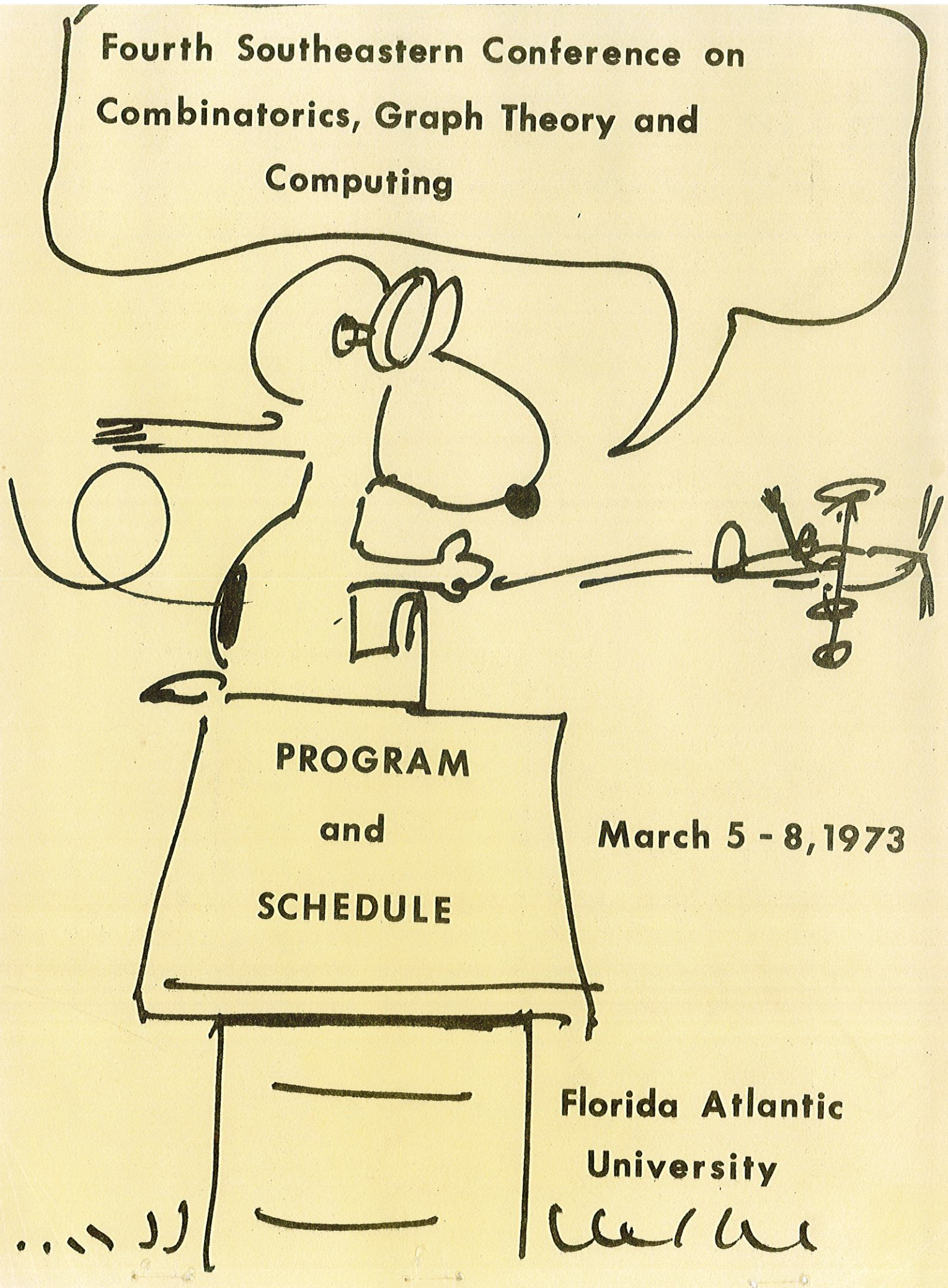
Fourth Southeastern Conference on Combinatorics, Graph Theory and Computing



PROGRAM

and

SCHEDULE

March 5 - 8, 1973

Florida Atlantic University

DAY	MONDAY		TUESDAY		WEDNESDAY		THURSDAY	
ROOM / TIME	259	207	259	213	259	207	259	207
8:30	REGISTRATION FROM 8:00		M. Doob [15]	A. Gewirtz [17]	J. Stuparyk [32]	B. Eisenberg [33]	E. Wang [50]	F. Hwang [52]
9:00	OPENING CEREMONIES		E. Kramer [16]	I. Rosenberg [18]	F. Burkowski [34]	M. Albertson [35]	L. Cummings [51]	J. Kalbfleisch [53]
9:30	J. Wallis		R. Collens [19]	V. Chvatal [21]	L. Billard [36]	R. Levow [38]	W. Wallis	
10:00	Hadamard Matrices		J. DiPaola [20]	F. Gould [22]	W. Hoskins [37]	F. Hadlock [39]	Block Designs	
10:30	K. Butler [1]	B. Alspach [3]	W. Mills		L. Lovász			
11:00	E. Luginbuhl [2]	L. Shader [4]	Covering Problems		Hypergraphs			
11:30 To 1:30	← L		U	N	C	H →		
1:30	J. Wallis		W. Mills		L. Lovász			
2:00					Factors of Graphs			
2:30	N. Pullman [5]	K. B. Reid [7]	J. Klerlein [23]	J. Dolch [25]	H. Williams [40]	R. Alter [42]		
3:00	K. Schmidt [6]	P. Hell [8]	C. Wagner [24]	S. Goodman [26]	C. Zarnke [41]	L. Adelson [43]		
3:30	B. Anderson [9]	P. Kainen [11]	R. Grimson [27]	S. Hedetniemi [29]	L. James [44]	P. Morris [46]		
4:00	R. Mullin [10]	C. Cadogan [12]	J. Dillon [28]	W. Kuhn [30]	P. Kritzinger [45]	G. Haggard [47]		
4:30	W. Wallis [13]	W. Richardson [14]		L. T. Ollman [31]	D. Geller [48]	R. Stanton [49]		
EVENING	Cocktails		Poolside Cocktails/Dinner Theater		Conference Banquet			

1

ENUMERATION OF FINITE TOPOLOGIES

KIM KI-HANG BUTLER*

Pembroke State University, Pembroke, N. C. 28372

GEORGE MARKOWSKY

Harvard University, Cambridge, Mass. 02138

Let X be a set with n elements. Let $T_0(X)$ ($T_0^c(X)$, $T(X)$, $T^c(X)$) be the set of all T_0-topologies (connected T_0-topologies, topologies, connected topologies) that can be defined on X. Let $T_0(n)$ ($T_0^c(n)$, $T(n)$, $T^c(n)$) be the number of T_0-topologies (connected T_0-topologies, topologies, connected topologies) defined on X. Let $\bar{T}_0(n)$ ($\bar{T}_0^c(n)$, $\bar{T}(n)$, $\bar{T}^c(n)$) be the number of non-isomorphic T_0-topologies (connected T_0-topologies, topologies, connected topologies) in $T_0(X)$ ($T_0^c(X)$, $T(X)$, $T^c(X)$). The enumeration of $T_0(n)$ and $T(n)$ have been considered by many authors for certain small values of n.

In this paper, we shall present some formulas relating: (i) $T_0(n)$ with $T(n)$; (ii) $T_0^c(n)$ with $T^c(n)$; (iii) $\bar{T}_0(n)$ with $\bar{T}_0^c(n)$; (iv) $\bar{T}(n)$ with $\bar{T}^c(n)$; (v) $T_0(n)$ with $T_0^c(n)$; and (vi) $T(n)$ with $T^c(n)$. We accomplish this by considering an equivalent problem. Namely, if $P(X)$ ($P^c(X)$, $Q(X)$, $Q^c(X)$) denotes the set of all posets (connected posets, quasi-ordered sets, connected quasi-ordered sets) that can be defined on X.

The authors are especially interested in the subject because of the overlap here of many areas of finite mathematics. Concepts from graph theory, lattice theory, matrix theory, and set theory are all relevant. In addition, a natural source of applications arises in, of all places, semigroup theory.

2

A TABLE OF KNOTS AS CLOSED BRAIDS

E. Luginbuhl* and R. S. D. Thomas

The University of Manitoba

ABSTRACT. A mathematical knot is usually represented as the structure that results from tieing a knot, in the ordinary sense, in an open string and joining the two ends of the string. A primary goal of knot theory has been to solve the problem of determining whether or not two given knots can be deformed continuously each into the other. It is very difficult to compare even the simplest of knots, for example, to show that the trefoil cannot be deformed continuously into its mirror image. All attempts to find a general solution to this problem have proved unsuccessful.

In this paper the knots of the standard knot table are represented by wreaths, defined as equivalence classes of closed braids. Various deformations of these closed braids are defined and a procedure is described to select a representative with a minimum number of strands from each equivalence class. This table of representatives can then be used for comparison with other such representations of knots. The authors make no claim of uniqueness, but their results may be useful and have been obtained systematically.

3

Title: A characterization of finite permutation groups that are doubly set-transitive but not doubly transitive.

Author: Brian Alspach

Simon Fraser University

Abstract: A permutation group G on S is k set-transitive if for any two subsets, say A and B, of S of cardinality k, there exists a permutation $\sigma \in G$ such that $\sigma(A) = B$. G is k-transitive if for any two k-tuples $a_1,\ldots,a_k$ and $b_1,\ldots,b_k$ there exists a permutation $\sigma \in G$ such that $\sigma(a_i) = b_i$, $i = 1,\ldots,k$. Using Berggren's characterization of finite symmetric tournaments and a graph-theoretic interpretation of double set-transitivity, we characterize finite permutation groups that are doubly set-transitive but not doubly transitive.

4

Abstract

On the Existence of Finite Central Groupoids of all Possible Ranks

Leslie E. Shader

The University of Wyoming

An algebraic system $\langle G,\cdot\rangle$ such that $(x\cdot y)\cdot(y\cdot z) = y$ is a Central Groupoid. The correspondences between finite central groupoids; finite directed graphs with the property that for every pair of vertices (A,B) there is a unique path of length 2; and 0,1 matrix solutions of $A^2 = J$, are discussed by Knuth.

In this paper the question of existence of solutions of all possible rank is answered. Also the problem of finding all solutions for the case of 16×16 matrices is discussed. (This is the first unsolved case.)

5 A THEOREM OF RADON AND HADAMARD DESIGNS

by

A.V. Geramita and N.J. Pullman*
Queen's University, Kingston, Ontario

A Hadamard design of Williamson type is an $n \times n$ matrix of n signed variables (each occurring only once in each row) whose rows are pairwise orthogonal. Using a completely combinatorial argement, J. Wallis showed that such designs exist only for $n=1,2,4$ and 8. Subsequently T. Storer obtained an algebraic proof by using a theorem of Hurwitz.

This problem suggests the following generalization:

Let a "partial design" be an $m \times n$ matrix $W(m,n)$, of n signed variables (each occurring only once in each row) whose rows are pairwise orthogonal. Let $h(n)$ denote the largest m for which a partial design $W(m,n)$ exists. Clearly $1 \leq h(n) \leq n$ and, by J. Wallis' theorem, $h(n) = n$ only when $n=1,2,4$ and 8. The purpose of our paper is to determine $h(n)$ for every n.

We do this by employing a generalization of a theorem of Radon (related to Hurwitz's theorem) and a theorem of J.F. Adams concerning vector fields on spheres.

6 INCOMPLETE HADAMARD MATRICES

K. W. Schmidt
Computer Centre
University of Manitoba
Winnipeg, Manitoba, Canada R3T 2N2

A direct method exists to find $K \leq n/2$ missing columns of a general Hadamard matrix of order $n = 4*t$, t odd. The method can always be applied if $K \leq 5$. For $K = 6$ this algorithm has also been used; successfully, but has been proved only for $t \neq 0 \pmod 3$. Some examples are given showing that the method can complete a Hadamard matrix, even when only $n/2$ columns are known.

7 THE NUMBER OF GRAPHS ON N VERTICES WITH 3 CLIQUES

K. B. Reid *
L.S.U., Baton Rouge, Louisiana 70803

A clique in a graph G without loops or multiple edges is a maximal complete subgraph of G.M. Rosenfeld asked (American Math. Mon., 78, (1971), 49-50) for the determination of $c(n,k)$, the number of nonisomorphic graphs on n vertices having exactly k cliques. Clearly, $c(n,1) = 1$. and Rosenfeld showed that $c(n,2) = [n^2/4]$, where $[x]$ denotes the greatest integer less than or equal to x. In this work we show that

$$c(n,3) = \begin{cases} (1/240)(n)(n^2-1)(n^2-9) + p(n,3), & n \text{ odd} \\ (1/240)(n)(n^2-4)(n^2-6) + p(n,3), & n \text{ even,} \end{cases} \quad \text{where}$$

$$p(n,3) =](1/72)(n)(2n^2 + 3n - 6)[\text{ , and }] x [\text{ denotes}$$

the integer in the interval $(x - 1/2,\ x + 1/2]$.

* This work will appear in the "Journal of the London Math. Soc."

8 Decompositions of graphs and hypergraphs using products -application to Ramsey numbers.

Pavol Hell, University of British Columbia

Given edge-disjoint decompositions of G and H into factors of a given kind, one can often construct a decomposition of $G \times H$, $G \otimes H$, $G \circ H$, etc. with similar properties. If moreover

$$G = \left(\times_{i \in I} G_i\right) \cup \left(\otimes_{j \in J} G_j\right) \cup \left(\circ_{k \in K} G_k\right) \cup \cdots$$

(as is the case for $G, G_i, \ldots$ complete graphs), then we can lift the decompositions from $G_i, \ldots$ to G as well.

A few applications will be mentioned, in particular, we shall obtain some lower bounds on Ramsey numbers; however, it is hoped that this note will prompt some further applications.

9

A Combinatorial Problem Arising in Topology

Bruce A. Anderson

Arizona State University

The following problem occurs in a study of the lattice of all topologies on a finite set.

BASIC PROBLEM. Does there exist a 1-factorization of K_{2n}, the complete graph on 2n points, such that the union of every distinct pair of 1-factors is a Hamiltonian circuit?

It has been shown that such a 1-factorization exists if there is a prime p such that either $2n = p+1$ or $n = p$.

In this paper we show that there is a Room Square whose column 1-factorization is of the required type on K_{16}. Thus K_{28} is now the first unsettled case. We show that, up to relabelling, there

i) is exactly one way to get the required 1-factorization on K_{2n}, $1 \leq n \leq 5$;

ii) are at least two ways to get the required 1-factorization on (a) K_{12}, (b) K_{2n}, if there are primes p,q such that $2n=p+1=2q$.

We compute symmetry groups which show that certain 1-factorizations of K_8 analyzed by W. D. Wallis in his recent monograph on Room Squares are really members of infinite families of 1-factorizations of various K_{2n}.

10

On the Size of (r, 2)-Systems (ABSTRACT)

R.C. Mullin

University of Waterloo

An (r, λ)-system is a collection F of subsets (called blocks) of a finite set V of elements (called varieties) which satisfies the following:

(i) every variety occurs in precisely r blocks.

(ii) every pair of distinct varieties occurs in precisely λ blocks;

(iii) $v = |V| \geq 2$, and $r \geq \lambda > 0$.

It has been shown that there exists a least integer $V_0(r, \lambda)$ such that if $v > V_0(r, \lambda)$, then the system consists of λ copies of V and $r - \lambda = n$ copies of each elements of V as blocks of size 1. Such designs are called trivial. We show that if for some non-trivial $(r, 2)$-system D on v varieties satisfying $v > \max. \{4, n^2 + n + 1\}$, then all intersections of distinct blocks of D are of cardinality $\leq n/2$, where $n = r - 2$. Applications in the estimation of $V_0(r, \lambda)$ are illustrated.

11

Lie Product of Graphs

by Paul C. Kainen

Department of Mathematics

Case Western Reserve University

Cleveland, Ohio 44106

In this paper, we define a new product for graphs in analogy with the Lie product of matrices. Let G be an ordered graph, or orgraph, that is, a graph $|G|$ together with a fixed linear ordering of its vertices. Then G is uniquely determined by its 0-1 adjacency matrix. This allows us to define, for two orgraphs G and H, each with n vertices, the Lie product $[G,H]$. This product has the "advantage" of not increasing the number of vertices; however; it has the "disadvantage" of depending on the particular ordering of the vertices of G and H. We investigate the elementary properties of the product, and suggest some applications.

12 INVERSE DIFFERENTIAL OPERATORS OVER GRAPH-LATTICES.

Charles Cadogan

University of the West Indies

The general form of a class of inverse differential operators over polynomials defined on partitions of positive integers is obtained. The coefficients in the polynomials are numbers of general graphs and the application of the operators to these polynomials determines a counting series for multigraphs on the respective partitions. The basic structure, the graph-lattice, commands a vital role in establishing the final form of the operator.

13

A Survey of
Complementary Room Squares and Related Topics

W.D. Wallis* & R.C. Mullin

Several recent results obtained both theoretically and by means of the computer are given in this up-to-date summary. These squares are important because of their roles in several constructions which require their use. In particular, a pair of complementary squares of side 129 would solve the Room square existence problem.

14

The Structure of Binary Matroids

W.R.H. Richardson
University of Waterloo

The structure of binary matroids is studied by defining a composition between two binary matroids which is based on the symmetric difference of their underlying sets. A theory of composition-decomposition is formulated in terms of chain groups. The composition is shown to be the constructive analogue of Tutte's notion of connectivity. If a binary matroid M has an n-separator then it can be decomposed into smaller matroids whose symmetric difference is M. Hence every binary matroid can be constructed by starting with copies of the "six" binary matroids with infinite connectivity and by using only the symmetric difference composition. Every construction scheme of a binary matroid yields a way of representing the matroid by a graph.

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Proceedings of the Conference on Interdisciplinary Research in Computer Science (Winnipeg, June, 1970). Editors: M.G. Saunders, R.G. Stanton, 284 pp. $ 10.00

Proceedings of the Louisiana Conference on Combinatorics, Graph Theory & Computing (Baton Rouge, March, 1970). Editors: R.C. Mullin, K.B. Reid, D.P. Roselle, 474 pp. $ 15.00

Proceedings of the Second Louisiana Conference on Combinatorics, Graph Theory & Computing (Baton Rouge, March, 1971). Editors: R.C. Mullin, K.B. Reid, D.P. Roselle, R.S.D. Thomas, 570 pp. $ 18.00

Proceedings of the 25th Summer Meeting of the Canadian Mathematical Congress (Lakehead University, June, 1971). Editors: W.R. Eames, R.G. Stanton, R.S.D. Thomas, 656 pp. $ 18.00

Proceedings of the Manitoba Conference on Numerical Mathematics (University of Manitoba, 1971). Editors: H.C. Williams, R.S.D. Thomas. $ 18.00

Proceedings of the Third Southeastern Conference on Combinatorics, Graph Theory & Computing (Boca Raton, Florida, March, 1972). Editors: F. Hoffman, R.B. Levow, R.S.D. Thomas, 488 pp. $18.00

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Administrative Assistant
Department of Computer Science
University of Manitoba
Winnipeg, Manitoba
R3T 2N2

15

BLOCK DESIGNS ON A GRAPH

Michael Doob

The University of Manitoba

The concept of a block design on a graph is a relatively new one. It is a combinatorial structure which can be considered as a generalization of both the concept of graph decompositions and that of a balanced incomplete block design. Some of the properties of a BIBD carry through to a block design on a graph. Others, such as Fisher's inequality, do not. Some infinite families of designs on a graph will be given with particular emphasis on the complete bipartite graph. In that case necessary and sufficient conditions for construction of these designs can often be given.

16

ON SOME DISJOINT STEINER SYSTEMS

Earl S. Kramer* and Dale M. Mesner
University of Nebraska, Lincoln, Nebraska

Spyros S. Magliveras
State University of New York, Oswego, N. Y.

A Steiner system $S(\ell,m,n)$ is a system of subsets of size m (called blocks) from an n-set S, such that each ℓ-subset from S is contained in precisely one block. Two Steiner systems (possibly isomorphic) have intersection k if they share exactly k blocks. The possible intersections among $S(2+i,3+i,9+i)$'s, $i = 0,1,2,3$ are determined, together with associated orbits under the action of the automorphism group of an initial Steiner system. The following are results: (i) the maximal number of mutually disjoint $S(5,6,12)$'s (or $S(4,5,11)$'s) is two and any two such pairs of disjoint $S(5,6,12)$'s (or $S(4,5,11)$'s) are isomorphic; (ii) the maximal number of mutually disjoint $S(3,4,10)$'s is five and any two such sets of five are isomorphic; (iii) a result due to Bays in 1917 that there are exactly two nonisomorphic ways to partition all 3-subsets of a 9-set into seven mutually disjoint $S(2,3,9)$'s. Results on a search for disjoint $S(2+i,5+i,21+i)$'s, $i = 0,1,2,3$, will be mentioned. Finally, a design which is a 3-dimensional generalization of Room squares will receive attention.

17

AN EXTENDED SIMPLEX ALGORITHM

A. Gewirtz*
Brooklyn College of the
City University of New York

A.W. Tucker
Princeton University

The simplex algorithm finds optimal basic feasible solutions x and y to the dual linear programs:

$$\text{Maximize} \quad c^T x = M$$
$$\text{subject to} \quad Ax \leq b \qquad b \geq 0$$

and

$$\text{Minimize} \quad b^T y = m$$
$$\text{subject to} \quad A^T y \geq c$$

Various methods remove the condition $b \geq 0$. In this paper we describe an efficient extension of the simplex algorithm itself which removes the condition. The extnsion does not involve the objective function. It finds a basic feasible x-solution. More generally it yields a means of finding a basic solution of a system of linear inequalities

$$Ax \leq b$$

without restrictions on A, x or b.

18

Zero-one programming and binary developments.

I.G. Rosenberg
Centre de Recherches Mathématiques
Université de Montréal

Consider the following 0-1 polynomial problem: minimize $F_0(X)$ s.t. $F_k(X) \leq 0$ $(k \in K)$, $F_\ell(X) = 0$ $(\ell \in L)$, $X \in \{0,1\}^n$, where all F_i's are polynomials with integer coefficients. Using binary developments and computations in GF(2) (based on the sum mod 2 and the multiplication on $\{0,1\}$) the characteristic Boolean functions of the constraints can be found either by an iterative procedure (which may involve storage and retrieval problems) or by direct formulae. The optimal solution can be obtained in p steps (where $0 \leq F_0(X) < 2^p$); in each step we must only test weather a certain expression in GF(2) is identically 0, and only in the last step we must simplify an expression in GF(2).

Expressions in GF(2) may also provide interesting reformulations of some combinatorial problems. Although there is little hope that this method (as any Boolean method) will be easily applicable in the general case, it may be useful in some applications (e.g. in problems with many symmetries or sparse matrices); and we feel that this somewhat unconventional approach deserves more attention.

19

A LISTING OF BALANCED INCOMPLETE BLOCK DESIGNS

R.J. Collens

University of Manitoba, Winnipeg

ABSTRACT: We present a computer generated listing of Balanced Incomplete Block Designs. The designs in the listing are classified by the cardinality of the variety set, v, and the block size, k, of the design. The listing incorporates a substantial number of previously known designs as well as many new designs constructed by the author. The listing is an extension and an amplification of a previously published listing.

20

A list of (v,b,r,k,λ) designs for $r \leq 30$, $6 \leq k \leq 1/2\ v$

Jane W. DiPaola
Florida Atlantic University

Jennifer Seberry Wallis
W.D. Wallis
University of Newcastle, NSW

This list gives the status of the construction of balanced incomplete block designs having $r \leq 30$, $6 \leq k \leq 1/2\ v$. In each case reference is given to a valid construction. Multiples, that is designs $(f,tb,tr,k,t\lambda)$, of designs (v,b,r,k,λ) which exist are omitted but multiples of designs which are unknown or non-existent are included.

Whenever a balanced incomplete block design (v,v,k,k,λ) exists then the

(i) derived design $(k,v-1,k-1,\lambda,\lambda-1)$ and the

(ii) residual design $(v-k,v-1,k,k-\lambda,\lambda)$ also exist.

If $\lambda-1$ or 2 and a (v,v,k,k,λ) does not exist then neither does a design with the parameters of the appropriate residual design.

Designs for $k \leq 5$ are omitted since all exist except for $(15,21,7,5,2)$.

21

V. Chvátal, Centre de recherches mathématiques
Université de Montréal

Let S be a set of linear inequalities

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i \qquad (i = 1,2,\ldots,m)$$

that determine a polytope P. The Edmonds polytope $E(P)$ of P is the convex hull of lattice points inside P. The inequality $\sum a_j x_j \leq b$ belongs to the elementary closure $e^1(S)$ of S if there are nonnegative reals $\lambda_1,\lambda_2,\ldots,\lambda_m$ with

$$\sum \lambda_i a_{ij} = a_j \ (j = 1,2,\ldots,n) \ , \ \llcorner \sum \lambda_i b_i \lrcorner \leq b \ .$$

We set $e^k(S) = e^1(S \cup e^{k-1}(S))$.

THEOREM. Given any P there is a smallest k such that $e^k(S)$ determines $E(P)$.

The smallest such k is the rank of P. The rank of P seems to measure the difficulty of each extremal combinatorial problem: matching problems have rank at most one, vertex-packing problems have unbounded rank. We study vertex-packing problems from this viewpoint and present a conjecture on series-parallel networks.

22

A Combinatorial Algorithm for Computing Saddle Points of Nonconvex Programs

F.J. Gould

University of Chicago

A saddlepoint theory for nonconvex optimization problems has been constructed in such a way as to parallel the convex theory conjoined with the usual Lagrangian formulation [see Arrow, Gould and Howe, "A General Saddlepoint Result for Constrained Optimization," Institute of Statistics Mimeo Series No. 774, Department of Statistics, University of North Carolina at Chapel Hill, September, 1971]. The saddlepoint computation can be performed by triangulating the nonnegative orthant of R^n and labeling vertices in a special way depending upon the zero contours of the partial derivatives of the saddle function. This will produce a pseudomanifold consisting of labeled (n+1)-tuples. The labeling is also such that a unique boundary n-tuple will exist with only one label missing. Under mild regularity assumptions on the saddle function a form of Sperner's lemma will guarantee that a finite graph-type path will lead to a completely labeled n-tuple, and such a set is shown to produce an approximation to a saddle point. Refinement of the triangulation leads to convergence. This paper discusses the mechanics of the computational procedure.

23

On Characterizations by Forbidden Substructures

D. L. Greenwell, R. L. Hemminger, and J. B. Klerlein*

Vanderbilt University

Let P be a partially ordered class under $\subseteq$. If $G \in P$ we shall call G a structure, and if $H, G \in P$ and $H \subseteq G$ we shall call H a substructure of G. Let H be a subclass of P. We say that H can be characterized in terms of forbidden substructures if there exists a subclass F of P such that $G \in H$ if and only if there does not exist an $F \in F$ with $F \subseteq G$.

Theorem: Let P be a partially ordered class under $\subseteq$. A subclass H of P can be characterized in terms of forbidden substructures if and only if for all $G \in H$, $H \subseteq G$ implies that $H \in H$.

The paper includes numerous examples relating to this theorem.

24

Covers of Finite Sets

Carl G. Wagner
Assistant Professor of Mathematics
University Of Tennessee
Knoxville, Tennessee 37916

A cover of a finite set S is a set of nonempty subsets of S, the union of which is S.

This paper is a survey of solved and unsolved enumeration problems connected with special classes of covers, among them partiti ons, partitions of type k, minimal covers, unnested covers (called "set systems" by Hartmanis and Stearns), and k-unnested covers. Some connections between covers and binary relations will also be indicated.

25

NAMES of Hamiltonian graphs

John P. Dolch
Computer Science Department
University of Iowa
Iowa City, Iowa

A computer was programmed to produce the NAMES of all Hamiltonian graphs through 8 points. The NAME of a graph is a canonical form represented by a string of digits.

This paper discusses some of the techniques and problems encountered and suggests avenues for future exploration by computers of this linear representation of relationships.

A lexicographical listing of the 383 NAMES of 7 point Hamiltonian graphs is presented.

26

HAMILTONIAN SUFFICIENCY CONDITIONS BASED ON SUBGRAPH STRUCTURES

S. Goodman*

S. Hedetniemi

University of Virginia

Charlottesville, VA 22901

Five sufficiency conditions and one necessary condition for a graph to be hamiltonian are given. Two relatively weak necessary and sufficient conditions are also presented. All of these theorems are in terms of subgraph structure and do not require the fairly high global line density which are basic to the Pósa-like sufficiency conditions.

27

Partition Generating Functions

R. C. Grimson
Elon College

In this paper we examine a class of plane and k-partite partition problems suggested by sums such as

$$\sum_{m_1,\cdots,m_k=0}^{\infty} x_1^{m_1}\cdots x_k^{m_k} \sum_{m_1,\cdots,m_r=0}^{\gamma(M_r(m_1,\cdots,m_k))} y_1^{m_1}\cdots y_t^{m_t}$$

where γ is an increasing function which is nonnegative integer valued on the nonnegative integers and where $M_r(n_1,\cdots,n_k)$ is defined by the following two properties: (a) it is symmetric in $n_1,\cdots,n_k$; (b) if $n_1\leq\cdots\leq n_k$, then $M_r(n_1,\cdots,n_k) = n_r$ $(1\leq r\leq k)$. These sums are evaluated (an expression is known for the above sum in the case $\gamma(x) = x$). Thus, as one example, we enumerate partitions $N = m_1 + \cdots + m_t + n_1 + \cdots + n_k$ (with exactly $t + k$ positive parts or with some parts = 0) such that for all j, $m_j \leq$ certain specified functions of the n's. Some interesting questions are raised.

28

The generalized Langford - Skolem problem.

J. F. Dillon, Department of Defense, Fort George G. Meade, Maryland, 20755.

For integers s and n greater than 1 the sequence $x_1, x_2, x_3, \ldots, x_{sn}$ is called an (s,n)-Langford sequence (resp. (s,n)-Skolem sequence) if each integer 1, 2, 3,..., n occurs exactly s times in the sequence and for each i, $1 \leq i \leq n$, successive occurrences of i are separated by exactly i (resp. i-1) terms of the sequence. The existence of an (s, n-1)-Langford sequence implies the existence of an (s,n)-Skolem sequence.

In this paper we obtain some new necessary conditions on (s,n) for the existence of such sequences. We also suggest a further generalization of these problems. Our nonexistence theorems are seen to apply to this new problem which, unlike the Langford problem, can be solved in the case (s,n) = (3,8).

29

On Hamiltonian Walks in Graphs

S. Goodman
S. Hedetniemi*

University of Virginia
Charlottesville, Virginia
22901

A hamiltonian walk in a graph G is a closed walk of minimum length which contains every point of G. An eulerian walk in a graph G is a closed walk of minimum length which contains every line of G. In this paper we establish several relationships between hamiltonian and eulerian walks. We also derive a number of bounds on the length of a hamiltonian walk.

30

INVERSE LINE GRAPHS AND HAMILTONIAN CIRCUITS

William W. Kuhn
St. Joseph's College
Philadelphia, Pennsylvania

Computer algorithms are given to find the line graph and the inverse line graph of a given graph. The inverse line graph algorithm terminates if the given graph is not a line graph.

Some remarks on finding Hamiltonian circuits by considering circuits in the inverse line graph are given.

31

On the Book Thicknesses of Various Graphs

L. Taylor Ollmann
Louisiana State University
Baton Rouge, Louisiana 70803

A graph can be "drawn in a book" by ordering the vertices along the book's spine and drawing the edges on pages of the book so that no two edges on the same page intersect except at their end points. The book thickness of a graph is the minimum number of pages necessary to draw the graph. Several results and open questions on the book thicknesses of various graphs are presented.

32 THE NUMERICAL SOLUTION OF INITIAL VALUE PROBLEMS WITH STATE DEPENDENT RETARDATIONS

F. J. BURKOWSKI J. STUPARYK

Department of Computer Science
University of Manitoba
Winnipeg, Manitoba

This paper presents a numerical technique for the solution of initial value problems of the form

$$y'(t) = F(t, y(g_1(t,y(t))),\ldots, y(g_n(t,y(t)))) \quad . \qquad (1)$$

Our study places special emphasis on a particular case of (1), the equation

$$S'(t) = -r(t)\, S(t)\, [I_0 + S_0 - S(\tau(t))] \qquad (2)$$

where $\tau(t)$ *is the unique solution of*

$$-\int_{\tau(t)}^{t} \frac{\rho(x)\, S'(x)}{r(x)\, S(x)}\, dx = m \quad . \qquad (3)$$

The functions $\rho(x)$, $r(x)$ *and the constant* I_0, S_0 *are known. Such equations provide us with a macroscopic description of the fluctuations of biological populations under the stress of an infection. The importance of such a model is due to the inclusion of a "threshold" effect described by (3). Thus, a susceptible individual in the population does not become infectious until his resistance to the disease has been broken down by sufficient exposure to the other infectious members.*

33

Bernard Eisenberg
Kingsborough Community College
of
The City University of New York

It is well known that the Chromatic Polynomial of a polygonal graph (a connected graph with precisely one circuit) on n nodes of which p are in its only circuit, is given by

$$M_G(\lambda) = (\lambda - 1)^{n-p}((\lambda - 1)^p + (-1)^p(\lambda - 1)).$$

It is claimed that this expression characterizes a polygonal graph.

34 THE NUMERICAL DERIVATION OF PERIODIC SOLUTIONS OF DIFFERENTIAL EQUATIONS WITH STATE DEPENDENT DEVIATIONS

F. J. Burkowski

Department of Computer Science
University of Manitoba
Winnipeg, Manitoba

In recent years, functional differential equations of the type

$$y'(t) + ay(t - \tau(t, y(t))) = F(t) \qquad (1)$$

have become important in the biological sciences. Such an equation can be used to describe the spread of infection in a population of susceptible organisms. If both τ and F are periodic in the argument t with period T, it is often the case that (1) will have periodic solutions. This paper presents an algorithm which can be used to numerically generate such periodic solutions.

35 A PROPERTY OF IRREDUCIBLE GRAPHS

MICHAEL O. ALBERTSON
Swarthmore College
Swarthmore, Pa. 19081

We call a graph irreducible if it is planar, five chromatic and has the fewest possible number of vertices of all graphs with these two properties. If G is irreducible then any possible alteration of the edges of G contained in any given five cycle of G will result in the altered graph being four colorable.

36

THE TRIANGULARIZATION OF THE MATRIX OF COEFFICIENTS OF A SET OF DIFFERENTIAL-DIFFERENCE EQUATIONS

L. Billard
University of Waterloo

We take a set of differential-difference equations whose matrix of coefficients has not only lower triangular elements but also elements in the upper off-diagonal, and whose solution by recursive techniques is seemingly impossible. For illustrative purposes, we consider a stochastic epidemic model. By redefining the model, we show how a lower triangular matrix of coefficients can be obtained. Thus, a solution can be readily found using recursive methods.

37

A SMOOTHING ALGORITHM USING PIECEWISE POLYNOMIAL FUNCTIONS

W.D. Hoskins* and G.E. McMaster

Department of Computer Science

University of Manitoba

The exact fitting of m-th order polynomial splines to periodic functions only known at a discrete set of uniformly spaced values, is extended into an economical algorithm for relaxing the exact interpolation requirement and obtaining approximations in a least squares sense for sequences of increasingly higher order splines.

38

BOUNDARY COLORATIONS

Roy B. Levow

Florida Atlantic University

Suppose that the Four Color Conjecture is true, and let G be planar graph drawn in the plane so that its outer boundary is a k-cycle. The restriction of a 4-coloration of G to the bounding k-cycle is called a boundary coloration. In this paper we report results of a computer aided investigation of boundary colorations. Of particular interest are the characterization of those sets of colorations of a k-cycle which are boundary colorations for some planar graph and the implications of such results for a conjecture of Albertson and Wilf concerning the minimum cardinality of such sets.

39

OPTIMAL GRAPH PARTITIONS

Frank O. Hadlock

Florida Atlantic University

Of practical importance are the related problems of partitioning the vertices of a graph so as to maximize or minimize (subject to block size constraints) the number of edges with endpoints in different blocks. Viewed as coloration problems, an algorithm for finding optimal solutions is given that proceeds by sampling each of a collection of subspaces (of the space of colorations). Within each subspace, an operator produces a local optimum from the sample point.

40

COMBINATORIAL AND NUMBER THEORETIC PROPERTIES OF SOME SEQUENCES SIMILAR TO LUCAS SEQUENCES

H. C. Williams

University of Manitoba

ABSTRACT. Let r, s, t, be integers such that

$$f(x) = x^3 - rx^2 - sx - t$$

is a polynomial with non-zero discriminant. Let ρ_1, ρ_2, ρ_3 be the three distinct zeros of $f(x)$ and put $\alpha_i = W_1 + V_1\rho_i + U_1\rho_i^2$ $(i = 1, 2, 3)$, where W_1, V_1, U_1, are given integers. Define

$$W_m = D^{-1}\begin{vmatrix} \alpha_1^m & \rho_1 & \rho_1^2 \\ \alpha_2^m & \rho_2 & \rho_2^2 \\ \alpha_3^m & \rho_3 & \rho_3^2 \end{vmatrix}, \qquad V_m = D^{-1}\begin{vmatrix} 1 & \alpha_1^m & \rho_1^2 \\ 1 & \alpha_2^m & \rho_2^2 \\ 1 & \alpha_3^m & \rho_3^2 \end{vmatrix},$$

$$U_m = D^{-1}\begin{vmatrix} 1 & \rho_1 & \alpha_1^m \\ 1 & \rho_2 & \alpha_2^m \\ 1 & \rho_3 & \alpha_3^m \end{vmatrix}, \qquad \text{where} \quad D = \begin{vmatrix} 1 & \rho_1 & \rho_1^2 \\ 1 & \rho_2 & \rho_2^2 \\ 1 & \rho_3 & \rho_3^2 \end{vmatrix}$$

The combinatorial and number theoretic properties of the sequences $\{W_m\}$, $\{V_m\}$, $\{U_m\}$ are described. It is shown that these sequences have many properties similar to those of the well known sequences of Lucas.

It is also demonstrated that the theory of these sequences can be applied to the solution of some Diophantine equations and to the solution of certain cubic congruences.

41

COMPUTER IMPLEMENTATION OF DELAUNAY'S ALGORITHM OF ASCENT

H. C. Williams

*C. R. Zarnke

University of Manitoba

ABSTRACT. A description is given of a Computer programme which can often be used to find all integer pairs (x, y) which satisfy the Diophantine equation

$$F(x, y) = 1,$$

where $F(x, y)$ is a monic binary cubic form with negative discriminant The programme makes use of a specially modified version of Delaunay's Algorithm of Ascent.

42

Characterization of Maximal Snakes in I(d)

L.E. Adelson, R. Alter* and T.B. Curtz

University of Kentucky

Lexington, Kentucky 40506

Let I(d) be the graph formed by the vertices and edges of the d-dimensional cube. A d-dimensional snake is a simple circuit C formed from the vertices and edges of I(d) so that any edge of I(d) joining two vertices of C is, in fact, an edge of C. The problem of determining the maximum length of a d-dimensional snake, s(d), is quite difficult. It is known that for all $d \geq 6$,

$$\frac{7 * 2^{d-2}}{d-1} \leq s(d) \leq 2^{d-1} - \frac{2^d - 12}{7d(d-1)^2 + 2}.$$

Also, s(d) has been computed for all $d \leq 6$. The largest known snake in I(7) is 48. In this paper, a criterion for two d-dimensional snakes of the same length to be equivalent is established and all non-equivalent snakes for $d \leq 6$ are determined. New results on s(7) will also be included.

43

Computation of d-Dimensional Snakes

L.E. Adelson*, R. Alter and T.B. Curtz

University of Kentucky

Lexington, Kentucky 40506

Let I(d) be the graph formed by the vertices and edges of the d-dimensional cube. A d-dimensional snake is a simple circuit C formed from the vertices and edges of I(d) so that any edge of I(d) joining two vertices of C is, in fact an edge of C. In this paper we describe the computational techniques used to discover and characterize all snakes for $d \leq 6$. A description of the man-machine interactive computer program used for hunting for long snakes is also included.

44 A True Random Number Generator

for the IBM 360 Machine

Lee O. James*

A. Neil Arnason

Computer Science Department
University of Manitoba
Winnipeg, Manitoba

A procedure is developed for generating strings of binary bits such that each bit is assigned a value independently of all previously generated bits, and has probability ½ of being assigned the value 1, probability ½ of being assigned the value 0. These strings can then be used to form real numbers, independently and uniformly distributed on [0, 1], in either single or extended precision. Unlike pseudo-random number generators, this method gives infinite cycle length and effectively perfect statistical properties, including independence of the digits within each random number.

The procedure is based on sampling regions of active core in the time-shared environment. Independence is assured by sampling a constantly changing core at pseudo-random locations. The correct probability of 0 or 1 is obtained by using a rejection technique. Theoretical justification of these properties is given.

The procedure is slow, but important applications for the generator are outlined.

45 ON THE DEFINITION OF MERGE METHODS BY BINARY MATRICES

by

J. W. Graham and P. S. Kritzinger*
University of Waterloo

In order for the conventional polyphase and cascade merge methods to go to completion in as few a number of cycles as possible, particular initial string distributions are necessary. We show how these string distributions can be arrived at using binary matrices which define a larger class of merge methods known as compromise merge methods. We then continue to exhibit different classes of binary matrices which give rise to efficient merge methods with entirely different initial string distributions. We indicate some of the properties which have been proven, of these matrices, and discuss the practical implications of the methods.

46 CHARACTERISTIC POLYNOMIALS OF ALL TREES

Philbert Morris

The University of the West Indies

This paper describes the Computer implementation of known methods of calculating the characteristic polynomials of the adjacency matrices of all non-isomorphic trees.

A brief description is first given of the generation of a list of all non-isomorphic trees.

The calculation of the characteristic polynomials has been completed for trees with 13 or fewer nodes.

47 "Minimum-edge Graphs with given hyperoctahedral automorphism group"

Gary Haggard
University of Maine at Orono
Orono, Maine 04473

Donald McCarthy
St. John's University
Jamaica, New York 11432

Andrew Wohlgemuth
University of Maine at Orono
Orono, Maine 04473

ABSTRACT. Given a finite abstract group G, whenever n is sufficiently large there exist graphs with n vertices and automorphism group isomorphic to G. Let e(G, n) denote the minimum number of edges possible in such a graph, and $e_c(G, n)$ the minimum number of edges for a connected graph of this type. To date, e(G, n) and $e_c(G, n)$ have been completely determined only for a limited class of groups; namely, when G is the identity group id, any symmetric group S_k, any dihedral group, or a cyclic group of order 3. The present paper extends the known results to include the hyperoctahedral groups, i.e. groups of the form $S_k[S_2] = H_k$ for $k > 1$. We show that when $k \neq 3$ $e(H_k, n)$ is undefined for $n < 2k$, and equals $k + e(id, n-2k)$ for $n = 2k$, $2k+1$ and for $n \geq 2k+7$. For each value of k this leaves five values of n unaccounted for; these cases are settled whenever k is even and ≥ 8. Although the pattern when $k = 3$ differs from the above, $e(H_3, n)$ can be determined for all n. Some results on the connected case are also obtained; in particular, $e_c(H_k, n) = n-1$ for $n \geq 3k+1$ and for all k.

48

REALIZATION WITH FEEDBACK ENCODING

DENNIS P. GELLER
HUMAN SCIENCES AND TECHNOLOGY GROUP
SCHOOL OF ADVANCED TECHNOLOGY
SUNY BINGHAMTON
BINGHAMTON, NEW YORK

FOR A FINITE STATE MACHINE M TO REALIZE A MACHINE M' WE USUALLY PRECEDE M BY A MEMORYLESS INPUT ENCODER TO TRANSLATE INPUTS INTENDED FOR M' INTO THE INPUT ALPHABET OF M. IN THIS PAPER WE INTRODUCE A MODIFICATION TO THIS PARADIGM BY INTRODUCING FEEDBACK FROM THE STATE OF THE REALIZING MACHINE TO THE INPUT ENCODER. THE RESULTING FORM OF REALIZATION DEPENDS IN A VERY STRONG WAY ON (GRAPH) STRUCTURAL PROPERTIES OF THE TWO MACHINES. THE CHARACTERIZATION THEOREM, GIVING NECESSARY AND SUFFICIENT CONDITIONS FOR ONE MACHINE TO REALIZE ANOTHER IN THIS WAY, INVOLVES A NEW CLASS OF MAPPINGS BETWEEN DIGRAPHS. WE THEN CONSIDER THE PROBLEM OF REALIZING A GIVEN BEHAVIOR BY A MACHINE WHICH ADMITS A DISTINGUISHING SEQUENCE. WE SHOW THAT THIS PROOBLEM CAN BE REDUCED TO ONE OF COLORING THE LINES OF A BIPARTITE GRAPH, AND GENERALIZE KŐNIG'S THEOREM TO PRODUCE A CANONICAL LINE COLORING FOR EACH BIGRAPH. THIS COLORING YIELDS A MACHINE WHICH REALIZES THE GIVEN BEHAVIOR, AND WHICH CAN IN GENERAL BE MORE EASILY MODIFIED, BY CHANGING OUTPUT SYMBOLS, TO ADMIT A DISTINGUISHING SEQUENCE.

49

LABELLING OF BALANCED TREES

R.G. Stanton* and C.R. Zarnke

University of Manitoba

The concept of labelling a tree was originally introduced by B. Grunbaum at the 1968 Waterloo Conference on Combinatorics. Basically, one is given a tree with n *nodes, and is required to assign the labels* $1,2,3, \ldots, n$, *to these nodes in such a manner that, if the length of the edge joining nodes* i *and* j *is defined to be* $|i-j|$, *then the total set of lengths represented is just the set of numbers* $1,2,3, \ldots, n-1$.

In this paper, we shall consider a type of tree that we call "balanced"; for purposes of labelling, a balanced tree is defined to be one of two types. We assume that we start from two given trees S *and* T; *then a Type I balanced tree is obtained if we attach to every node of* S *a tree which is a copy of* T, *whereas a Type II balanced tree is obtained if we attach to every node of* S, *with a single exception, a tree which is a copy of* T.

We shall show that if S *and* T *are two given trees that are already labelled, then it is possible to label the balanced trees of either Type I or Type II that are formed from* S *and* T.

50

A NEW CLASS OF FINITE CYCLIC PERMANENT GROUPS

EDWARD T.H. WANG
University of Waterloo

A permanent group G is a group of non-singular matrices such that per (AB) = per (A) per (B) for all $A,B \in G$. It has been shown by Beasley and Cummings that when the underlying field is infinite of char 0 or $p > n$, every matrix in a permanent group must have the 1-diagonal structure in the sense that it has exactly one diagonal with all entries non-zero, e.g., a triangular matrix. The question was left open for $p \leq n$.

The purpose of this note is to answer this question by showing that for $p \leq n$, the matrices in a permanent group need not have the 1-diagonal structure. In fact, for each prime $p \geq 3$, and for arbitrary field F of char p, we shall show that the cyclic group $\langle A_p \rangle$ generated by the $p \times p$ derrangement matrix is a permanent group of order $2p$.

We also construct, by using the derrangement matrix and the de Bruijn-Erdös matrix, a non-cyclic permanent group the matrices of which do not necessarily have the 1-diagonal structure.

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MAXIMAL PERMANENT GROUPS

Larry Cummings

University of Waterloo

Computation of the permanent is generally difficult but of some interest in work involving SDR's and in the dimer problem. One obstacle to computation is that unlike the determinant, the permanent is not multiplicative.

A permanent group is a group of $n \times n$ matrices on which the permanent function is multiplicative. Obvious examples include the groups of upper and lower trangular matrices under multiplication and the monomial groups.

Recent work of L.B. Beasley and the present author gives a characterization of nonsingular permanent groups which contain the diagonal group over fields of characteristic 0 or of characteristic $p > n$. Here we exhibit $\binom{n}{2}$ new maximal $n \times n$ permanent groups over these fields.

52

A GROUP-TESTING PROBLEM

F. K. Hwang
Bell Laboratories
Murray Hill, New Jersey

Consider a set I of n items $I = \{I_1, \ldots, I_n\}$ where I_i can be treated as an independent binomial random variable with probability p_i of being defective and $q_i = 1 - p_i$ of being good, $i = 1, \ldots, n$. The problem is to use a sequence of group tests to verify whether I is good (containing no defective) and if not, then find one defective. An optimal procedure is one which minimizes the expected number of tests to solve the above problem. In this paper, we give such a procedure. We also give an optimal procedure to find all the defectives according to their ordering in a given sequence of items.

53

SOME COMBINATORIAL PROBLEMS IN STATISTICAL INFERENCE

by James G. Kalbfleisch* and D.A. Sprott
University of Waterloo

ABSTRACT

The development of high speed computers has made it possible to carry out exact statistical methods where previously it was necessary to rely upon large-sample approximations. The purpose of this paper is to illustrate some of the interesting combinatorial and computational problems associated with these exact methods. We consider an exact test for the equality of slopes in r log linear models when the underlying probability distribution is Poisson. To obtain the significance level, it is necessary to compute a sum of products of certain combinatorial coefficients $c(s_i, t_i)$. The sum is taken over all sets of values $t_1, t_2, \ldots, t_r$ with Σt_i fixed and $\Pi c(s_i, t_i)$ sufficiently large. Here $c(s_i, t_i)$ is the number of ways to obtain a total t_i in s_i rolls of a k-sided die.