to be represented by equivalence classes of the permutation group. Thus their enumeration is equivalent to listing representatives of these classes. A transversal of a certain quotient group gives an explicit enumeration of a certain type of green graph. Another type of equivalence, exchange equivalence, is defined and it is shown that every connected green graph is exchange equivalent to one of the type mentioned above. An effective algorithm is then given to generate all the graphs in an exchange class from an exchange class representative. This enables one to enumerate all connected green graphs. With this procedure green graphs may be listed in any order for which the computer has sufficient memory to store the list. Details are contained in [1] and [2].

[1] Rosensteel G., Ihrig E., and Trainor L.E.H., "Group Theory and Many Body Diagrams I Classification and Structure", Proc. R. Soc. Lond A., 344 387 (1975).

[2] Ihrig E., Rosensteel G., Chow H., and Trainor L.E.H., "Group Theory and Many Body Diagram II Enumeration Methods and Number Approximations", Proc. R. Soc. Lond. 348 339 (1976).

Cyclic Arrangements of the non-Identity Elements of a Group

Richard Friedlander*, University of Missouri-St.Louis

92 Basil Gordon, University of California, Los Angeles

Michael Miller, Rand Corporation, Santa Monica, California

In his solution of the Map Coloring Theorem for all compact 2-dimensional manifolds except the sphere, G.Ringel was led to the following group-theoretic problem. Under what circumstances can the non-identity elements of a group G of order n be arranged in a cycle $a_1, a_2, \ldots, a_{n-1}$ such that the quotients $a_1^{-1}a_2, a_2^{-1}a_3, \ldots, a_{n-1}^{-1}a_1$ are all distinct? If such an arrangement exists, we call it a Ringel sequencing (R-sequencing) of G. In this paper, we make the conjecture that a necessary and sufficient condition for a finite abelian group G to be R-sequenceable is that its Sylow 2-subgroup is either trivial or non-cyclic. The necessity of this condition is easy to prove. The sufficiency is much more difficult. We prove it in the following cases:

1) The Sylow 2-subgroup of G is $(Z_2)^m$, where $m > 1$ but $m \neq 3$.

2) G is of odd order with a cyclic Sylow 3-subgroup.

3) The Sylow 2-subgroup S of G is $Z_2 \times Z_{2^k}$ $(k>0)$, and $|G/S|$ has at least one prime factor $\equiv 2 \pmod 3$.

We also prove the conjecture in a number of other more special cases, and obtain some results for non-abelian groups.

On the Unimodality of Certain Partition Functions

93 A. M. Odlyzko, Bell Laboratories

Given a sequence $1 \leq a_1 \leq a_2 \leq \ldots$ of positive integers, let $P_k(n)$ equal the number of partitions of n into distinct elements of the finite sequence $a_1, \ldots, a_k$; i.e., $n = a_{i_1} + a_{i_2} + \ldots + a_{i_r}$, where $1 \leq i_1 < i_2 < \ldots < i_r \leq k$. We discuss conditions under which $P_k(n)$ is a unimodal function of n.

94 RAMSEY THEORY AND THE DIMENSION OF POSETS

William T. Trotter, Jr., University of South Carolina, S.C.

The dimension of a poset P is the smallest positive integer t for which P is the intersection of t linear extensions of P. Alternately, the dimension of P is the smallest positive integer t for which P is isomorphic to a subposet of Euclidean t-space $\mathbb{R}^t$ equipped with the product ordering. A poset P is called an interval order when it is isomorphic to a poset of the form $(\mathcal{C}, \lhd)$, where $\mathcal{C}$ is a collection of closed intervals of $\mathbb{R}$ with $A \lhd B$ if and only if $a<b$ in $\mathbb{R}$ for every $a \in A$, $b \in B$. Although a poset of length two can have arbitrarily large dimension, Rabinovitch proved that the dimension of an interval order of length n is at most $1 + \log_2 n$. Rabinovitch, Bogart, and Trotter proved that for every t_0, there exists an interval order whose dimension exceeds t_0. In this paper, the connection between Ramsey theory and the dimension of interval orders is used to show that there exists a constant c_1 so that the dimension of an interval order of length n is at most $c_1 \log_2 \log_2 n$. Ramsey theory is also used to show that the dimension of the interval order consisting of all intervals with integer endpoints from $\{1,2,3,\ldots,n\}$ is at least $2\log_2 \log_2 n$.

Subsets of a finite set that intersect each other in at most one element

Donald E. Keenan, Texas A&M University 95

This paper generalizes some theorems by DeBruijn and Erdos and by Ryser. Let $S_1,\ldots, S_m$ be m subsets of an n-set S and suppose $|S_i|\geq 3$ $(i=1,\ldots,m)$. Suppose further that $|S_i \cap S_j|\leq 1$ $(i\neq j;\ i,j=1,\ldots,m)$ and that each S_i has non-empty intersection with at least m-c of the remaining subsets for some fixed positive integer c. A Fisher type inequality of the form $m\leq n$ is proven using a theorem by Ryser and showing that no column of the incidence matrix may have a relatively large column sum. For the case m=n it is shown that for sufficiently large n the incidence matrix has constant line sums. When c=3 and the incidence matrix A has constant line sums the rows and columns of A may be partitioned into cycles in a natural way. It is shown that A then has a cyclic substructure and that its rows and columns may be permuted so that $AA^T=A^TA$.

New Solutions to the Rota/Mullin Problem of Connection Constants

96

J.M. Freeman
Florida Atlantic University

Multinomial-like coefficients are defined combinatorially, and then used to give explicit formulas for the constants connecting sequences of binomial type.

On Binomial Sequences of Binomial Type

97

R.C. Mullin, University of Waterloo

The theory of polynomial sequences of binomial type is interpreted in terms of sequences of combinatorial structures defined on partitions of finite sets. An associated theory of enumeration of "labelled" objects generated by "components" is discussed.

TREE SYSTEMS WITH APPLICATIONS TO NETWORK FLOW CONTROL AND ROUTING

98

F. Hadlock, Florida Atlantic University

The notion of a Tree System is introduced as a basis for routing and flow control in networks. Given an undirected network N in which path length is the number of nodes, a Tree System is a collection T of coordinatized spanning trees which preserves distance in the following sense. For any pair nodes u,v of N, there is a tree t in T such that distance in t between u and v equals the distance in N. Coordinatization of the trees is accomplished by an algorithm (due to Hadlock and Hoffman) so that in each tree, tree distance may be computed by the Manhattan or rectangular metric. The coordinatization along with the metric provide within each tree a notion of direction as well as a simple mechanism for determining contention between scheduled transmissions. Additional requirements for tree systems are discussed which would tend to distribute flow or ensure reliability. Heuristic techniques are presented for constructing tree systems for a given network.

Graceful Trees

99

K.M. Koh and T. Tan, Nanyang Univ., Singapore
D.G. Rogers,* Mathematical Institute, Oxford, England

Some methods of graceful labellings of larger trees from smaller trees are presented. In particular, it is shown that the complete m-ary tree is graceful.