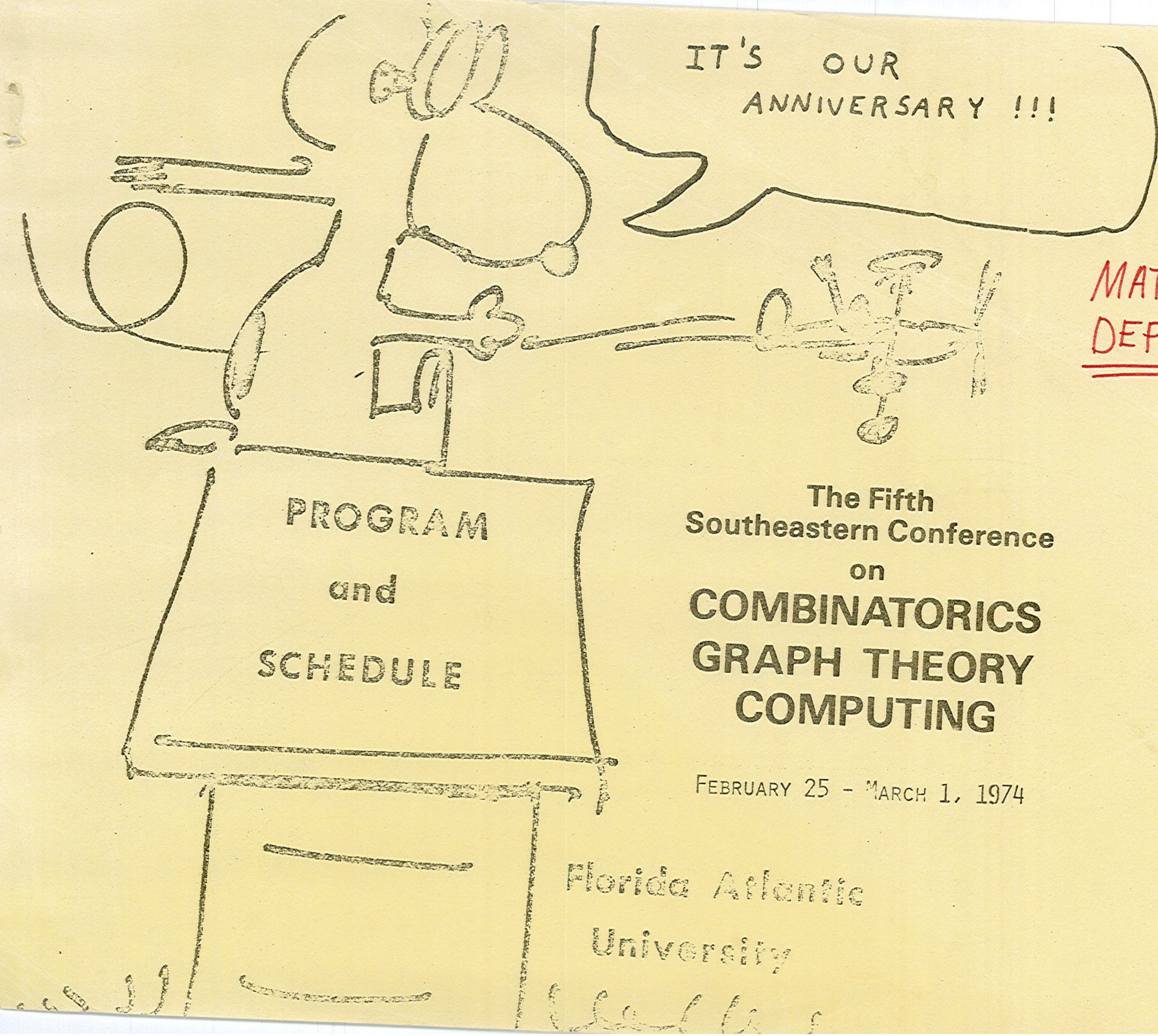

MATH DEPT. COPY

The Fifth Southeastern Conference on COMBINATORICS GRAPH THEORY COMPUTING

FEBRUARY 25 - MARCH 1, 1974

Florida Atlantic University

MONDAY 6-7:30 Cocktails at the Hoffman's, 4307 NW 5th Avenue
TUESDAY 7:30 Banquet----Sheraton---Cocktails Available 6:30
THURS 6:00 Beer Party at Freeman's, 741 Azalea

	MON		TUES		WED		THURS		FRI	
	207	Gold Coast Room	207	GCR	207	GCR	207	GCR	207	GCR
8:30	Registration (from 8 AM)		E.F.Ecklund[8]	L.B.Richmond[10]	A.B.Cruse[21]	F.O. Hadlock[23]	C.R. Cook[29]	S. Vanstone[31]	J.L. Lassez[45]	F. Hoffman[48]
9:00		Opening Pres. Creech	R. Alter[9]	C.R. Cook[11]	R.B. Killgrove[22]	F. Jaeger[24]	F. Escalante[30]	K.B. Reid[32]	C.E. Haff[46]	B.A. Anderson[49]
9:30									P. Morris[50]	
10:00		D.H. Lehmer		E. Lehmer		J.J. Seidel		J.M.S. Simões-Pereira	B.L. Hartnell[47]	N.S. Mendelson
10:30									M. Chein[51]	S. Lin[52]
11:00									G. Chaty[53]	R.C. Mullin[54]
11:30		D.G. Corneil		J. Doyen		C. Berge		P. Erdös	J.H. Ahrens[55]	R.J. Collens[57]
12:00									A. Bouchet[56]	R.A. Kingsley[58]
12:30										
1:00										
1:30		D. H. Lehmer	M.O. Albertson[12]	V. Pless	H.J. Ferch[25]	J.J. Seidel	J.P. Hutchinson[33]	J.M.S. Simões-Pereira		
2:00	J. Freeman		J.L. Jolivet[13]		B.D. Beach[27]		A. Rosa[35]			
2:30	J. Spencer[1]		G.J. Simmons[14]		E. DuCasse[26]		R.M. Wilson[36]	S.T.Hedetniemi[34]		
3:00	L. Shader[2]		A.H. Baartmans[15]		E. Sachar[28]	C. Berge	J.S. Wallis[39]	P.J.Slater[37]		
3:30	M. Doob[3]	D.G. Corneil	F. Burkowski[17]	J. Doyen			C.C. Wang[40]	R.B.Levow[38]		
4:00	B. Alspach[4]	L. O. James[5]	E.S. Kramer[16]	R.S.D. Thomas[18]			J.W. Di Paola[43]	P. Kainen[41]		
4:30	S.E. Goodman[7]	H.C. Williams[6]	W.H. Mills[20]	D.S. Johnson[19]			S. Kundu[44]	K.P.Bogart[42]		

TITLES AND ABSTRACTS OF INSTRUCTIONAL LECTURES

DR. C. BERGE, University of Paris
Hypergraph Theory

DR. D. G. CORNEIL, University of Toronto
Computational Complexity

DR. J. DOYEN, University of Brussels
Precise title to be announced

DR. P. ERDŐS
Probability Methods Applied to Combinatorial Problems

DR. D. H. LEHMER, University of California
Computation in Combinatorics

DR. E. LEHMER, University of California
An Outcropping of Combinatorics in Number Theory

DR. N. S. MENDELSOHN, University of Manitoba
The Golden Ratio and van der Waerden's Theorem

DR. J.M.S. SIMOES-PEREIRA, University of Coimbra
Precise title to be announced

DR. J. J. SEIDEL, University of Eindhoven
A Survey of Two-Graphs

Self-Dual Codes

by Vera Pless - Project MAC, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139.

If F is a finite field with q elements, let F^n denote the direct product of F with itself n times regarded as a vector space as usual. An (n,k) linear error-correcting code over GF(q) is a k-dimensional subspace of F^n. For even n, an (n, n/2) code C is self-dual if C is self-orthogonal with respect to the usual inner product. Self-dual codes are important both practically and mathematically for the following reasons. First, many of the best algebraic codes (for example, the Golay codes) are self-dual. Second, it is often possible to compute the weight distributions of self-dual codes since they must satisfy Gleason polynomials. Third, many t-designs, including rare 5-designs, have been found in abundance in self-dual codes. The group of a code is the group of monomial transformations leaving it invariant. Fourth, many self-dual codes have interesting groups; the only known 5-transitive groups are groups of self-dual codes. Also the Golay codes are known to be intimately related to the new Conway simple group CO_1. Fifth, it has recently been shown that "good" self-dual codes exist over any GF(q) as n gets large. Even more recently, all self-dual codes over GF(2) for lengths up till 24 have been completely classified.

GOOD k-COLORINGS OF THE VERTICES OF A HYPERGRAPH.

Claude BERGE
University of Paris

Let $H = (E_i / i \in I)$ be a hypergraph with vertex-set X. A good k-coloring is partition of X into k classes such that, for all E_i, the number of classes which meet E_i is equal to $\min\{|E_i|, k\}$.

Clearly, for k=2, this is the "bicoloring" of Erdős and Hajnal; for $k \geq \max|E_i|$, this concept reduces to the "strong k-coloring" of H; for $k \leq \min|E_i|$ this concept reduces to the k-partition of X into transversals. The "equitable k-coloring" introduced by the author to study totally unimodular matrices is also a good k-coloring.

First, we shall show that a balanced hypergraph has a good k-coloring for all k. (In particular, a bipartite multigraph has a good k-coloring of its edges for all k). We study the values of k for which a product of balanced hypergraphs has a good k-coloring. We give some sufficient conditions for a graph G to have a good k-coloring of its edges for all k.

In a previous paper, we proved that the complete h-partite hypergraph has a good k-coloring of its edges for all $k \geq$ the maximum degree or $\leq$ the minimum degree. J.C. Meyer has extended this result by showing that for all k, there exists a good k-coloring of the edges.

1 A NEW LOWER BOUND FOR RAMSEY NUMBERS

Joel Spencer
MIT

The lower bound for the Ramsey numbers $R(k,k)$ is of the order $2^{k/2}$, due to Paul Erdős. We use a new method of Lovász to improve the lower bound by a factor of 2.

2 Monochromatic Configurations and 2-Colorings of the Plane

Leslie E. Shader
University of Wyoming

In "Euclidean Ramsey Theorems" Journal of Combinatorial Theory (A) Vol 14, pp. 341-363 (1973) Erdős, Graham, Montgomerey, Rothschild, Spencer and Strauss, consider 2 colorings of the Euclidean plane and attempt to determine if certain configurations of points have a monochromatic copy for every 2-coloring. For such a configuration A with this property we say $R(A,2,2) \equiv R(A)$ is true, otherwise $R(A)$ is false. Erdős et al, show that $R(T_d)$ is false for T_d, the equilateral triangle of side d, while $R(B)$ is true for B any 30°-60°-90° triangle. Moreover, for all other triangles C it is not known, whether $R(C)$ is true or false. In this paper, it is shown that for every right triangle D with sides $2d$ and $3d$, $R(D)$ is true.

3 ON THE ALGEBRAIC STRUCTURE OF MAGIC GRAPHS

Michael Doob
University of Manitoba

A magic graph has been defined by J. Sedlacek to be a graph with a real valued edge labeling such that distinct edges have distinct labels and the sum of the labels on the edges incident to one vertex is the same for all vertices. We shall look at a more general situation; namely, we shall assume that the edges are to be labeled with elements from an abelian group, and that the sum of the labels on the edges incident to a vertex v is to be a prescribed value $r(v)$ that may differ for different vertices. We shall see that we may construct all possible labelings, and to do this we need only know the structure of the circuits of the graph and that of the elementary 2-subgroup. Upon application to integral domains, the circuit properties will lead us to certain matroids which in turn will give a dimensional structure to the labelings. Upon application to the reals, we shall characterize certain classes of these graphs including zero magic, semi-magic, and trivial magic graphs. We shall see that the constructive methods employed can lead to a translation of some problems concerning magic graphs making them more amenable to solution.

4 On generating the graphs with a given permutation group as their automorphism group

Brian Alspach
Simon Frasier University

Abstract

Consider S_n, the symmetric group of degree n, for a fixed value of n. Let $\mathcal{G}_n$ denote a set of representatives of the conjugacy classes among all subgroups of S_n. We discuss an algorithm for determining the graphs of order n having their automorphism group equal to G as G runs through $\mathcal{G}_n$.

5 MAPLE

L. O. James* and C. R. Zarnke
University of Manitoba

A language called MAPLE, intended for developing combinatorial program packages is described. APL handles problems in matrix manipulation very effectively. MAPLE provides a core language similar to APL; it also has facilities for extension by the definition of new operators and data types. This permits the writing of packages capable of handling other mathematical objects in a natural fashion. Examples are taken from such an extension that permits manipulation of graphs..

6

Some Algorithms for Solving a Cubic Equation Modulo p

by H. C. Williams, C. R. Zarnke
University of Manitoba

ABSTRACT

Recently two different algorithms for solving

$$(1) \qquad x^2 \equiv a \pmod{p} \quad ,$$

where p is a large prime, have been given by D. H. Lehmer and Daniel Shanks. Both of these methods require approximately $\log p$ operations in order to solve (1). In this paper it is shown how these algorithms may be extended in order to solve

$$(2) \qquad x^3 + ax^2 + bx + c \equiv 0 \pmod{p} \quad .$$

These extended algorithms require $0(\log p)$ operations to solve (2). The relative speed of both algorithms is discussed.

7

ON THE OPTIONAL HAMILTONIAN COMPLETION PROBLEM

P. J. Slater
Cleveland State University
Cleveland, Ohio

and

S. E. Goodman *
S. T. Hedetniemi
University of Virginia
Charlottesville, Virginia

The Optional Hamiltonian Completion Problem can be stated as follows: let the points V of a graph G be partitioned into a set V_0 of optional points and a set V_1 of non-optional points; determine the minimum number of new lines which when added to G result in a graph which has a cycle containing every point of V_1. The (non-optional) Hamiltonian Completion Problem which has been studied recently by several authors assumes that the set V_0 of optional points is empty. In this paper effective algorithms are presented for solving the Optional Completion Problem for trees and unicyclic graphs. It is then shown how to use these two algorithms to solve the non-optional Hamiltonian Completion Problem for cacti.

Consecutive k-th power Residues and Non-residues

8

by E. F. Ecklund, Jr.,

University of Manitoba

A. Brauer has shown that for positive integers k and m, all sufficiently large primes possess a run of m consecutive k-th power residues modulo p. A prime which fails to have m consecutive k-th power residues is called exceptional. For the non-exceptional primes, p, let $r = r(k,m,p)$ denote the first occurence of m k-th power residues modulo p: $r, r+1, \ldots, r+m-1$. Set

$$\Lambda(k,m) = \text{supremum}\{r(k,m,p) \mid p \text{ non-exceptional}\}$$

$\Lambda(k,m)$ is finite when there is an upper bound for the first occurence of m consecutive k-th power residues modulo p which is independent of p.

In this paper, we investigate two related functions, $\Lambda^*(k,m)$ and $\Lambda'(k,m)$, which weaken the consecutive residue condition of $\Lambda(k,m)$ in the following ways:

$\Lambda^*(k,m)$ represents an upper bound on the first occurence of m consecutive k-th power residues or m consecutive non-residues all belonging to the same coset with respect to some k-th power character.

$\Lambda'(k,m)$ represents an upper bound on the first occurence of m consecutive k-th power residues, or of m consecutive k-th power non-residues.

The computation of $\Lambda^*(k,3)$ and $\Lambda'(k,3)$ is reported.

REMARKS AND RESULTS ON PALINDROMIC NUMBERS

9

R. Alter* and T. B. Curtz

University of Kentucky

A palindrome is a number which has the same sequence of digits reading from the left as from the right. Let N' be the integer obtained by writing the digits of the integer N in reverse order, and let $N + N' = S_1$, $S_1 + S_1' = S_2, \ldots, S_{k-1} + S'_{k-1} = S_k$. If there exists an integer k for which S_k is a palindrome then the integer N is said to be palindromic. It has been conjectured that all integers are palindromic. Although this conjecture has not been settled in the decimal system it is false in the binary system. (The number 10110 base 2 is nonpalindromic). With the aid of a computer some new results on this conjecture have been established.

10

THE ASYMPTOTIC COMPUTATION OF THE AVERAGE ACTIVITY OF A TREE IN A ROOTED MAP

R. C. Mullin and L. B. Richmond*

U. of Waterloo U. of Manitoba

Using Tutte's definition of internal activity, an asymptotic relation for the average value of this quantity for a tree in a rooted map with n edges is obtained. An asymptotic relation for the number of twin-tree-rooted maps with n edges whose root bond contains $k+1$ edges is also obtained. The method used is to determine the asymptotic behaviour of the generating function near its singularity and then to apply a Tauberian result of Hardy-Littlewood to determine the asymptotic behaviour of the Taylor coefficients. We conclude with a numerical determination of the constants arising in several of the asymptotic results obtained.

11

REPRESENTATIONS OF GRAPHS BY N-TUPLES

Curtis R. Cook

Oregon State University

An n-tuple graph has a set of distinct ordered n-tuples of non-negative integers as its point set. Two n-tuples are adjacent if they agree on one or more positive-valued coordinates. A plane graph is the special case of the n-tuple graph when the points are ordered n-tuples of positive integers. Every graph is n-tuple and plane graph representable. Intersection graphs are related to n-tuple graphs whose points are binary n-tuples.

The dimension and height are two parameters that measure the "minimalness" of the n-tuple and plane graph representations. Various upper and lower bounds for the dimension of n-tuple and plane graph representations of a graph G are related to the chromatic number of the line and clique graphs of G, to the maximum degree of a point in G and to the largest induced $K_{1,n}$ in G. The height of an n-tuple representation is bounded above by two and the height of a plane graph representation is bounded below by $\beta_0(G)$.

12 UNAVOIDABLE SETS IN TRIANGULATIONS

Michael O. Albertson
Smith College

An unavoidable set in a triangulation is a list of subgraphs some one of which must appear in every triangulation. A new unavoidable set is demonstrated. Upon deletion of one subgraph from the list, it is shown that the list is no longer unavoidable and a smallest triangulation that has no member of the revised list is presented. Various other properties of triangulations are discussed.

13 Pseudo-hamiltonian cycles in simple graphs.

J. L. Jolivet
Centre Universitaire Du Mans

Let G be a simple connected graph of vertex-set X and edge-set E, with $|X| = n$. A pseudo-hamiltonian cycle of G is a sequence $C = (x_{i_1}, x_{i_2}, \dots, x_{i_{n'}})$ of vertices such that :

1) Every vertex x of X belongs to C.

2) For all $k = 1, \dots, n'-1$, $[x_{i_k}, x_{i_{k+1}}] \in E$ and $[x_{i_{n'}}, x_{i_1}] \in E$

We shall denote $s(G) + n$ the minimum cardinality of such a sequence and so :

* If G is hamiltonian $s(G) = 0$

* If G is a chain $s(G) = n - 2$

This index "mesure", in a certain way, how a graph is not hamiltonian. Several results on s (G) will be given, which generalise the corresponding theorems in hamilton graph theory ; in particular the well known theorem of Dirac will be extend as follow :

Theorem : If G is a simple connected graph with a minimum degree $d \geq \frac{n-p}{2}$, where p is an integer such that $0 \leq p \leq n - 2$; then $s(G) \leq p$.

14 Synch-sets: A Variant of Difference Sets

Gustavus J. Simmons
Sandia Laboratories

This paper investigates the problem of designed optical encoding discs such that the difference between the in-synch signal and the maximum value over all of the out-of-synch signals is maximized. In abstraction of this problem, an $S(k,\lambda)$ synch-set is defined to be a set of k non-negative integers for which not more than λ pairs have a common difference. Clearly, an (n,k,λ) difference set is also an $S(k',\lambda)$ synch-set where $k = k'$, however, for a specified largest element it is generally possible to achieve $k' > k$ or equivalently the largest element in a synch-set for a fixed k can be less than for the corresponding difference set. For example, for $k = 10$ the least upper bound for a difference set is 81 for the (91,10,1) set while (0,1,6,10,23,26,34,41,53,55) is a S(10,1) set which realizes an upper bound of 55.

Best possible constructions are given for most $k \leq 10$.

15

DESIGNS ADMITTING A REGULAR AUTOMORPHISM GROUP

A. H. Baartmans
Southern Illinois University

Let D be a design with parameters (v,b,r,k,λ). A design D is said to be regular of degree n if there exists an automorphism group G of D of order n such that no nonidentity element of G fixes any point or any block of D. A generalized (n,r,λ) difference set in a collection of sets $D = \{D_1,\dots,D_t\}$ where each $D_i = \{a_{i_1}, a_{i_2},\dots,a_{i_{r_i}}\}$ is a set of r_i residues mod n such that $\sum_{i=1}^{t} r_i = r$ and for each $x \not\equiv 0 \pmod n$, there are exactly λ pairs a_{i_m}, a_{i_p} $1 \le i \le s$ such that $a_{i_m} - a_{i_p} \equiv x \pmod n$.

<u>Theorem 1.</u> Let $D = (P,B,I)$ be a design with parameters (v,b,r,k,λ). Let n divide (v,b) and Let D be regular of degree n with group G. Let G have s point orbits and t block orbits. Let a_{ij} be the number of points of point orbit P_i incident with a given block in the j^{th} block orbit, then (i) $\sum_{j=1}^{t} a_{ij} = r$. (ii) $\sum_{i=1}^{s} a_{ij} = k$, (iii) $\sum_{j=1}^{t} a_{ij}(a_{ij} - 1) = \lambda(n-1)$, (iv) $\sum_{j=1}^{t} a_{ij}a_{kj} = \lambda n$.

<u>Theorem 2:</u> A design D with parameters (v,b,r,k,λ) is regular of degree n with cyclic group G iff

(i) There exists a matrix $A = (a_{ij})s{\times}t$ such that the a_{ij} satisfy the conditions of Theorem 1.

(ii) There exist (n,r,λ) generalized difference sets $D_1,\dots,D_s$ with $D_i = \{D_{i1}, D_{i2},\dots,D_{it}\}$ satisfying

(a) $|D_{ij}| = A_{ij}$

(b) $\bigcup_{j=1}^{t} (D_{ij} - D_{kj}) = \lambda Z n$

(c) n is the smallest positive integer such that $(D_{1j} + n, D_{2j} + n,\dots,D_{sj} + n) = (D_{1j}, D_{2j},\dots,D_{sj})$.

16

INDECOMPOSABLE TRIPLE SYSTEMS

Earl S. Kramer

University of Nebraska

A t-design $(\lambda;t,d,n)$ is a system $\mathfrak{B}$ of sets of size d from an n-set S, such that each t subset of S is contained in exactly λ elements of $\mathfrak{B}$. A t-design is indecomposable (written $IND(\lambda;t,d,n)$) if there does not exist a subset $\mathfrak{B}' \subseteq \mathfrak{B}$ such that $\mathfrak{B}'$ is a $(\lambda';t,d,n)$ for some λ', $1 < \lambda' < \lambda$. A triple system is a $(\lambda;2,3,n)$. Recursive and constructive methods (several due to Hanani) are employed to show that: (1) an $IND(2;2,3,n)$ exists for $n \equiv 0,1 \pmod 3$, $n > 4$ and $n \neq 7$ (designs of Bhattacharya are used here); (2) an $IND(3;2,3,n)$ exists for n odd, $n \geq 5$; (3) if an $IND(\lambda;2,3,n)$ exists, n odd, then there exist an infinite number of indecomposable triple systems with that λ.

Several problems and conjectures will be discussed.

17

Some New Computational Results Involving Davenport-Schinzel Sequences

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Department of Computer Science

University of Manitoba

The paper presents:

a) Some new DS sequences

b) A report on the computation and analysis of DS sequences subject to a regularity condition.

There will be some discussion pertaining to the optimization of the computer program involved.

18

GARSIDE'S BRAID-CONJUGACY SOLUTION IMPLEMENTED

R.S.D. Thomas and B.T. Paley
University of Manitoba

ABSTRACT. While the determination of conjugacy in the three-strand braid group is easy by a method known for fifty years, in the groups or more than three strands it is not. The latter problem was solved in principle by F.A. Garside ["The braid group and other groups", <u>Quart. J. Math.</u> Oxford (2), 20(1969), 235-254.]. Garside showed that the process he defined led in a finite number of steps to a standard set of representatives for a braid's conjugacy class. The number of steps is so large, however, that without the use of an electronic computer even rather imple examples are out of the question. This paper outlines the authors' changes in the process made in order to render it viable in a real computer. The paper concludes with an example of the resultant algorithm's use.

A motive for considering braid conjugacy is that such conjugacy is a sufficient condition for the equivalence of closed braids in combinatorial topology. The solution discussed here depends on finding routes through a graph related to Cayley diagrams. The whole is made possible by taking advantage of LISP, the recursive list-processing computer language. Combinatorial topology done by computing on graphs.

19

WORST CASE BEHAVIOR OF GRAPH COLORING ALGORITHMS

by

David S. Johnson
Bell Laboratories
Murray Hill, New Jersey

There are many applications in which it is desirable to construct graph colorings using as few colors as possible. Due to the complexity of the task of finding the chromatic number $X(G)$ of a graph G, many algorithms have been proposed which, though not guaranteed to use the <u>minimal</u> number of colors, do construct their colorings <u>quickly</u>, and, for randomly generated graphs of moderate size, appear to use <u>close</u> to the minimal number of colors. In this report we <u>examine</u> the prominent algorithms of this type, and show that, in fact, they can behave quite poorly. Let $A(G)$ be the number of colors used by a given algorithm on graph G. We show that the ratio $A(G)/X(G)$ is unbounded for the algorithms of Welsh and Powell, Wood, and Matula, including the SLI algorithm which <u>is</u> guaranteed to use no more than 5 colors if the graph <u>is</u> planar. In the course of our constructions we obtain upper bounds on the size of the smallest graph for which $A(G)/X(G) = n$. For the SLI algorithm we need no more than $O((3n)^{6n})$ nodes, whereas for the rest of the given algorithms we need no more than $O(125^n)$, and, in one case, no more than $O(n)$.

20

ON THE COVERING OF TRIPLES BY QUADRUPLES

W. H. Mills

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Let S be a finite set of n elements. Let $C(n,4,3)$ denote the minimum number of quadruples such that every triple of elements of S is contained in at least one of them. By a result of Schönheim we have

$$C(n,4,3) \geq \left\lceil \frac{n}{4} \left\lceil \frac{n-1}{3} \left\lceil \frac{n-2}{2} \right\rceil \right\rceil \right\rceil ,$$

where $\lceil x \rceil$ denotes the smallest integer that is at least x. For $n \not\equiv 7 \pmod{12}$ we show that $C(n,4,3)$ is actually equal to this lower bound. For $n \equiv 2,3,4,5 \pmod 6$ the result is already known and is due to Hanani and Schönheim. For $n = 6$ it is almost trivial. For $n = 13, 25,$ and 30 constructions were obtained by computer. For the other values of n, $n \equiv 0 \pmod 6$ and $n \equiv 1 \pmod{12}$, the result can be proved by induction. For $n \equiv 7 \pmod{12}$, $n > 7$, the value of $C(n,4,3)$ is unknown.

ON EXTENDING INCOMPLETE LATIN RECTANGLES

21

Allan B. Cruse
University of San Francisco

An *incomplete latin rectangle* of type (r,s,t) is a rectangular array of r rows and s columns in which a subset of the rs places are occupied by integers from the set $1, 2, \ldots, t$, and in which no integer occurs more than once in any row or column. Such a design may be uniquely represented by a (0,1)-matrix of size $r \times s \times t$ (displayed in three-dimensions). We obtain necessary and sufficient conditions under which such a matrix may be extended (by adjoining additional "faces") to an $n \times n \times n$ permutation cube, thus providing an embedding of the incomplete latin rectangle in a complete latin square of order n. Our conditions generalize a well-known combinatorial criterion on extendibility of latin rectangles due to H. J. Ryser (Proc. A.M.S. 2 (1951), 550-552). Our proof employs a classic theorem of D. Konig on sums of permutation matrices, and exploits properties of certain convex polytopes.

Subsquares Complete Latin Squares of Order 12

22

R. B. Killgrove*, San Diego State University
and
Ed Milne, Naval Postgraduate School, Monterey

If a finite projective plane has some quadrangle from which every point and line can be reached by extensions of joining and intersection, then we say the plane is singly generated. (See Killgrove, "Completions of Quadrangles in Projective Planes", Can. J. Math., Vol. 16 p. 63). If there is no such quadrangle, we say the plane is nonsingly generated. In the latter case the Latin squares of the digraph complete representation are decomposable into subsquares; such Latin squares are called subsquare complete. (See Hiner, Notices of AMS Vol. 17, p. 758).

A computer program has been written to find low order subsquare complete Latin squares. The input consists of the possible first three rows of such a square. The subsquare complete Latin squares containing subsquares of order 2 and subsquares of order 3 are of interest for projective planes of orders 9, 12, and higher. The order 9 case was resolved in the paper by Killgrove mentioned above. Order 12 has four inputs, two yielding no square, one square, new at this time, the remaining yielding the group table for A_4, and yielding the one due to Hiner. (Same reference.)

Minimum Forests of Bounded Trees

23

Frank O. Hadlock
Department of Mathematics
Florida Atlantic University
Boca Raton, Florida

Given a vertex weighted graph and an upper bound on the weight of trees, the problem dealt with is that of finding a minimum spanning forest of bounded trees. Polynomial bounded algorithms are presented for special cases and a branch-and-bound algorithm is given for the general case.

ON VERTEX-INDUCED FORESTS IN CUBIC GRAPHS

24

F. Jaeger
Université Scientifique et Médicale

All graphs will be finite, undirected, with no loops or multiple edges.

Let G be a connected cubic graph with n(G) vertices ; let s(G) be the maximum number of vertices in a vertex-induced forest of G.

We show that $s(G) \leq \left\lceil \frac{3n(G)-2}{4} \right\rceil$ (1)

We then study some cases for which equality holds in (1) :

If the vertices of G can be covered by two vertex-disjoint vertex-induced trees of G, then equality holds in (1).

Corollary : If G is the dual of an hamiltonian maximal planar graph G, then equality holds in (1), and the vertices of G^* can be covered by a connected cactus subgraph C of G^* such that :

$\left\lceil \frac{n(G^*)-1}{2} \right\rceil$ faces of G^* have their boundary in C.

We then present some conjectures related to the previous results.

IMPLEMENTATION OF INFORMATION RETRIEVAL SCHEMES

25

H. J. Ferch
University of Manitoba

A language for use in implementing experimental information retrieval systems is described. The facilities of the language include the ability to create user-defined data structures and operations. Emphasis is placed on the use of the language in the implementation of various combinatorial-based searching schemes.

An Enumeration of a Class of Multiple-Valued Functions

26

by

Edgar DuCasse
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Brooklyn College, C.U.N.Y.

This paper presents an application of combinatorics to computing. A large part of the appeal of multi-valued devices in digital systems theory lies in their ability to realize a relatively large number of functions compared to binary logic gates. There are m^{m^n} m-ary functions of n variables, so that there is only the comparitively small number of 2^{2^n} n-input binary functions. However, if a logic designer is interested in systems which emit no temporary false signal values, many of the m^{m^n} functions available to him are useless when m>2[1,2]. This is because any multiple-valued function which assumes values f(x) and f(y) such that $|f(x)-f(y)|\geq 2$ for some pair of arguments x and y satisfying $|x-y|\leq 1$ must assume every value intermediate between f(x) and f(y) during its transition between these two values. This transition will produce a temporary false signal value at the output of the network, thus rendering the system useless for the designer's purpose. In this paper we enumerate the multi-valued functions useful to the logic designer and compare these results to the number of binary functions.

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STORAGE ORGANISATION FOR A TYPE OF SPARSE MATRIX

27

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One class of mechanical translators for programming languages, called LR-recognizers, uses a large matrix to store the description of the language being translated. This matrix is rather sparse (usually about 5% non-zero entries) because not all possible symbols may occur in any position in the language. Various storage mechanisms have been discussed in the literature, varying from storage of only the non-zero entries to decomposition of the matrix. These organisations are questionable either because access to entries is expensive (as in the first case) or because the resulting recognizer is extremely complex.

A storage organisation which combines ease of access with storage efficiency is described in this paper.

ROW SPANS OF PROJECTIVE PLANES

28

Howard Sachar
Lafayette College

This paper investigates the F_p span of the rows of an incidence matrix of a finite projective plane. We obtain results on the dimension and minimum weight (in the sense of algebraic coding theory) of both this span and its orthogonal.

29

ADJACENCY GRAPHS

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Curtis R. Cook
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ABSTRACT: Let A be an arbitrary (0,1) matrix. The points of the matrix graph of A are identified with the 1's of A and two points are adjacent whenever the corresponding 1's lie in the same row or column. In this paper we consider the adjacency graph, the matrix graph of the adjacency matrix of a graph G. Two characterizations of adjacency graphs are given; as a corollary of one of them we show that the adjacency graph is connected if and only if G is connected and contains an odd cycle. We also obtain bounds for various graph parameters, e.g. chromatic number, partition number, point and line independence numbers, point and line covering numbers, of adjacency graphs in terms of the maximum degree, line independence number and number of lines of G. We also give a homomorphism from the adjacency graph of G onto the line graph of G.

30

Forbidden Structures of Some Planar Intersection Graphs, by Fernando Escalante.

National University of Mexico

A characterization of the type described in the title is given for some intersection graphs such as total graphs, entire graphs, n-th power of graphs, interval graphs and proper interval graphs.

31

A Bound For $v_0(r, \lambda)$

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$v_0(r, \lambda)$ is the least positive integer such that if D is an (r, λ)-design defined on a set of varieties V and $|V| > v_0(r, \lambda)$, then D is a trivial design. In the paper, we establish that

$$v_0(r, \lambda) \le \max\{\lambda+2,\ n^2+n+1\}$$

where $n = r - \lambda$.

32

Arc Disjoint Paths in the Transitive Tournament

K. B. Reid, Louisiana State University

The transitive n-tournament is the tournament with n nodes and no cycle. A k-path in a tournament is a path of length k. Denote by $p(n,k)$ $(1 \le k \le n-1)$ the largest integer so that there exist $p(n,k)$ arc disjoint k-paths in the transitive n-tournament, each path starting with the transmitter. This paper determines the value of $p(n,k)$ for certain n and k, $1 \le k \le n-1$.

33 On Words with Prescribed Overlapping Subsequences

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A question which arises in the study of polymer sequences in biochemistry leads to the following combinatorial problem. Given a positive integer i; given $f_1, f_2, \ldots, f_q$, each a sequence of letters from an alphabet on n letters, each sequence of length at least i; how many words w are there such that

a) f_j is a subsequence of $w (j=1,2,\ldots,q)$

and b) either f_j and f_k are disjoint subsequences of w or they overlap in precisely i letters $(j, k, = 1, 2, \ldots, q, j \neq k)$? There are no such words unless certain consistency conditions are met, and when met, the number of words can be counted precisely. The answer is obtained by counting Eulerian paths on appropriate directed multigraphs.

34 INDEPENDENCE GRAPHS

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The independence graph I(G) of a graph G is defined to be the intersection graph on the independent sets of points in G. Independence graphs possess a number of interesting properties, among which are the following:

1) I(G) contains subgraphs isomorphic to the line graph $L(\bar{G})$ and clique graph $K(\bar{G})$ of the complement $\bar{G}$ of G;

2) two graphs G and H are isomorphic if and only if their independence graphs I(G) and I(H) are isomorphic;

3) the automorphism groups of G and I(G) are isomorphic; and

4) a rather surprising result concerning the chromatic number of a graph. A dominating set of a graph is a set of points S having the property that any point of G not is S is adjacent to at least one point in S. The independent dominating number i(G) of a graph G is the minimum number of points in a dominating set which is also an independent set of points. For any graph G, $\chi(G) = i(I(G))$.

A number of other results are also presented, including a generalization of 4) above and several necessary or sufficient conditions for I(G) to be Hamiltonian.

35 NEARLY KIRKMAN SYSTEMS

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Alexander Rosa, Department of Mathematics, McMaster University

It appears to be an advantage to use the graph-theoretical terminology. By a Δ-factor of the complete graph K_v we mean a factor of K_v whose each component is a triangle. Let N(v) be the maximum number of edge-disjoint Δ-factors of K_v. One has trivially $N(v) \leq [\frac{1}{2}(v-1)]$; if equality holds then any set of $[\frac{1}{2}(v-1)]$ edge-disjoint Δ-factors of K_v is called a nearly Kirkman system (NK-system) of order v. If $v \equiv 3 \pmod 6$ then an NK-system of order v is the same as a Kirkman system of order v and is known to exist for all such positive v.

If $v \equiv 0 \pmod 6$ one finds easily $N(6) = 1$ and also $N(12) = 4$ so that NK-systems of orders 6 and 12 do not exist. On the other hand, $N(18) = 8$ so that $v = 18$ is the smallest order for which an NK-system (which is not a Kirkman system) exists. An infinite class of NK-systems is given by the following theorem:

Theorem 1. An NK-system of order $v = 6n$ exists whenever n can be written as a product of two integers r and s such that $r \equiv 1 \pmod 3$, $r \geq 4$ and $s \equiv 1 \pmod 2$.

One can also prove

Theorem 2. If there exists an NK-system of order v then there exist an NK-system of order t.v where $t \equiv 3 \pmod 6$.

We conjecture that an NK-system exists for all orders $v = 6n$, $n \geq 3$.

36 A Few More Squares

Richard M. Wilson
The Ohio State University

Difference methods and finite fields can be used to increase the known lower bounds on the number of mutually orthogonal latin squares of order n for several values of n.

37 Irreducible Point Independence Numbers and Independence Graphs

Peter J. Slater
Cleveland State University

An independence graph H is a graph which can be obtained as the intersection graph on the collection of independent subsets of the point set of some graph G. After developing several concepts associated with a new parameter for a graph, the irreducible point independence number of G, denoted $\alpha^*(G)$, a characterization of independence graphs is given. Several results about independence graphs are given, including $\chi(G) = \alpha(H)$ and $\psi(G) = \alpha^*(H)$, where χ and ψ denote the chromatic and achromatic numbers, respectively.

38 Coloring Planar Graphs with Five or More Colors

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Let G be a plane graph bounded by a k-cycle with vertices $v_1, \cdots, v_k$. Let $n \geq 5$ be a given integer. Let G_1 be the section subgraph induced by $v_1, \cdots, v_k$, and let G_2 be the section subgraph induced by $v_1, \cdots, v_k$ together with any vertices adjacent to at least n of the vertices $v_1, \cdots, v_k$.

(i) If $k < 2n - 4$ then every n-coloration of G_2 can be extended to an n-coloration of G. In particular, every $n - 1$ coloration of G_1 can be extended to an n-coloration of G.

(ii) If $k < 3n - 9$ then every n-2 coloration of G_1 can be extended to an n-coloration of G.

(iii) Every n-3 coloration of G_1 can be extended to an n-coloration of G.

The bounds given in (i), (ii), and (iii) are best possible.

39 Orthogonal Designs

Jennifer Seberry Wallis
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An orthogonal design of type $[s_1, \ldots, s_\ell]$ on the variables $x_1, \ldots, x_\ell$ is a square orthogonal matrix with each variable x_i occurring s_i times in each row. We will discuss some necessary conditions for existence of such designs and show their relationship with recent work on Hadamard matrices.

40 Permutations with Fixed Index and Number of Inversions

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Let $a_1 a_2 \ldots a_n$ be a permutation of the set $\{1,2,\ldots,n\}$. The index of the permutation $a_1 a_2 \ldots a_n$ is defined as the sum of all subscripts j such that $a_j > a_{j+1}, 1 \leq j < n$. The number of inversions of the permutation $a_1 a_2 \ldots a_n$ is the number of pairs (a_i, a_j) such that $1 \leq i < j \leq n$ and $a_i > a_j$. Let $A_n(j)$, $0 \leq j \leq \binom{n}{2}$, be the set of permutations on $\{1,2,\ldots,n\}$ with both index and number of inversions being j. We show that $|A_n(j)| = K_j$ for $n \geq N_j$, where K_j and N_j are constants. For small values of j, the constants K_j and N_j are computed and given in the paper.

41 On (Unique)^2 Coloring

Paul Kainen

Case Western Reserve University

In this paper we investigate which surfaces S contain a unique graph G such that the chromatic number, k of G is maximal for all such graphs and such that G is uniquely k-colorable. For example, K_7 is the only uniquely 7-colorable graph on the torus.

42
The Number of Graphs of Edge Connectivity Exactly One

ABSTRACT

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In this paper the technique used by Doubilet, Stanley and Rota to compute a formula for the number of connected graphs is extended to a partial solution of the problem of finding a formula for the number of graphs with edge connectivity n or more. For n=2 the problem is solved completely by applying a computation due essentially to Cayley of the enumerator by degree sequence for labelled trees on a fixed vertex set, and subtraction gives the number named in the title of the paper. This computation provides a parallel to Riddell's computation of the number of blocks, or graphs of vertex connectivity 2 or more.

43
A Regularity Problem for Incidence Structures.

Jane W. DiPaola

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Let E be the set $\{1,2,3,\cdots,e\}$. Let $T^*(e)$ represent the cardinality of the largest family $\mathfrak{F}_m$ of equicardinal subsets of E such that for $X_i, X_j \in \mathfrak{F}_m$, $i \neq j$

$$\frac{|X_i| \cdot |X_j|}{|X_i \cap X_j|} = e$$

Evaluate $T^*(e)$ for fixed e. Partial results are known.

44 THE MINIMUM DIAMETER STEINER TREE IN A GRAPH

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ABSTRACT. Let G be a finite connected graph with n vertices and S, a specified subset of nodes in G. A minimum diameter Steiner tree is a tree subgraph of G which contains all specified nodes and has minimum diameter. An algorithm requiring $O(n^3)$ elementary operations is given for finding a minimum diameter Steiner tree.

The corresponding problem where the number of edges in the tree is minimized instead, is known as Steiner minimum weight tree problem. At present, it is strongly believed though that finding a minimum weight Steiner tree connecting an arbitrary node set S requires an exponentially (in n) large amount of computation.

45 Some combinatorial properties of trees.

J. L. LASSEZ

Purdue University and Universite de Paris VII

In a series of papers the author has investigated the relationship between prefix codes, automata and free submonoids, many results have a nice interpretation in terms of trees and their patterns. We present here several results concerning pseudo periodic trees, overlapping patterns of a tree, self overlapping trees, conjugate trees and combinatorial properties such as trees which are patterns of every of their subtrees. The proofs which use automata or monoid theory will appear elsewhere.

46 A Ranking Problem in Graphs

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An algorithm is presented for ranking the maximal forests of an edge-weighted graph G. The ranking is relative to any non-decreasing real function of the edge-weights of G. The algorithm has linear convergence rate in contrast with those for solving related combinatorial ranking problems, such as finding nth-best paths in a graph, which have exponential convergence rates. The theorems and algorithm apply as well to ranking the bases of a matroid.

47 ON UNICYCLIC GRAPHS WITH EXACTLY TWO ISOMORPHISM CLASSES OF SPANNING TREES

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In this paper we characterize those unicyclic graphs (where the unique circuit is of odd length) whose spanning trees can be partitioned into two isomorphism classes. In order to establish this result, we first obtain certain necessary conditions for spanning trees of a unicyclic graph to be isomorphic. The application of this information to those graphs which have exactly one circuit (which is of odd length) and precisely two isomorphism classes of spanning trees yields the desired result.

It is noted that the results in this paper can be extended, by similar techniques, to give a complete characterization of finite graphs whose spanning trees can be partitioned into two isomorphism classes.

48 Maximal Orbits under Affine and Holomorphism Groups

Frederick Hoffman
Florida Atlantic University

In earlier work (part of it jointly with Welch) the author has investigated orbits under actions of groups of linear transformations and isomorphism groups on power sets of vector spaces and finite groups, respectively, and under affine groups on vector spaces over GF(2). Recent increased interest in vector spaces over fields of odd characteristic and on Holomorphism groups has led to further investigation into orbit structure under actions of affine groups and holomorphism groups. Partial answers to the general questions of existence of maximal orbits are given. The proofs are both combinatorial and group theoretic in nature.

49 Extending Starters From Subgroups

Bruce A. Anderson

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Suppose G is an Abelian group of odd order, H is a non-trivial subgroup of G, and S is a starter on H. Several ways of extending S to a starter T on G are examined. It is shown that the Mullin-Nemeth method of producing strong starters can be used to find strong starters on any finite product of cyclic groups of prime power order, so long as all the primes are odd and none is a Fermat prime. This simple construction is shown to be a special case of the multiplication theorem of Gross.

50 On Self-Complementary Graphs and Diagraphs

Philbert Morris
University of Waterloo and University of the West Indies

Read found, in 1963, the number of self-complementary graphs on $4n$ nodes equals the number of self-complementary diagraphs on $2n$ nodes. This paper seeks a simple correspondence between these two sets.

51 SOME RESULTS ON CONNECTIVITY CLASSES IN DIGRAPHS

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(Notations not defined here may be found in [B] .).

We consider digraphs without multiple arcs and loops, and the following five kinds of connectivity.

A digraph $G = (X, V)$ is strongly connected (sc) if : $\forall x, y \in X$ $\exists$ path from x to y and from y to x ; it is semi-strongly connected (ssc), or unilateral, if $\forall x, y \in X$ $\exists$ path from x to y or from y to x ; it is lower quasi-strongly connected (lqsc) if $\forall x, y \in X$ $\exists z \in X$ and paths from z to x and y ; upper quasi-strongly connectedness (uqsc) is dually defined and a graph is quasi-strongly connected (qsc) if it is lqsc and uqsc. Finally, a graph is connected if $\forall x, y \in X$ $\exists$ chain between x and y. So each digraph is in exactly one of the six connectivity classes c_i, $i = 0, 1, \ldots, 5$, naturally defined. (c_5 consists of all sc digraphs, c_4 of all ssc and not sc, and so on).

In this paper we complete some results concerning the most generally studied classes : c_0, c_4 and c_5;

The most important result, obtained with the aid of J. BERSTEL [J.B.], is the determination of the minimum and maximum numbers of arcs in a digraph of order n in c_i. We also characterize these digraphs (For c_0, c_4, c_5 see [HNC], Chap. 3).

We determine the possible classes of $\overline{G}$ knowing the class of G_j and the C_i-maximal graphs (those which are in c_i, but change of class by adjonction of an arc) are characterized. Finally using a well-known result of H. E. ROBBINS [R], we characterize the graphs with an antisymmetric qsc or ssc orientation.

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52 Construction of 2-Balanced (n, k, λ) Arrays

by

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Let P denote the set of positive integers $\{1,2,\ldots,n\}$ and P^λ the union of λ copies of P. Following [1], we call a set with k elements a k-set. If $S \subseteq P^\lambda$, let $\|S\|$ denote the sum of all elements in S. A 2-balanced (n, k, λ) array is a collection of k-sets S_i $(i=1,\ldots,m)$ such that

1) $\bigcup_{1}^{m} S_i = P^\lambda$

2) Each k-set S_i can be partitioned into two parts, $S_i = P_{i1} + P_{i2}$ such that

$$\|P_{i1}\| = \|P_{i2}\| = \tfrac{1}{2}\|S_i\| .$$

In this paper, we show that necessary conditions for the existence of 2-balanced (n, k, λ) arrays are also sufficient by a direct method of construction. Furthermore, we show how 2-balanced (n, k, λ) arrays can be used to construct a class of neighbor designs used in serology, or to give coverings of complete graphs by k-cycles.

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53

NUMBER OF STEPS IN ACIRCUIT DIGRAPHS

G.Chaty[*],M.Chein[**],P.Martin[***],G.Pétolla[****]

We consider a finite digraph $G=(X,U)$ without multiple arc and without circuit: X is the vertex-set, U the arc-set. Let $p(G)$ be the minimal number of paths in G which make a partition of X. We give the following definition of the k-number of steps in G: $s_k(G)$ is the minimal number of arcs that it is necessary to add in order to obtain an acircuit digraph G' with $p(G')=k$, $1 \leqslant k \leqslant p(G)$. $s_1(G)$ was defined in [CM] and studied in [CM] and [CC]. We complete the results about $s_1(G)$ and generalize some of them to $s_k(G)$, $1 \leqslant k \leqslant p(G)$:

(1) Let $\rho(G)$ be the length of the longest elementary paths of G.

$$p(G) - k \leqslant s_k(G) \leqslant n - \rho(G) - k .$$

(2) Let G_i be the connected components of G, $1 \leqslant i \leqslant c$.

$$s_k(G) = \sum_{i=1}^{c}(s_{k_i}(G_i) + k_i) - k, \text{ with } k_i = \min(k,p(G_i)).$$

(3) If G has no cycle, then: $s_k(G) = p(G) - k$.

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54

A NOTE ON SELF-COMPLEMENTARY DESIGNS

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Let V be a finite v-set of objects called varieties and F a family of nonnull proper subsets (called blocks) of B. For each subset A of S, let $\lambda(A)$ denote the number of blocks containing A. Then the above system is a regular t-wise balanced design D if for any two x-subsets A and B of S, for given $x \leq t$, $\lambda(A) = \lambda(B) = \lambda_x$. We point out that not all blocks have the same cardinality. We say that D is (strongly) self-complementary if the blocks of D can be partitioned into pairs $\{\{B_i, B_i'\}\}$ such that B_i and B_i' are complementary, that is $B_i \cap B_i' = \phi$, $B_i \cup B_i' = V$. Schellenberg has shown that every self-complementary BIBD is a 3-design. We generalize this result and show that every self-complementary regular 2m-wise balanced design is also a self-complementary regular 2m+1-wise balanced design. Applications of this result are used to construct several 3-wise balanced designs, and to show the non-existence of certain regular self-complementary designs.

55 Equivalence Classes in Directed Graphs and Matrix Reduction

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Let M be an $n \times n$ matrix of zeros and ones which has at least one 'solution' (one set of n independent ones). 'Reduction' of M is the process of eliminating all those ones which cannot be part of any solution. This is a step in Gotlieb's method for time table generation. The presented algorithm avoids the direct use of J. Lions' 'tight sets'. Instead it achieves reduction more efficiently by introducing an associated directed graph G of n nodes. The task then becomes essentially the determination of equivalence classes of mutually connected nodes in G. Using appropriate data structures this is shown to be possible in at most $al + bn$ steps (l is the number of links in G which is equal to the number of ones in M). The algorithm is finally worked into an efficient method for the determination of transitive closures in directed graphs. Extensive computational experience is reported in respect to all the methods in this article.

56 COVERINGS BY PATHS OF THE ARCS OF A TRANSITIVE DIGRAPH WITHOUT CIRCUIT.

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A digraph is a couple $G = (X,U)$ with a finite set X of vertices and a set $U \subset X \times X$ of arcs . For every arc $u = (x,y)$ put $s(u) = x$ and $b(u) = y$. A path of G is a sequence of its arcs $u_1 . u_2 \dots u_n$ such that $b(u_i) = s(u_{i+1})$ for every $i = 1,2,\dots,n-1$. This path is called a circuit if $b(u_n) = s(u_1)$. The digraph G is said to be transitive if the following condition is verified:

$$\forall x,y,z \in X : (x,y) \in U \text{ and } (y,z) \in U \Rightarrow (x,z) \in U .$$

Let P be a set of paths of G . We shall say that P is a p-covering (resp. a disjoint p-covering) of G if every arc of G belongs at least to one path in P (resp. belongs to one and only one path in P) . Denote c(G) the minimal number of paths in a p-covering of G and d(G) the minimal number of paths in a disjoint p-covering of G . It is obvious that $c(G) \leqslant d(G)$. G. Chaty and M. Chein have conjectured that $c(G) = d(G)$ if G is transitive and without circuit . We prove that conjecture .

57 Near Block Designs and BIBDs

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A near block design (NBD) is a pairwise balanced design with two block sizes, k and $k+1$. Techniques for constructing such configurations are discussed and used to create balanced incomplete block designs which were previously unknown.

58 Invariants of Symmetric Balanced Incomplete Block Designs

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Two new sets of invariants for BIBD's are described. The first of these is the Smith Normal Form, and the second a generalized intersection number. The invariants are computed for several designs with parameters (36, 15, 6) and (45, 12, 3), enabling us to obtain new non-isomorphism results among these designs. These results are an extension of work presented by the authors at the Third Southeastern Conference.