

* CR is the Gold Coast Room.

Room 207 is entered from the second floor lounge of the University Center.
Coffee available in the other half of the Gold Coast Room. :
POOLSIDE COCKTAIL PAR'IYfor Conference participants at HOLIDAY INN-LAKESIDE 6PM, Monday, February 17, 1975
CONFERENCE BANQUET at 7:30PM at the HOLIDAY INN-LAKESIDE in the Baroness and Marquis Rooms, Tuesday, February 18, 1975 (Drinks will be available in the Baroness Room from 6:30.)


MONDAY, February 17, Professor Richard M. Karo will speak on "The Computational Complexity of Some Graph-Theoretic Problems.• TUESDAY, February 18, Professor Ernest E. Shult will speak on "Some Combinatorial Problems in Finite Group Theory,•
WEDNESDAY, February 19, Professor Richard M. Wilson will speak on "t-Designs and Linear Algebra.•
THURSDAY, February 20, Professor Paul Erdos will speak on "Extremal Problems in Graph Theory and Combinatorial Analysis,



Some Linear Algorithms for recs

$$
\begin{gathered}
\text { S. Mitchell } \\
\frac{\text { S. Hcc:ctnicr.1i }}{\text { S. Goodmail }}
\end{gathered}
$$

University of Virginia Charlottesville, Va. 22901
-• Let Tn be an arbi rary tree with n vertices. We present algorithms which solve a variety of problems on Tn in time that is linear inn. In particular, these algorithms find the Hamiltonian completion number, tho domination nurnl>er, a maximum b-matching, and point and line covering numbers.

## G, O!i\II . ; P . 1 PTI,G.rl'ThLAA

 of a siJ:lple s e:ucntial !":-0,:ra:afj.c. on) v constitutc6 hv .irit. 'MCt:ica) st.:itc-ron::s),. $\cdot$ hi, :', r.tlnimizes trc tctal nmh: r of rc-r.ist.c-rsto $1, \mathrm{c} 11$ scd.

Let Ir re a ;,ot r.:-cursive sC'ciu ;, © of .irit',,,<'ticaJ.stnt.-l'C':it. $=1$ G• $£ X, U)$ tllt"2cirn,it clir.rap! 1 :is,oc:atcd as follodnp, : X is tl-c set of v:ifal,Jcs an-1 fr,v)e U jf there is a statcn.-nt $x=f(\cdots, Y, \ldots) \quad$ fo $T T$ •/In ir-mkinc-ntatic-nor if is a J.ir, $c:$ :rorder

 $!'-1 \mathrm{x} / \mathrm{d}(\mathrm{x})-=0 / \quad ;(: . \mathrm{ii}) \quad \mathrm{b}(\mathrm{x}) \cdot \mathrm{r} .>()() .>.-/ \mathrm{y}, \mathrm{xz} ; \mathrm{y}=>\mathrm{Vz} \quad:(\mathrm{x}, \mathrm{z}) \quad \mathrm{U} \quad ?^{\circ} \mathrm{GV}$.
(
 algorith: 1 for this last prorkM. '3v usinr. Fl11I 's results 1 -anrovc, 1 thi, t ti-e nroi-,em of ca:; , uting $g$ such th:it r. (G)=r.ltnr $(G, "\{)$,$) is a ;,olvncr..ia! co!111, )etc nro'-Jtrr. . Y:inaHv '"'<'$ obtaine. $1 \mathbf{S}=$ res Jlits for particular p, raphs.
. so:-:z ru:su:,::s ABOUT TIBB MrnIMAL CARDINAL ( p (G) ) OF A PATII SZT rn A G! IAPH Ga ( $X, U$ ) CARRYING OUT A PARTITION OF $X$, AND THE NU?\{E:C:R OF JM:!"S (S (G) ) HI ACIRCUIT GRA?H::J.

## $\frac{M-H \quad 6 \text { MRR }}{\text { G. }}$

U1 HVERS!T2 PARIS. VI
INSTITUT $1 \neq$ PROGRJJ•U•JATIIDN
In :a graph $G$ a ( $X, U$ ) , $S(G)$ is the $m$ !.nimum $c: f$ the ardir.al of a set of arcs Vc:X: "' snch that $G(X, U v V)$ iS an ac:trct:it graph and verifies $p$ (GI) c $\mathbf{r}$
.. We study t?le relation b<?tween:;:, (G) and S (G) , g!ve some decomposition theorems i order to compute those .invariants ill so::ie special clas:; o graphs.

This work follows M. Cfil.IM: , G. CiiATY last year con:fcre:ice paper , ai:d would not have exist without s. HBDET!H:EMI , S. GCO MA. study about tte $¥$.amilto $l=$ Co pletion Problem.

Behrend's thcore for sequences containing no k-clerncnt arithnetic p ogression of a certain type. 'l'.(;.Brown, Si:non Fraser University.

Let $k$ and $n$ be positive integers, and let $d(n, k)$ be the maxi -.:.." cen ity in $\{0,1,2, \mathrm{kn}-1\}$ of a set.containing no arithmetic progression of $\%$ cer $s$ lol'ith fi!'.'st term $a=: E a_{i}{ }^{k} l$. anO cor.. $0: 1$ d'iff-ercnce $d=1: \&: k^{i}$, ,.., here
 is called a k-dia9on.11. Thus $d(n, k)=; c::!;-. J$, where.;:,, is a subset $c: \quad[0, k::)$ which has largest cardinal while not containing a:-.y k-diac; onal. (Note t:"lat $k$ integers form a k-diagonal if and only if their $k$-ary r«prescntations C'1.n be ; , ut ir.to the rows of a matrix in such a ay that eac colu.: O=th\& atrixi reaCi s iron top to botto, is either iii ... i, for so=e i dcpc Ci g c te colt=, :or $012 \ldots k-1$. For exa. ple, $\{2,5,8\}$ is a 3-diac;onal which contains ::o 2-diagonal.) Setting $S k=\lim _{\mathbf{n - x . 0}} d(, k)$, we show that $\underset{\mathbf{K - ;} ; \mathbf{c},}{L} B k$ is either 0 or 1 . (At present the only known values of $S k$ are $1-B 2 \stackrel{\text { n-;,co }}{=} 0$. It' is also try!e that 81 !: $62=: \cdots=: B k=: \cdots=: 1$.

## (a) Catalan and "Total Informatioi1" Ni.::nbers Louis Shapiro, Howard U:,iversity

Let $C=\{1,2,5,14,42, \cdots\}$ denote the Catalan nu bers. We prove th following pro?ositions



A short, natural proof is given of ooch i cn:ies an as an pplication we can show that two interpretations of :J:e sequer.ce $1,1,2,4,9,21,51, \cdots$, the "total infroma::cn nu. bers ", $\infty$, ii'lcicle.

RE IARKS A."D RESULTS RELATED TO Ti-IE MCRGAN WARD CONJtCTu"RZ

## Rona\}d Alter <br> University of Kencucky

A second order linear recurrence s. a. l. r.) is a sequence \{an\} of integeri: satisfying $a n-2=$ Man+l - Nan where $H$ and $N$ arc fixed integers and at least.: $\cap n a_{n}$ is nonzero. Morgan ard conjectured that in every nor.degenerate' s. o. 1. r. no integer can occur ;:;ore th.:,.'" five tines. This pa?er is c., ncerr.ed Yith the proble;:; of characterizing those integers in as. o. 1. r. which occ r ore da." $\backslash$ once.

## Steve Clark, inf., James 'Spriggs

Dept. of .ath., U. of Maryland, College Park, MD 20742
Let ${ }^{n 2}$, for $n$ a nonnegative integer, be the full bin.i.ry tree with $n+1$ levels; i.e. $2^{11}$ branches. In this note we derive a fo=ula. for the largest number $F(m, n)$ of braac!-ies a subtree of n 2 can have yet 1 pot contain a subtree ho::ieo.r.orphic to m 2 . We find $F(m, n)=I: l l i$ (J), our search for this bound was inspired by a stat ment of ${ }^{-}$S. Shelah who micd that a certain related function $G(m, n)$ was bounded by $n \&$ fit, for some functio g. We shall prove $F(n-., n)=G(m, n)$. Thus our theorc:w shows that $C(: n, n)$ is never greater than $n: u$ and, for fixed $m, m>l$, is asymptotic to $n-1 /(m-1)$ !.

A !la.I ar.d short proof is given to a theor.:m of Sabiclussi asserting that, .rt.th for xceptional cases, the group of auto orphisms of a connected gr;i.ph is iso.:icrphic to the group of auto:norphisms oi its line graph. This proof easily extends to give the corresponding. .results for pseudographs

## = d dipseudographs

## (j)

k-B1ccks ar.d U1trablocks in Graphs, DAVID \./.MATULA, Soutilern Methodist Univers_i_!r. For $k ; 1, l_{\text {, }}$ k-block is a maximal k--poir.t conrected subgraph of a graph. bloc $\bar{k}$ - for any $\mathrm{k}^{-}$, hich•does not contain a (k!-1)-'.lock is ter:r.ed an ultrabloc:<. Although both k-blocks and ultrablocks can extensively ave.lap, it is sholn cnarthair total number $n$ a given graph is moderate. Specifically, the total number cf ultr\&blccks in appoint graph $G$ where the largest point connectivity of any subgraph of $G$ is $-r$ is given by $\{(p-1+1) / 2 J$ for an $p>T \geq 2$. A similar result is presented fork-blocks.
(I.2.)

The Char.c:cterization of Certa $n$ Sets of Graphs Using il Generalized Closure Oocntion, $\backslash 1!I L T A ;-1 \mathrm{H} .-$ DAY, Sc uth 2 rr . il:thodist University. - Jardine and Sibson invt:stigated flat cluster r.iethcds and characterized them in terms of certain sets cf graphs called ind'.cator familias. In this paper I characterize indicdtor fa;nn ics in tems of certain closure operations defined on a set of gn::hs. This is a special case of a ore general result in which $r$ characterize certai subsets of lattice elements in terms of closure operntions defined or a comple e lattice•

1Nicholas
Jardine and Robin Sibson, Mathematical
Taxonomy. Jvhn kiley $t$, Sons Ltd. (19m-:-----
(R) Routing fer Solid aste Collection

Frank o. Hadlock and Frederic\% !toffman, Florie.a Atla:-.tic u .:.v rsi..ty
Tlis is a preliminary $r$ port on the dcvelop ent of a libr $=Y$ cf routed block patterns for both-sides-of-the-street solid w3ste co!l ctioll. The routed patterns arc to be Ltsed as the basi for a o ula= I>f>: oacll to routing. The procedure involves several algoric s, so c orc gencrutcd. Patterns not. c:o:lforming to pos i;:)le st._:cc pote.-r s ctrc gcncrutcd. Patterns not. c:o:1forming to pos i;:)le st.-:cc potte:-r.s lion addition, pc:1ttcrns which can be routcC as u:1ions of .in? ior panterns st osc with bridg s) we liminated. .he as tin constraints, to minimize turns the shall discucs scme of che algo ithms used, as well as som properties of the sraphs involved.

This research is being carried out und $r$ a grant frora the ?AU-Y!U Joint Ccr.te: for Enviror -ncr.tal and Urban :>roblc:i:s.

## @


1 i. :-:.:G: od; ) ci.... i.1 o;i the finite set E in 2 :1 1 ::oi.t i : .-:-:'lic;-

 r.'.acc-oicl ct::si=:ns or _r;rLw po':1cr i:r.uc;: incluc.0 the !=:o•;: i"J:1i œ p-;:c-

:Evc:r.y rar_ : 3 ::iatro.-:.c. de sicn , i, th ind :c a princ por:er is, esi:: 11-
tially, either (i) a (J. $\left.+q+q 2 t^{0}, 1+q, 1\right)-31: B D ; 0::$ (ii) a
(l \# q2t-+ 1, $\quad+q, 1$ ) EIED; or (i.ii) a c=-01.!p d.ivisitle c.es::.o: r,itn $A-=1$ a,..d $1+q^{2 t}+1 \quad$ [TOU.PS,_ec.ch o:f s.i.zc $p^{s} \quad p^{s}, 11$ <n.:-:C:

'ih Cll.::JCS 0 a r.nu.cn. 1 ;er !:;Cr.-::.c 013 BDts, $\because$ :,it:1 !,> C::.:-:cto:-s o:" fo ::1:
 v:.
pirio

The e::i-tZ!ncc of -c.mc (iii) desi.-, s is a.:-iu... sclvcd ul..o.';J.. :.1.
Ir. su:.'!m2...ry, ei..'"\&:y i:-2J1.1:, 3 :12.t.roid-C. s.i.c:;:1 of ::,:::r.:c :cc.: $:$.e::-i:::t-;::-::is ua2.;.:ootn a ne"1"'f'ect r.a:-::roid des5 (a : atrvit. i ,:-;:lic:1 the $:$ :.1.2..-:--c:: each gi-ve:r. :.' $\mathrm{l}^{*}$ :la,le t:le sa:-.ia ca:rdillalit:, . Simila:: results f'.:. ld for c 11 ra."'llm i:1ighcr t:ila..'13 •

Let $n$ be a positive intei;er. A composition of $n$ is

## a $k$-tuple of positive integers $a_{1} \ldots$ ak such that

$\mathrm{al}+\cdots+\mathrm{ak}=\mathrm{n}$. A co,Jposition al $\cdot \boldsymbol{\bullet} \cdot \backslash$ is a lexicog, aphic $\mathrm{co}:$ :iposition if fo::- every cyclic permutatic, n " of ( $1, \ldots, \mathrm{kl}$ there exists an integer $j, 1 \quad j$. $\$ k$, such t:iat $a 1=a, r(i\}$ for $1<j$ and aj $>a_{, r(j)}$.

Let GF(2) denote the f:!.e:td consisting of two eleme:1.ts and let $\mathrm{V}_{\mathrm{m}}$ de:1.ote the m <li en!:>ional vector space over GF (2). A deBruijn sequence of orde::- $m$, is a sequence [xi) of lengtll $2^{m}$, xi , GF(2), such that every element o. $\mathrm{V}_{\text {lil }}$ appears as $\mathrm{xk}, \mathrm{xk}+\mathrm{i}{ }^{\prime} \ldots, \mathrm{xk}+\mathrm{m}-1$ for some k with the subscripts taken modulo $2^{m}$.
. I-ithis paper the authors ive an algorithr.. 1"or generating a cowplcte list of t.e lexi ographic compositions of n , a d then use this list in an algorithm to generate a c.eBr..ijj:1. sequer. : :e of ordar $n+1$ •

$$
@_{\text {fd.rold Fredricksen, Free Sets }}^{\text {IDA-CRD, }}
$$

:::>rinceton, N. J.•
A su:n-: rce æe':: S 1s a collection of integers ,:hich satisfies the condltion that if $1, j \in S$ then $i+j t S(i, j$ not necessarily distinct). A proble:n.of Schur is to place .the int<:>gers 1,2, ..., k into $k$ SUUll-free sets with mk <'.Slarge es possible. Schur ;;hewed th.:it e. K! is an upper bound for mk.. For k 1, 2, 3 extremal solutions e.re easy to find by hand, viz. $m k=1,4,13$ in the three, respe tive cases. Via e. co:mp,.1ter backtrack search naur..ert sho,,ed $\mathrm{ff} \mathrm{fi}_{4}=41$.

By a construetivc teci1nique, Schui lo:, er bour.ds $:_{k}$ by showing k+1 3::ik ${ }_{+}$. Both upper and lower 'cou:.cs have l:>:eeni:npro ',ed. 'i'hc up;:,e:- bound co (e-1/2)k! by Whj_tehead and the :Lvwer bound to (89) ${ }^{n} / 4$ by Abbott and l-lcser. 'l'he latter $1 ;$ :akes :u:e of Ba.umert•s ialution ; 0 the k 4 case.
-The author gives a pl cement of he int.e crs 1-138 into five su. -f:-ee s cs usincs $3^{\circ} \mathrm{bec}:$ 'ed-\ Pu backt!"t..ck siri.ilar to 3awr.c:-t ts. T'nis ir.-,, roves the lower bcund slightly to $\{277)^{n / 5}$. ?he nur.iber 5 would sec to be rr.uch larger t:ian 136.

## J. F. Dillon.

## Departrr.ent of Defense

Fort George G. Meade, XD 20755
R. L. \}leFarland and J. A. Maiorana have independently constructe:l a large family of <li.f.ference sets in the elementary 2.beli.:in 2-gr:-oups of squi:.are order; w. call t'nese- difference se:s
 'Jy l'. Kes:wa Hcnon, R. J. Turyn, et al. We have ,:ecently give:1 a "p.:i.rtial spread" construction, a s;,ecial case of -which takes c!le follo1,ing "cyclocomic" form.

THEOREM. Let CC be the grot.: o ?f $\left(2^{\mathrm{m}}+1\right)^{\text {th }}$ powrs i:1 t:ie finite field $K: \overline{\mathrm{GF}}(22 \mathrm{~m})$. Then the t.:nion of $2^{\text {n}},-$ cosets of co cons::ituces differer.cc et. the addir:ive group K .

In chis paper ,,.econsirler a very special family of ciffer11nce sets which are given by this theorem and which ara closely related to $n$ interesting pair of error-corr cting codes We shew that these new difference sets are not eqt.:ivalent to any in F.- - -'IIIYM. We obtain this result and others on inequivale.nce by er.:.?loying c rtain affine invariants which we develop here and which are useful in the more general stt.:dy of Boolean functio::is.

1A'fIN' RECT.11.'-:-GES.

## 

G E. $\mathbf{H}^{\prime} \mathbf{M}^{\prime \prime}$-2star
Mat.hs:atics arrl Ccn:puter Scie.-:œ Depa..'"tre."It
\&an. don Univ sity
An $r$ by $n$ rectanqul..c.::> array is cc.zzed a Ic.-;ia rcctc.ngZe if

## alt tha intagers $7_{1}, 2$....., $n$ appaar in ecch $r \zeta$, and ff $r$ Cisir:.c= ,-f;eg\#"''s occur. in aach c:olunrr: .. In this papOr, ise p;::eSant a py,cof of an e:::pZicit fomiAZa ('Zient 19t;1) for evZuctirg the relhbe

of three $b!^{f}{ }_{n}$ La.tin rectangZas. Fsing ."/'UZti-prea ise in tager c.ritmu:itic, a tabz.e is aonst:"Ucted giving the number of rectangles for r. 40 .

## (d) •A DCRANG!:MENT P.I.O3LEM

## Rich3rd Askey, I,io1..ra:I Is mull, Thanaa Ismail

!'v!atile:':')a:.Jcs Research Center, University of Wisconsin,. Madison, Wisco sL,

The classical derangement pro':-lem may be described as follows
! i k-people leave U-eir hats, one per person, at a di??ck room and the hDls ae returned al random, how many ways can each i:,crson receive oi,ly one hat and

- it is not his own and what is the probability of this happening. The number is
 Ev:-1 a:id Gillis conside:-..!d the !ol!ow!ng \;Cnerallzat!on of this problem. In addition to checking hats some paople also check coats, gloves, scarves $\propto$ any numb:?r of other items. Each person collects the same numbec of Jtc:r,s he checked. 'ine prc.blem is to find the number of ways of doing this so that nobody receives ar.y piece of his own clothing. Using M11dMaho11's Master?hecrem, we give a sim;,le proo! to Even ar,d Gillis' result. We also prove that when $k$ peop!e each check $n$ items the proba!!;!lity thnt no perso" receives ar:y p!Ccc of his own, fer $k$ large and $n$ fixed Is $e^{-n}\left(1+0\left({ }^{1}\right)\right.$ for $k>1$. Tœ e:ror. term is $O\binom{1}{k}$ for $n>1$ and $O\left(\frac{1}{(k+1)!}\right)$ for $n=1$ and both. are best possibl .-


## A CENSUS OF 3-CLASS ASSOCIATION SCH.E ES

## . Dale M. Mesner <br> Univer:ftY of Nebraska <br> Lincoln, Nebraska 68508

Let $X$ be a finite set, $/ \mathrm{XI}=\mathrm{V}$. • An m-class association scheme on Xis a set $\{R 0, R 1, \ldots, R m\}$ of symmetric binary relations on , where $R 0=\{(x, x) l x$ e: X\}, each $(x, y)$ is in exactly one of $R 0, \cdot . ., R m$, and the following holds for i,j ,k.e: $\{0,:, 2,3\}:$ if $(x, y)$ e: $R_{i}$, then the integer PJ-i. $=I\{z$ e: XI $(x, z)$ e: R., $(y, z)$ e: Ric\} $\mid$ is constant (depends only on $i, j, k)$. $p_{\text {ifi }}$ is denoted $n_{i .}$. $2-c l$ ss schemes have been exiensively studied. This talk deals with $m=3$, descrioing a table of all parameters $v \leq 200, n_{i}, p_{J^{\prime} K}$ which satisfy the major conditions known to be necessary for the existence of 3-class association schemes. For $\mathrm{v}<100$ the table includes parameters for about 700 known schemes, 200 impossible, and 200 unknown. For $100 \cdot<\mathrm{v}<200$ the total is about 2300 and the proportion of unknown schemes is la::-ger. Inspection of the table reveals some new families of potential schemes.

## 3-class association schemes

Rudolf .Mathen<br>Deaartrnent of computer Science<br>University of 'l'oron

Necessarv conditions are derivec for the existence of 3 -class associ;tion sche es. These results are bnsed on parameter relations, eigenvalue multiplicities anc the Xrei con<litio. Certain parameter coc inations are ruled o t by countin,;i clicrues or cycles incident with a vertex ir. the corresponding grap!1s of associates.

The known-3-class association schePles are -::hen
classified into union and prodcct sche es, $s$ he es derived from syrr.metric bJ.ock desiqns, projective geo;n€:trie:s il.:'lc !-loore o.ra) Jhs and as syf!1!Tletric and exceptional sche!'.es. E'or each class the properties of. the schemes e.nd construction tr.ethods are given. Finally, applications are discussed relating association schemes to gr aphs and .balanced incomplete block designs•

Firs: Order Graph Acceptors
Curtis R. Cook, Oregon State University
In this paper we define acceptors for classes of gra?hs generated by iirst order context-free: graph gra,r.r.::in:. "r!": o:.: Seate acceptor recognizes the sets oi graphs generated by first order graph gra, T-ars ar.d the finite state acceptors recogni=e sets of graphs generated by first ord r graph gra::-.. ars NhosP. productions are sequences $0:$ : rewriting $r$ les. We show the
equ5.valence between the ce-:er:r.ini.stic and r.an-C:eter:::i.nist.::.c
acceptors. Finally wa considP.r the closure $0:$ : th sats $\mathrm{o}:$ : graphs under various graph operations such as line graph, block graph, clique graph, etc. and the closure under se opcra ions.

Cha=act izations ?f Line D pscudoz=aphs

$$
\begin{aligned}
& \text { J\&eph B. } \cdot \text { KIerlcin } \\
& \text { Western C rolin } t_{\text {r. ! ver si cy }} \\
& \text { O..llo... } \mathrm{h}<=\mathrm{c} \text {, IC } 28 \mathrm{i} 23
\end{aligned}
$$

In chis ;>>aper we ;>rctc:s.c a r.cw de:f.:initio:-. o a line ciipscuCoz.a? •,hie:, i.,;,i.:-..1!. the. old dct:nition. :reflects al.L types of adjac ncies. e the;n proSt:It t.O :yy;its of ch.:ir ctcrizatio:,;s. Tr.e irst c'na.racteriz:cs .4.i.: Ci;:>scudcsr.,.,is $\ddot{-}$ :cr.=.s i t 0 1 wprop $r$ p.:irt!tions oz the vrcx set. The secvj \& a !or idGc CYoC sul;Gigro? characterization $f 0=$ the class ! disra, s whic are line $\mathrm{Gi}, \mathrm{s} \mu \mathrm{Gog}=\mathrm{a}$ s.

A Combinatorial and Algebraic Machine Ai<lod Computation System

## (CAHA)

Tr.e CAMAC system is a renk flexible, interactive computer system .based on Joiin Cannon's Croup cor.1putcr system.

A group is given to the computer by either generating permutations or generators and relations. The system on handle my groups at ance ad stores the infonnation computed about the group in a table which is referred to by the rare of the group. There are commands to compute the order of the group, the index of a subgroup, orbits, conjugacy classes, and special subgroups like norr.1alizers and stabilizers. There are also prograrr,s -..hich compute vector-orbits of a pcrr:iutation group. Sore major algorithi:::s used tv implement these co=ands will be discussed. A main algcothm used for groups given by generators and relations is the Todd-C,,xeter algo.ithm and an important aliorithm for handling pcrutation groups is the Sims algorith. The type of problems already solved by the system will be given, and future capacilities will be indic.ited.

## Paul K Stockmeyer

College of I,illio.m and fory
The famous gmph reconstruction conjecture asserts that except for small trivial cases, every $n$-point graph is uniquely determined by its $n$ subgraphs on $n-1$ points, each obeained by deleting ore point and its incident edges. The corresponding conjecture for tcurnaraents ;as first considered by Harary ad Palmer [c:onatsh. Nath. 71 (1967) 14-23), who verified the ccejecture for non-strong tourna,ients with more th"n 4 points. The truth of the conjecture for strong tournaments wes cast into serious doubt by Beineke ad Parker [J. Combinatorial Theory 9 (1970) 324-326], ;ho displayed a pair of 5 -point tournaoents with the sme collection of subtourn ents, and three pairs of 6-point counterexamples. (Counterexample pairs on 3 and 4 points were already k,1own.)

An exhaustive coii:puter search has noc, c.etern,ir:ed that

1) the conjecture is true for 7-point tournaments, and
2) there are exactly two distinct pairs of ccunterexamples among.the 6,880 tournents on 8 points. In both cases, each tournament is the converse cf the other, a property noticed by Baineke - d Parker of all c?u:iterexaDples on 4 a 6 ?Oines.

The maxinum number of 3 or 4 -cycles that an $n$-tournaicent can have is well known. The problem of ma,i:i:nizing the nu:uber of S-cycles is more difficultas a S-tournament can have as nany $\boldsymbol{\infty}$ ::rree S-cyclcs.

In this paper we consic!er the class cf toumaments ;;1th the followir.g prope.-cy (which we call semi-transiti•.ity):
if $a \cdot x$ ad $a \cdot y$, and if $x+b+y$, then $a^{4} b$.
Since it is easy to minirize the number cf 5 -subgraphs havir.g mo 5 -cycles, and since a semi-transitive 5-tourna ent can have ro core than wo S-cycles, the problem reduces to maximizing the nußber of 5 -subgraphs with t ;; S -cycles, w.fic.-i we call Cs's.

The rgular semi-transitive n-::ourna®en:: :;as ( $\mathrm{n}^{5}$-ion ${ }^{3}+9 n$ )/ 1920 Cs's. We :ho., that any sernitransitive tournament oust "contract" oo a regular tournament, and we compute the nu.bber of $\mathrm{c}_{5} \cdot \mathrm{~s}$ in any se:ii-transitive tournaffieut in tems of regular ::ourna::ient co which it contracts.

In this way establSsh an upper bound for the $\mathrm{n}=$ ber of $5 \cdot \mathrm{~s}$ which is of the order of $n / 1920$..
(a)

Anf:111C1 Pl1t11:.rity of Csyley uia€"ra.nf: Plrn:Ir ?,:.:esenta.tions. Ol vira Rs.riaport Strasser, SK at Sto:-y BrJok, and .Hfney $\cdot 7,!$,evir Rut,;;ers, Er=s $\cdot i ; i c i$, NJ. Let!' (x|'••, xt; r/1, ..,$r$ वa) $(x ;:<)$ be the nresentaticn o: a grou:-,, ar.c. $G$ the Cayley dia,cram of t ,e pre,a@ntaticn If G c.ln be crosen p:.ar.er, pcir.t-sy:::-,etric (:d th t:ic $\ldots$...;c c-iuntercloc;.r,ise succs:s:no: of eqces l-ertcx), a-; 1-Jcally fi-

 Co:1di tios are irr.posed on the wart1 $R$ wfic1 c:c nc,ocsoary to =de ? $=$ si,-ida2. plan::r.Th\$e nllo the ssir;ninL to a of a i;ni»ua ele:-:-2;,t $S_{\text {t }}$ cf the sy,.:=.et-ic $=\approx$ ?
 lowinc toree conditios are equivalent (Theorec 1): (1) cf inisal x-le th $u$ Cer auto:corphi\$:: cf the free
 tr.e pre,entatio:..) It is shown that $S$. Con! ists of $h .-1$ di_sjoi::t cycles if a, ori.y if $\mathrm{Pi}: ;$ a to or?hic image in Ft of a free rod oft a standard special plc.: $\otimes \mathrm{r}$;,reSi'?ntation $\mathrm{P}^{\prime}$ and a free group of rank h . (Theores: 2.) The values of tro $\therefore$ :., egers, q and r., which cor::;,letely c:.aracterize $\mathrm{P}^{\prime}$ are read off the cet of ., n:cs n (Thecre:. ;.) Po:- giver, t (numbe of eene:-at..,rs) and c (co:-t!!!G.ible nober of defi!"li::g releto:-:, ), all special planar presentations and their iso orphy classes are fou. $\uparrow$ d.

Department of Computer Science
Univ rsity of Waterloo, Waterloo, Ontario, Canada
. $u$ ultilist files arc widely used for information retrieval with Boolean expressions. Two algorithms, one by Hsiao and Harary and another, called the Trace algorithm, may be used to advantage with such lists_. We present a probabilistic analysis of these two algorithms for several situations. For a rumber of com on cases, it is shown that the Trace algorithm is more efficient.
for the constru tlon of .a re"lization cf ,!! that h.i.s a
iltoni"n •path starting at a specified vertex (if such a
realization exists). This algorithm can also Le used to construct
a realiz"tion vith a hamiltonian cycle.
..

A NODE-ELIMINATION METHOD FOR FINDING ALL
Silil'IE CYCLES IN A DIRECTED GRAPH
by I-Ngo Chen

## Syracuse University \& University of Alberta

A method for finding all sir.iple cycles in ${ }^{\prime} A$ di::--ected graph is presented. The method is based on the idea of node-elimination. To eliminate a node from a graph, we simply join each entering edge to every leaving edge of the node. It is proved that a graph, after eliminating a node from it, wili recein all the simple cycles. The algorithm first transforms node attribute to edge attribute, then proceeds to eliminate nodes from the graph, one by one, until finally only one remains. During the process, simple cycles will appear, gradually, as selfloops of some nodes. Thus, upon the termination of the process, all the simple cycles of the graph can be found. Infonnacion about the graph is stored in a matrix. A $\overline{A t}$ the beginning of the process, en entry of the matrix may•store nc edge, or a single edge. After the process starts, an entry rr.ayhave to store sequences of edges.. This infornation may be respresented by sequences of symbols where different symbol represents different edge, or they ay be represznted by a finite nuruber through a certain coding scheme. In the first case, variable entry length has to be provided, while in the second case, extra space and time are required for tracing the cycles. Upper bounds for both cases are given;

## SEQ,UE,,CDIGS CF CERT.U: :: DIHEDRAL GROUPS

S. A. Ancie:-so:1
AriJona State University

Su?pose $G$ is a finite group of order $n$. A sequencing of $G$ 1s an ordering e, a.1, a2, ... , an-1 of the ele:::cnts of $G$ such that the partial ?roduc $s$ e, ea1, ea1n2, •••, eala2...an-l are distinct. Sec;uenc1n3s are t..sefu:1: $1 n$ constructlnc; certain types of Latin Squares and in finding certain decompositions of co plete directed graphs. Little f known about sequencin5s of non-Abelian groups and mcst exai:lples known have been found by computer.. I'le consider a. techclque for sequencing certain dihedral groups.
@
SOXE OBSERVATIONS IN FINITE TRANSVERSAL THEO Y
University of Calsary and Tel-Aviv University
A kno,-,, necessary and s fficient condition for a finite family $\mathrm{F} \cdot\{\mathrm{F} 1, \mathrm{~F} 2, \ldots, \mathrm{~F} 8\}$
of finite cets, to have ' t ' O disjoint: transversals is $\mathrm{J}=\boldsymbol{= 1} \mathrm{F}_{\mathrm{i}_{\mathrm{j}}} \mathrm{J} 2.1 \mathrm{k}, \mathrm{k}-1,2, \cdots$ a.
Obviously this condition is sufficient for F to have ?ro?ercy B. Attcmpcing to stre gthen this condition by requiring only tw disjoint maxi:::al partial transversal.
obtained the surprising result that if \& fa. ily F hns two disjoint $m$.xi al pa:-U;.l l•..ns_versals then it. has also t"o disj-,int tra:-:sversals. This spoils the above vay to find a st onger condition.
 fer $F$ to have property $B$, it is not necessary, but it is best possible, in the $\cdot \mathrm{en} \cdot \mathrm{e}$, th.t $\left|0 .{ }_{1} \mathrm{~F}_{\mathbf{i} \mathbf{i}}\right| \mathrm{k}$ is.not suffi ient.
$\qquad$

Wolfe, Warren
Queen's University, Canada
"Amicable Orthogonal Designs"

Orthogonal designs are generalized Hadamard arrays and have been used to construct weighin matrices. Pairs of designs, $X, Y$ where $x y^{t}=Y^{t}$ are particularily useful in generating large designs.

Using Clifford algebras, we have obtained limits on the number of variables which can occur in such a pair, extending the Pullman - Geramita result on sets of anti commuting, orthogonal matrices. We also have results which describe the nature of designs which can occur in such a pair.

## @

A Process DccomDosition Thc,orcm. John C. Hansen, Univc: sity of Nissouri-Rolla - A necessary and sufficient condition for the parallel decon csition of $a$ process is given in terms of parti ioning of the state space of that process. A process is definod as a triple ( $S, f, I$ ) where $S i s$ a state space, $f$ is a successor function in that space, and $I$ is a subset of $S$ which defines the initial states of the process.

State Granhs anrl the Assir.nment ProhleM for Asynchronous Seo, uential 1-'achines
f:dr,ur nur.asse
11rool lvn Collere, c.tr.N.Y.

Sequential machines can be classified as heinp either synchronous or <1svnchrc,nolls. Seouentiill rr.cichjncs ?.T'; ca 1] ed svnchrnnous if th ir one ation ii controllerl bv clockinp evices whose outnuts svnchronize sirnals in their c-ircl:its, an:'! clsvr;chrcnous if thev function without these timinr evices. Asvnchronous circuits have the advantape o be nr ahlc to utili e basic <lcvicc snecd since it is unnecess rv fry thern to -;ait for synchronizinr: nulscs. Unfortunatelv, pua!'antecinr, p-:-cne':" onerati<'.'Jl of asvnchronous circuits in the oresence of uneoual $t$ ar:smission delavs of sipnals is a diffic lt task. Of nirticular il"lnortrlnce in this connection is the wav in which nares are assip.ned to the internal states of the machine. This oaper studies the relationshi? between state p.ranhs of asynchronous sequential machines and some of the standard methods of coding their internal states.

THE BELL POLYNJMIALS A. $\backslash$ 'DMK $\backslash X I M: . ~ \ L G O \ . J ' P S$
OF Tile SFNIGRoup of rxxmLy STOCHASTIC \}.'AIRICES

> K Ki-Han Butler
> Alabama State Univerșizy

Let be the set of all $\mathrm{n} \times \mathrm{n}$ doubly stochastic: matrices.
Let $g(r \ln )$ be the set of all maxiEal groups of $n$. Let $g^{*}()$
be the set of all nonisomorphic: maximal groups of . Let be
the nth Bell number. Let Yn be the nth Bell polynomial.
Let $t(\mathrm{Yn})$ be the number of te:-ms contained in Yn . The mai."l.results

l\{f, .M!IIONIAN CIRCUITS, CYCLIC Pl'.'Rl\{UTATIONS, A.'ID EQUIDISTRIBUTED FINITE OiUllTS

Ab5tr-ac:t: Cci:iputing a $\infty$ :i.t.inuous map $T: X+X$ en a finite word leneth computer which can distirl(;u!.sh points in $X$ only up to a fini.tc partition $P=$ ( $P 1, \cdots, \cdots 11]$ of $X$ yields a "discrete" map $r: P \rightarrow P$. In upplication, for exrunplc, $P$ is a dyadic deco::;;:,osition of the unit square. p . D. LX has shown that m if X is a there is a per.nutation 'To: P +P that approximates T . This means t!1ut the n:a. $\mathrm{X}^{-}$ ima:i error is small if the diameter of the partition $P$ is s.ral.i. le show t.at if $X$ is co:1nected and if either $n$ has a small prime divisor or $T$ tas a fixed Jvi:-, $t$ ere is a pe:-nutation $t$ that aporoxi.11:i.tes $T$ in the above sense Jvi:-, $t$, $t$ ere is a. pe:-:nutation $t_{1}$ that approxi. 11 :.i.tes $T$ in the above sense.
If $X$ is a $k$-di, nensional polyhedron, $m$ a $k$-dirncnsional measure on $X$, and $T$ is an -p eserving hc:t.eor=orphism of $X$, then there is another -preserving hoMeo oran , i , p of $\mathrm{x}, \mathrm{Ti}$, that o.pproximatcs $T$ and has a finite orbit which enters each $\bar{P}^{\prime} 1$. exactly once. These results are i, roYed $\mathrm{b}, \cdot$ estnblishing sufficient conditions for Hal:liltoni! 111 circuits in a class of directed graph which $r$ flect the adjacency stnicture of $r$ and the action of $T$ on ?. Analogous $t$ corcms are esto.blishcd for pi.rtitions which l!.re "almost" equimeasured and for partitions with arbitrary measure distribution. Tbe above results are s:.Urpened forms of previous results of Oxtoby and Ula:n, K..tok o.nd Stepin, and the authqr.

Recent Progress on Path Nwr.bers of Digraphs

$\xrightarrow[\text {, rian R. Alsp3ch }]{\text {,ir.on Fraser University }} \quad$ and $\quad$| Nonnan J. Pullman |
| :--- |
| Queen's University | urnaby, Canada

Queen's University
Kingston, Canada

## , $\mathrm{nST}!\mathrm{UCT}:$

If $G$ is a digraph ( a dirGcted graph having no oops or multiple arcs) a path decomposition of $G$ is a mily of arc-disjoint, tircuit-frec paths which partitions :he arcs of $G$. The cardinality of a minimal path decomposition s called the path number of $G$ • A, alogous notions for mdirected graphs were studied. by Ercl5s; Gallai; J.ov\&s z; ; Harary. .nd Sch1,•e11k; and Stanton, Cowan and James.
<br>{e will discuss recent results of Chein; Mason? O'Brien } nd the authors. They concern bounds on the path numbers of -arious classes of digraphs and the determination of those ntegers which are path numbers of digraphs in a given class• everal open problems will be presented.

Let $S$ be a finite set $0:$ ? $n$ elp-ments. Let $C(n, k, 2)$ denote the minimal nuinber of k element subsets of S such that every pair of elements of $S$ is contained in at J.east one of them. We determine $C(n, k, 2)$ for $n<5 k / 2$.

For n.::; $2 k$ this was done by R. G. St ton,
,J. G. Kalbfleisch, ar.d R. C. 1t.i.:llin. For. $2 \mathrm{k}<\boldsymbol{n}$ _: $\mathrm{IF}_{\mathrm{k}} / 3$;,,e have $C(n, k, 2) 7$ except fo. the. case $7 k-3 n=1$ Fo. _ $7 \mathrm{k} / 3<n<12 k / 5$ we have $C(n, k, 2)=8$ except for $t$. e case $12 \mathrm{k}-5 \mathrm{n}=\mathrm{y}$ with k even. For $12 \mathrm{k} / 5<_{\mathrm{n}}<5 \mathrm{k} / 2$ we have $C(n, k, 2)=9$.

COVER SEQUENCES AND M. \PS

$$
\begin{gathered}
\text { by } \\
\text { Lena Qi311g } \\
\text { Arthur T. Foe }
\end{gathered}
$$

Temple University, Philadelphia, Pa.
A cover sequence of $a$ finite set $S$ is $a$ finite sequence of subsets of $S$ whose tmion is $S$. Let $f$ be a map fro:n TT,:,::.. o:-ito $S$. If one considers $f$ as a table, then each ixi can be t.F.ought of as a cover sequence of $S$.

T1le present work establishes the conditions on a collection 113: J of cover sequences of $S$ such that at least one fur.ctio: f can be defined fro:n TTt3 i onto S . Relations and applications to the t\},eory of decomposition for automata and to the .represl::ntative selecdon problem will he discussed.
(d) SCME $t$-DESIGNS FOR $t \geq 4$

Earl S. Kramer
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At-design (A;t,k,v) is a system of sets (called $\cdot b l o c k s)$ of size $k$ from av-set $S$, such that each $t-s u b s e t$ of Sis contained in exactly A blocks of. After selecting a permutation group $G$ acting on $S$, an $m$ by $n$ matrix $A$ of integers is constructed with the property that a ( $A ; t, k, v$ ) exists with $G a s$ an automorphism group iff $A$ has an $m$ bys submatrix $M$ where $M$ has uniform row sums A. For example, with $G=\operatorname{PGL}(2,17)$ and $J S I=V=18 \quad 3$ by 17 matrix A is searched and (A;4,8,18)'s are found for 51 different values of $A$ (not including complements). A nur.iher of ot er $t-d e s i g n s$ fort $=4$ and Swill be discussed.
torial d signs. A totally non-sy metric quasigroup
::;atisfies (i) $a b=b a$ only if $a=b$ and (ii) (ab) $a=b$,
$\mathrm{b}=\mathrm{a}(\mathrm{ba}) \quad$ Such quasigroups are shown to be idempotent
and to have cyclic multiplication ( $\mathrm{ab}=\mathrm{c}$ implies $\mathrm{bc}=\mathrm{a}$ ).
The ain results are I. .If there exists a totally no -
symmetric quasigroup $O \quad v$ elements there exists a balanced incomplete block design with parameters $v, v(v-1) / 3$, $\mathbf{v - 1}, 3,2$ and II. There exists a totally non-symmetric $q$ csigroup of ordtr $v=6 t+1$ whenever $6 t+1$ is a power of a prime.. An e::::ample shows the converse of $I$. to be false,

## aN UNICYCLIC GRAPHS IIAVING TWO DISJOINT t'. AXIMNIY.ATCHINGS

B.L. Hartnell

Uni¥ersity of Manitoba
We are. interested in obtaining information on thos.? unicyclic graphs rhich possess two disjoint maximal matchings. tn $t$ is paper, we characterize
:hose unicyclic graphs in which each of the trees rooted on the circuit
cave two disjcint maxilllal matchings, but the graphs themselves db not,

A PARI'ITICU!NG ISCMJRPH.ISM Aī11RITii?1 FOR DIREX:TED GRAPHS USING THE F MATRIX
R

> L.E. Druffel \& D.C. Schmidt,

Vanderbilt University;
J.E. Simpson, University of $\overline{\text { Kenturnctur }}$

A number of characteristics have been used to define conditions necessary for isomorphism. Traditionally, gcaph isomorphism algorithms attempt to find a permutation of the adjacency matrices of the two graphs. Druffel and Schmidt have reported a partitioning algorithm using necessary conditions based on the distance matrices. The algorithm is effective for a large number of grapns, including some regular graphs. However, using the minimu..il distance matrix for graphs with a maximum distance of two offers no improvement over the use of the adjacency matrix. Kelly has defined a m trix $F$ for which each
 re.moval of the vertex pair vi• Vj, Conditions necessary for isomorphism based o:n the $F$ mat.cix are similar to the distance matrix so that the partitiOtting algorithm may be applied using the F watrix. The algorit."un so applied is practical. for non-regular graphs.

## INVERSE V1FFERENT1AL" OPE'RATO•RS OVER GRAPH-LATTICES

## Charles C. Cadogan

University of the West Indies
The ge.n.ua. 1 601tm 06 a. c.ta.,H 06 -ltivu.se di66eltentia..t qpe1ta.to11.1;oveJt polynom-<.a.U de6ined on pa.1tt-i.ti.OM 06 po6i.t.lve .lntegeM .11;obta.ined. The coe6\{.lc-i.er..u. in the polynom.la. 16 all.ethe nu.mbeM v6 gene.1;,a.lgll.a.ph6and the application 06 the qpell.a.told> to the. 6 e polyncm.la. U detv:.m-i.nu a, counting 6 e11.lu 001;.mu.lt.lg11.a.ph6on th 11.upect.i.ve pa.11.t-ltidM. The bMic ,1;,t.'1.u.ctu.the, g1:.a.phla.ttice,corma.nd1; a, vital 1tole in e,,1;ta.bl-i.1,h-ltige 6-lna.1. 601tm 06 the ope a.toUt.
A. !laartmzns, K. Dar.hof and G. Frank SOUTHER/I IU.I1101S u:1:vrnsnv
kl aut;iimorphism a on $\mathrm{a}(\mathrm{v}, \mathrm{b}, \mathrm{r}, \mathrm{k}, ; 1$.) block design Dis. said to c'ict regularly or, if it permutes both the points and blocks in orbits all of the same length $n$ $1>1)$. A design admitting a regular automorphism submits to a relatively simple i rese:1tai:io:1.. !n previous work, Baartrnans and Oar.hof have develop<:d ,lecessary id sufficient conditions for a block design tc adr.iit a regular autonlorphism. lese conditions suggest a series of algorithms which can be used to determine if giv2r, design $\{$ knolm or unkn01m) admHs a regular automorphism of a given order. 1 effect, the problem of finding an unkr.own desi9n is replaced by the simpler and ire res:ricted problem of finding a regular representation for the design.

The algorithms \{implemented as computer prograrr.s) for carrying out the :>rocess e deso•ibed. Heuristic techniques are needed in several of the algorithms. i-he cblems of applying the heuristics--such as chosing an effective state-space rcesentati•Jn, finding a good evaluation function and reducing the number of eq, i $\backslash \cdot .$, . !Ot nodes--are discussed.

Vario s r J lts of applying tbe programs to specific design parameters are isted. The results indicate m:W repre!>entations for several desig:1s ;;;d :he >n-existancc of regular representations fqr other (known and unknown) designs.

A Computer Construction for Balanced _Orthogonal Matrices

Paul J. Schellenhc,rg
University of Waterloo
A balanced weighing matrix is a square orthogonal matrix of O's , l's and - -1 's such that the matrix obtain<ld by squarine entries is the incidence matrix of $a(v, k, A)$ configuration. The existence of $a(v, k$, ; $)$ ccnfig'.lration with appropri.:;te parameters does not imply the existeoce of a correspondio balanced weiGhing.matrix as is hown for $v=16$, $k$ a 6 and ). 2 -
-•
The structure of cyclically gener ted balarlc tl weiching matrices ilas bee:i the objact. of sc,veral investigations. Using these properties, a computer search is car.:-ied o:it for balanced weighing matrices corresponding
to ( $\mathrm{v}, \mathrm{k}, ; 1$. ) configurations with small valcr,s for v .
\'HHS'l' TOU RNM,:ENT

## Ronald D. Baker <br> Department of athematics <br> The Ohio Sta te University

Columbus, Ohio 43210
E. H. f,'.oore [!,mer. J. r.'iath. 18 (1896), 290-JoJ J defines a Whist Tournar::ent in $v$ players, Ylh [v], as an arrangement of v-1 rounds of play such that every player meets every other player once as partner and twice as onnonent, a round being a list of $1 / 4 \mathrm{~V}$ ta $\overline{b l e s}$ cxhaustim, the list of players.A table is a set of fou!' players with. a distinguished partitior. into pairs called tJartners, the other four pai"rs being known as op:,onents. It is shown that Wh $[v]$ exist for all $v=c 4 ;$, . except possibLy $v=1 J 2,152$, or 264. without the distinction between partners a.ld opponents a $W: 1-2$ is a re oJ.vab:!.e BIBD with k-=4 and $A=J$ •
a

A new rn<>thod for the systematic generation of Steiner quadruple systc;iis of order vis discussed. This method incorporates a state-space search, where the space of \& ates corresponds to the $s t$ of quadruple systems cf order $v$, and tha operators are bijections which are applied to subsystel:i,; of a quadruple system. The object of the sear:: :h is to obtain all pairwise non-isomorphic designs reachable from a specified start desiga. Formi.:las for tr.e num1. rs of subsystems in an order-v quad:::-uple system are derived, leading to a classification of the generated designs according. to th-a :1umbers of subsystem!:. th1=y co:-itain.

In particular the method is appljed to the case $v=, 1 €$. Properties of the generated set of d2signs are examined, givi.: g in!o=mation on the derived triple syster.iz, and also on the $s$ :ucture and number of higher order quadruple systems.

## (a) FINDING THE INDEPENDENT SETS OF A MATROID

Lambert S. Joel, Marjorie L. Stein and Christoph Wit:::gall

## National Bureau of Standards <br> \section*{Abstract}

Axioms for circuits and axioms for independent sets are two equivalent ways of defining matroids. However, all proofs of this fact in the literature rely either on another set of axioms or or additional concepts such as duality. In a new proof of this 2quivale..,ce, we derive the independence axioms from the circuit axioms by constructing an algorithm for generating the inc:epcn: ent sets of :he matroid from its circuits.

Skew-Tran6lation Generalized Quadran.;;.J.es

Stanley E. Payne
Miami University
(Ohio)
Abstract: Let $G$ be a group of order $S^{3}$. Let $L=\{H$. lo.:: $i \quad:: S+1\}$ be a collection of subgroups, each of order $s$, for vfiich $\operatorname{HiHj} \mathrm{n} \mathrm{Hk}=\{\mathrm{e}\}$, ir i, j, $k$ are distinct. Then the incidence structure! $S^{*} .=\backslash 1$ ', $\mathrm{B}, \mathrm{d}$ i,rith $P=G$ and $8=\{$ Higjg $E G, 0:: i:: S+l\}$ is a generalized qi..adrangle of order $\{s-J ., s+1)$. Let Ho be a norma.l subgroup of $G$. '!'hen $S^{*}$ may be constricted about the pivotal family $M o=\{1, g \lg \mathrm{E} G\}$ to oot,air. a skew-translation generalized quadrangle $S$ of order $\{s, S)$. In this way all knovn generalized quadranoles of orders $\{s, s)$ or $\{s-1, s+1$ ) raay be obtained. This construction provides a generaliz tion of the knoll! ! ones via coset geometries in a nonabelian group in a manner i.na:ogous to the construction given by Tha.s of translation generalized uaclrangles sing elementary abeli n groups.

## Roy B. L3Ow

Florida Atlantic University
At this Conference last year the authoc presented the conject $e$ that lf $G$ ie a planar grapr., P. iJ a subgraph, $n \rightarrow$ !', ar, d $O<r<n$ there i3:a tl:nc!:ion $i(H, r,, c)$ such that an 1 co:...orn of-li extends t:, an n-colori;,g oi C ! - nd only i:': $\div$ exfndD o -n n-coloring of: :.;d:v d ver:-..cs v, of G sr..tsfyir.s- c.:, 1.: f(!!, ., :.;
 $\ldots$, ...ul,t, !:o:i the inve\&tig.,,ticn of thi3 conjec-i:.:e :ill a : :-


## HDEPEDENOE ATIOS ANDTOGDA, GRAPHS 

(R)

Supylse Gis a graph with $V$ vertices that embeds on the torus. A set of vertices 1 is independent in Gif no p?ir of vertices in 1 is adjacent in $G$ Let ol(G) be the maxinm number of vertices in ay independent sel; and (.). (G) be ct. (G)/N. the 1rclenendance ratio. Set $U(1)=f ; M(G): G$ ebods $e n$ the torus 1 and $\mathrm{L}(1)=$ limit'i>oints of $\mathrm{U}(1)\}$ - Thee are essentially only four to oidal graphs w-th ndependa. ratios less than ./'i (A.lbertson "nd liutch:!.nso:!!, 11 T;13 Independence ijf.tio and Genus of a Graph"). Theorell. $\Rightarrow$ (1) C 1.

A Lowe - Bc, d on the Ctu. 'o,natic Number of' a' Sphe_re

> Gustavus J. Simmons
> Sanda Laboratories Albuquerque, NavMexico

The chrc. "Tlatic m:mber $x\left(s_{r}\right)$ of the three dime:nsional sphere, $S_{r}$, of radius $r$ is $d$ :?fined to be the :i.est number of sets partitioning the surface of $S_{r}$ such that no set contains two points at unit chm dal distance apart. Obvi;msly, $x(S)=1$ for $r<\frac{1}{2}$ am! $x\left(S_{i}\right)=2$. ::lim lans had shei-r. 1 that $x(S) ? 4 f^{\prime}$ or $\mathrm{r}::$; fj and had. conjectured ${ }^{W}$ that $\mathrm{x}(\mathrm{Sr})$ ? 4 for all $\dot{r}>\frac{1}{2}$, i.e. ${ }^{\mathrm{r}}$ for 0.11 na1 t.rlvial. case!., . The nif1 result of this p::per is a p:-:-oof of this conje:ct.i..e-w.ch. at t.lit: s=:ii ,7e่ l'Cnders •.umec-a?s:;ary the specitl a.:-gm,z::ts requir.:id for



## EARL G7x !yle-s ID, JR., Un1'-r..1ty of Pittsburgh

## Chro!n3t1c "olvrior.:1 1s f'or !)dt11e Cydes

A double evele is a e2ph with $n$ vertic2s ::-.21月n ed8es such t :1at the codr;c set can be p.:11 t.tioned into $t-10$ disjoint n -scts, edh of whilil is an edge set for an n-cycle an then vertices of t1e vertex set. R\&c.1menoc relations for chrotlat1c polyno..;1...Is of s:,:ec:ial c12sses of do'.lble cycles are given. These recurre:-i.c.,; :i.'(;laticns y11:! fast algorithms for ar,1.1plling chrol:!:-.tic polyr.ol?;:las for thee spe al clas:...as. Tcse $2.1 g:=>$ rith:ll::. ara r.=late.c.
 1:-t.t.r.

1lolonct:d Colo..:,:ir., s and Grar,h rf. nt11tio1'

## J. A. ?0116;•

Ur.iv ,., Hy _ O£ ; ;,. crloo

An und!rected er ph c=n, in i ncral, be ri nted in many ways.
Of particular interest are oriantations wh!ch have certain r;pecifi d properties. Theor os of H.E. RoLbina and C. St. ;,A. Nash-Williams, !or example, are concerned with the arc conn ctivity $f$ the rP.sulting orientation.

Let $G$ be a graph in which all vortcr. degree $h$ ve the sace parity, and let $k$ be a nonnega ive integer. i.e giv-, a neccs:."1') and eufficient condition for $C$ co have ar. orientation ir. which, at each vertex, the !n anJ ?uc Le\&recs $d$ ffer by e.Qctly $\mathbf{K}$, and shoJ hc,v thla condition is re...:.:d to botl. :irStzach'a tl,eorem ald tw fuc colur cinjedux •

# On Graohs of Ramsey Tvoe 

# OPTIHIIL RFI\RRANGEABIE GRAPHS 

| S.A. | Burr | AT\&T Long Lines |
| :--- | :--- | :--- |
| P. | Erdos | Hungarian Academy of Science |
| L. | Lovasz | Eotvos L. University |

P. Erdos Hungarian Academy of Science L. Lovasz Lotvos L. University

If $F$, $G$, and Hare graphs, write $F$ ( $G$, H) to signify $t h a t$.if the edges of.Fare colored $d$ and blue in any fashion, either the red subgraph off contains a copy of G or the blue subgraph contain 3 a copy of $H$. Various. properties of such graphs Fare studied.: For instance, for given $G$ and $H$, the minimum chromatic number $X$ of any graph F for which $\mathrm{F}-$-; $\{\mathrm{G}, \mathrm{H})$ can be determined, at least in principle. In particular, if $F\left(\mathrm{Kr}_{\mathrm{r}}, \mathrm{K}_{\mathrm{n}}\right)$, " $\mathrm{X}\left(\mathrm{E}^{\prime}\right)$ 2: r ( $\mathrm{m}, \mathrm{n}$ ), where $\mathrm{r}(\mathrm{m}, \mathrm{r}$ ) denotes the ordinary Ramsey number. Thus $6(\mathrm{~F}) \mathrm{r}(;: 1, \mathrm{n})-1$, where $6(\mathrm{~F})$ denctes the maximum degree of $F$. On the other hand if $F$ is a ;iiniroal graph for which F-4 ( $\left.\mathrm{K}_{\mathrm{o}}, \mathrm{Kn}\right)$, then $\mathrm{S}^{\circ}(\mathrm{F}) \pm \mathrm{mn}$, here ( $F$ ) denotes the minimum degree of $F$. Ea $h$ of the

## Qbove re lts is sharp.

Ramsey Numbers of Families of 2-Complexes
-
R. A. Duke

Georgia Institute of Technology
In the last few years a great deal of work has been done in what has come to be known as generalized Ramsey theory for graphs. A long !ist of "generalized Ramsey numbers" have alrnady been computed. In the most recent of a series of papers on this subject co-authornd by Frank Harary the area was extended to include generalized Ramsey numbers for pairs of 2-dimensional simplicial 2-complexcs. In that paper Ramsey numbers for several small pure 2 -complexes, or plexes, wre computed.

In most of the cases for graphs the critical colorings are fou;-:d among a few rather simple canonical types. The critical colorings found so far for 2 -complexes often involve Steiner triple systems or other designs.

In this paper values are obtained for the Ramsey numbers ol certain classes of pairs drawn from the families of 2-cornulexes described in the paper mentioned above. The techniques used include the utilizatio:1 of know:i values of the Ramsey numbers for graphs as well as various resuits from the study of block designs.

Bell \(\begin{aligned} R. K. Chung<br>Laboratories\end{aligned}\)<br>Murray Hill, New Jersey

Let $G$ bee finite graph with vertex set
$\mathrm{V}=\mathrm{MV} N$. We say that $G 1$; rearrangeable if fo:- alJ. choices of distinct vertices i1,12,..., it in $X$ and
J1,J2, $\infty^{\infty}, j t$ in $N$ t:iere exist vertex disjoin_t_paths between $i_{k}$ and J1, for all k. For exa.,:iple, a corr.plete bipartite graph with the vertex sets Mand r. is.rearrangeal::le. However, this graph Will usually have many ore ecges than is necessary for rea::-rangeability.

We determine the minimal.nu."ber of egges any rearrangeable graph may have for all choices of Mand N . We also discuss generalizations in which Vis strictly great $r$ than $M L$; $N$ anci/or $t$ is bounde:i by a predej;e=Leci value.

## ONSUDORAHMDER NDPELUIOE Di nuFS

by

rinite mophs $F$ and $C$ let $\operatorname{BF}(C)$ denote ne

 graphs, 1t 1s n\&u\& to eik them whather ar no the 'I $\$$-1titles F (C). FcJ, ae linearly independent hen G $1 \$$ restr!.ctet! to く For exom;le, 11"; • ( $\mathrm{K}_{1}$, KJ (-here $K$, dentes the compte gqph on ; verticeG) and $G$ is the ta-ily $o$-all (finite) filE then of course $N_{E}(T)-N \quad$ (T) $a_{1}$ tor all T<C. Slightly lcos tr1v1'lly, $1 \mathrm{f}:$; - ( $\mathrm{Sn}: \mathrm{n} 1,2,3, \infty$ ) (wher $S_{n}$ danotes the an n edges) and C 111ginls the family or ell trees then

$$
\mathrm{f}:(-i)^{\mathrm{n}+1} \mathrm{Ns}_{\mathrm{n}}(\mathrm{~T}\} \cdot 1 \text { tor all } \mathrm{T}\{(\mathrm{i} \text {. }
$$

It will be proved that such a linear deper.c ce can . ocar :.t d is t'lr.1te, io Fei hes an isolated point and G cont1ns all trees. Thie result hes Ic, >orar.t applic4t1ons 1 n recent voric O : L. to 4 sZ ard one of the authors.
-ABSTRACT. Althouch cethods exist in the literature for the cou::ierat:!.on of connected linear graphs by partition, these generally involve group theoretic expressions which do not lend thc.t:1selves readily 0 coi::put.i.tional techniques. The authors obtain recursive 11:ethods which are amenable to computation Ly v..rlous formula manipulation techniques.

# FINDING SIMPLE PATHS IN A GRAPH BY symbolic manipulation of boolean equations; 

## R. B. Worrell and B. L. Hulme

Sandia Laboratories
Albuquerque, New Mexico

Simple paths in a graph can be determined by the symbolic manipulation of Boolean equations. In this procedure a graph \{directed or undirected) is represented by a set of Boolean equaticris--one equation for each node. By a process of substitution, an equation is generated hich represents all ti:e simple paths from one node to another. Application of the distributive law to this equation then delivers a disjunctive normal form . wherein each term is one of the si ple paths. The Set Equation Transformation System (SETS), a software package for the symbolic manipulation of any Boolean equations, has been used to carry cut .this prccedure for se eral example graphs.

This work was supported by the United States Atomic Energy Cor.unission. $\qquad$ - A LATI'ICE ALGEBRA FOR FINDING

SD'.PIE PATHS AND CUTS IN A GRA.t-'"'H
Bernie L. Hu.line
Sandia Laboratories, Albuquerc;.ue,.N. M. 871.15
Ari algorithm for finding sLnple paths in a directed or undirected graph has been given by Fratta and Montanari and a similar algorithm for cuts has been presented by Martelli. Each of these authors has developed a different "regular algebrar, appropriate for the proble:n. In this paper we present an algebra, a.1.wost identical. to Martel.li's, in which both the si.I:lple path and cut algorithms are valid. We show that this algebra is a self-dual distributive lattice. Tnus, we unify the treatment of the simple po.ths and cuts within a. single algebraic structure and find tt.a.t they a:l'e naturally related by the concept of duality.
*nds work was suppoz:ted by the United...sts.t. s Atomic Energy Co:mnission

## -D.N. Jackson and J.W. Reilly

## University of \.,aterloo, Canada

This paper gives the exponential counting series for the nut: ber of homeomorphically irreducible labelled gra phs, with and witho:,t cultip:e loops and edges. The series obtained bears a i.trong resembla;ice to t!1e series given by Gilbert for the nur.:ber of gra;:,hs on a specified nlinber of vertices and edges. The derivation involves the application of the Principle of Inclusion and Exclusion together with a decoi7.position 0 : ::he configur.:.tions involved into multiple-loops and ultiple-edges with edge subdivision. A linear recurrence equation is obtained which pe ts the number of homeo::ior;,hically irreducible simple labelled graph.:; on $n$ vertices to be computed in $O\left(n^{2}\right)$. Tabulati $n$ fo: this case a:id the corresponding connected case are obta.ined for 0 s n s 20;
(Paper submitted for publication elsc here.)
Complexity of the Vari:ible-Lcngth Encoding Problem

$$
a
$$

Norbert Cot

Department of Electrical Engineering Stanford University

A prefix code is given. It cor.sist:s of $n$ word,s w, with symbols dr:.wn from an :ilphat;.ct $a j$. $T c$, e:ich letter $a$ is associated a cost $a_{\mathbf{J}}$ and to each codeword ${ }^{w}{ }_{i}$ a probability $p_{i}{ }^{\prime}$ Then the cost $c_{1}$. of a word equals $3^{\text {aj }}$ and the cost of the code C is such that $e=r \underset{i}{p} .{ }_{l}{ }^{C_{l}}$ :

The problem is to find the optimal code (minimal cost)., Karp formul:ited this problem in terms of int.?ger pro ramr.iir.g, which 1.eads to a possibly non-polynomial solution. With the following condition, Pi= Pk' V1, k, some algorithms are known requiring $O(n \log n)$ steps. Using some structural properties of the probler.i, we recently showed how to only use $O(n)$ steps for a binary alphabet.

In this paper, using some additional theorcr.is, we s!-,ow how to extend the result to a general alphabet. We then analize tile i::iplications to the general problem.

Co:1PUTER DECOMPOSITION Cf DIFFERENCE SETS
R.J. Callens

University of Manitoba
A computer algorithm which tries to divide a differenc
set of cardinality $n$ with differences each occurring , ti::ies into two disjoint $s ® s e t s A$ and $B$ of cardinality $\mathbf{k}$ anc! $n-k$
such that the differences of $\mathbf{A}$ and the differences of $B$ •
together s m to $), / 2$, will be preserrted.

# . AUTOMATIC GENERATION or KEYWORD DF;TECTION SOFTWARE 

$\xrightarrow{4}, \because, I$

## R.J. Collens <br> and <br> P.H. Dirksen

University of Manitoba
Winnipeg, Canada-
University of Waterloo

The construction of programs for the detection of
keywords is convnon to many computer applications. We describe -here a system for the automatic generation of such programs. The resulting keyword detection programs are both reliable and efficient.
(fI Minimum 'l'um Euler Circuits Frank 0. Hadlock, Florida Atlantic University

Given a block pattern which has been augmented to ltave only even vertices, a simple algorithm .i.s given which produces a minimum turn Euler circuit. An under lying assumption is that the augmented pattern has no edges of multiplicity> 2. With this assumption, a pattern decomposition is defined for which the number of components increases the number of turns in the rninir:mm turn circuit. .A branch and bound matching algorit is given for matching odd vertices so as to minimize not only the cost of matching them but also the number of components in the augmented pattern.

This research is being carried out under a grant fro:.. the FAU-FIU Joint Center for Environmental and Urban Problems.

## A CONSTRUCTION FOR BIBDs, RBIBDs

## AND 3-DESIGNS

S.A.LONZ

UNIVERSITY OF WATERLOO

## S.A. VANSTONE

ST. JEROME'S COLLEGE, UNIVERSITY OF WATERLOO

- A construction is given for balanced incomplete block desigr.s (BIBD) resolvable designs (RBIBDS) and 3-designs which makes use of substituting disjoint equicardinal sets from cne design for varieties of a second d sign.

Magic Pair-Wisc Balanced Designs
; '. .r. Sr-1-*
U.R.S. Murty
$\because \quad ;$ University of Waterloo

A pair-wise balanced design is said to be k-magic
if positive integer weights can be attached to its points
so that the sum of weights. of.points on any line is equal
to .k. A magic design is one which is $k$-magic for some $\mathbf{k}$.
Some constructions of such designs are prese ted. The problem of characterizing magic designs which are $£ 1: 1 b e d d a b l e ~ i n ~ t h e ~$ real plane remains unsettled,
iinimizing Weighted conflicts
by Paul G. Kainen

Case Western Reserve University
Cleveland, Ohio 4h1o6

If, in the ex5m scheduling proble;n, the nun:ber of exam per!oids is. fixe<; one tries to min mize the sum of the weighted conflicts. We examine this situation from two points of view, (1) Every weighted adjacency grap; is induced by a bipartite graph and we seek a minimal such bigraph. In terms of the example, what is the .ninimum number of students needed -o produce the weighted adjacencies? (2) If all weights are unity and there are onzy two colors available, how many conflicts can there ce:
R. Haggkvist and C. Thomassen
*)
University of Wcrterloo -
On Cycles in Di graphs
We determine tne minimum degree which guarantees a
strong digraph to contain cycles of all lengths and the
minimum number of edges guaranteing a hamiltonian digraph to contain cycles of all lengths. As applications of these results we determine for each $k$ the minimum number of edges required t.o insure that a digraph includes a cycle of length
k . The corresponding problem for undirected graphs has not
\#35 HAS BEEN CANCELED been solved completely for $k$ even.
R
A NEW BLOCK DESIGN
W. H. Mills

Institute for Defense Analyses Princeton, New Jersey 08540

A balanced incomplete block design with
$\mathrm{V}=106, \mathrm{~b}=371, \mathrm{r}=21, \mathrm{k}=6, \quad=1$ was found
by computer. This is apparently_the first BIBD
ever constructed with $=1$ and. $v 1-0,1(\bmod k)$,
although R. M. Wilson has shown that such designs
exist if $v$ is sufficiently large.

