Thirty-Sixth Southeastern International Conference

Combinatorics, Graph Theory & Computing

® Program and Abstracts \ Florida Atlantic University @ March 7-11, 2005

Invited Speakers

36th Southeastern International Conference
on Combinatorics, Graph Theory, and Computing

Monday, March 7, 2005
9:30 AM and 2:00 PM

Alexander Rosa
MacMaster University

**Ringel's Conjecture and Graceful Labellings; Forty Years Later
and Colouring Designs: Some Recent Results and Trends**

Tuesday, March 8, 2005
9:30 AM and 2:00 PM

Charles. J. Colbourn
Arizona State University

Covering Arrays and the Power of Apathy

Abstract

A *covering array* of *size* N, *strength* t, having k *factors* with v *levels* each, is an $N \times k$ array whose entries are chosen from a v-set V, with the property that every $N \times t$ subarray has every one of the v^t t-tuples from V at least once as a row. To understand why these are important, imagine testing a device with k inputs each having v possible values; each factor is an input, and each level is a value for that input. Instead of exhaustively testing v^k possible input combinations, we can instead use the N rows of the covering array to prescribe tests. Each row is a test, each column is a factor, and the symbol in an entry gives the value for the factor in that test. Testing is not exhaustive, of course, but all possible interactions arising from t or fewer inputs will be revealed by at least one test. Moreover, when $t \ll k$, we typically find $N \ll v^k$. Such covering arrays have found extensive application recently in interaction software testing; while not well suited to all such problems, they are useful in many. When one considers that inadequate software testing costs the U.S. economy $20-$60 billion annually, tools for generating test suites (covering arrays) are sorely needed. To minimize testing costs, covering arrays with the fewest rows N for a given t, k, and v are of most interest.

While covering arrays evidently hold much practical interest, our focus in this talk is on mathematical reasons to like them. Most algebraic and combinatorial constructions are patterned on those for orthogonal arrays of index one (equivalently, covering arrays with $N = v^t$) and on recursive combinatorial constructions. We outline four main directions of combinatorial research on covering arrays: direct constructions, recursive (product) constructions, heuristic search, and probabilistic techniques.

We review each of these approaches briefly, focussing on the cut-and-paste constructions. We then weave the threads above to describe recent research on covering array construction using hybrid constructions. The idea is quite simple. The usual strategy for applying recursive methods for combinatorial designs is to first find many small ingredient designs, and then apply the recursive construction. In these cases, the ingredients do not interact and can be found independently of one another.

As we show, the cut-and-paste recursions for covering arrays are different. The ingredient designs can (and often do) overlap. So we propose a decomposition approach. The cut-and-paste recursions specify potential decompositions of large covering arrays into smaller arrays, and the specific decomposition determines precisely how these smaller arrays interact. Our idea, therefore, is to first choose the decomposition of the larger array, and only then to search for the smaller arrays needed. In this way, the interactions among these smaller arrays can be used both to simplify the search, and to permit the array to be "tailor-made" for the role that it plays. Once the properties of the small arrays and their interactions is determined by the decomposition, we can sometimes use direct constructions (from finite fields, designs, or finite geometries) to construct them; and when we cannot, we can employ heuristic search techniques to find them by computer. Of course, the benefit in the approach is that computational methods seem much more effective when the array to be found is small. We propose a particular approach for strength two arrays that exploits "don't care" positions, demonstrating the power of apathy. We close with a list of combinatorial questions on covering arrays.

Wednesday, March 9, 2005
11 :00 AM and 2:00 PM

Mateja Sajna
University of Ottawa

An Invitation to Almost Self-complementary Graphs

A graph is called *almost self-complementary* (A.SC) if it is isomorphic to the graph (called an *almost complement* of X) obtained from its complement by remo\'ing the edges of a 1-fuct.or. The study of almost self-complementary graphs \\as first suggested by Abpach, aud i11itiatcd by Dobson and Sajna in a 2004 paper on almost sclf-c:ornplcmcntary drrnlant graphs. This paper rewabl the complexity of the problem of ASC graphs: while C\'ery automorphism of a graph is also an automorphism of its complement, the same may not be true for an almost complement; and while an isomorphism from n self-complementary graph to its complement exchanges the two edge sets, an isomorphism from au ASC graph to au almost complement need not prcsen·e the "missing" 1-factor and therefore need not exchange the edges of the graph with those ofthe almost complement. An isomorphism from an ASC graph to an almost complement, as well as an automorphism of au ASC graph, is called jafr if it preserves the associated !-factor. In this talk we shall present some recent results on \'arious types almost self-complementary graphs.

Part I: Constructing Almost Self-complementary Graphs

Several construction techniqur.s for ASC graphs will be introduced and used to prove existence results for i;ome infinite families of ASC graphs, in particular, regular, vertex-transitive, and circulant ASC graphs, distinguishing those that admit fair isomorphisms into all almost complement.

Part II: Homogeneously Almost Self-complementary Graphs

We shall focus on a special class of vertex-transitive ASC graphs called *homogeneously almost self-complementary;* that is, ASC graphs possessing a vertex-transitive group of fair automorphisms and a fair isomorphism into the almost complement that normalizes it. It turns out these are precisely the graphs that occur as factors of symmetric index-2 homogeneous factorizations of the graphs $K_{2n} - nK_2$. We shall present several constructions and existence results, including the classification of all integers n of the form $n = p^r$ and $n = 2p$ with p prime for which there exists a homogeneously almost self-complementary graph on 2n vertices.

Keywords: almost self-complementary graph, regular graph, vertex-transitive graph, circulant graph, homogeneously almost self-complementary graph, homogeneous factorization.

Thursday, March 10, 2005
9:30 AM and 2:00 PM

John Wilson

University of Toronto

Axiomatic Circuit Theory

Friday, March 11, 2005
9:30AM

Tran van Trung

University of Duisburg-Essen

Combinatorial Methods for Covering Arrays

We survey algebraic and combinatorial techniques for constructing covering arrays of strength t � 3. Some of these techniques *are* inspired from a recursive construction given in the 80's by Roux.

Friday, March 11, 2005
11:00 AM

Brief Session Dedicated to the Memory of Frank Harary

Organized by Gary Chartrand with John Gimbel and Jay Bagga

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1 On Menon-Hadamard Difference Sets in Groups of Order $4p^2$

Omar A. AbnGhneim*, Ken W. Smith, Ccnt:rnl J\:Iirhigan University; Pm1l E. Becker, .Jennifer I<. Mendes, Penn State, Erie

Menon-Hadamard difference sets in groups of order $4N^2$, with N a multiple of 2, 3, or 5, ,we the only known gennindy 11011ahdia11 difforence sets. Iiams showed that if N is a prime congruent to $1 \mod 4$, then only 6 groups of order $4N^2$ could admit Menon-Hadamard difference sets. I11 this paper, we prove that another two of these 6 groups can be eliminated. Our µri111ary tools are quotient images and complex group char<1cters.

2 A New Algorithm That Improves Network Performance by **Maximizing The Number Of Disjoint Paths**

\IVasim El-Hajj, Ghassen Ben Brahim, Chandrasekhar Achalla*, Dionysios Kountanis, Western Michigan Uuiversity

In this paper, we **will** be dealing with a network planning problem. It consists of optimally i11tcrco1mcd.ing a set of new switches to d pre-existing nct:work contigu-ra.l.ion. Wf. consicfor a nf.1,work with two types of nocfos: tlw switcl1f.s will be p!M:e<l in the backbone network nnd the network access points will be placed nt the edge network. fotcrconnecting .uew switches to a pre-existing uetwork

lws the advantage of not modifying thc current co1111ections. The goal of the proposed scheme is to improve the overall net.work performance in l.erms of: (I) increasing network protection and fault tolerance, (2) providing better Quality of Services (qoS), and (:1) decreasing the blocking probability. We propose an intel-ligent al:.';orithm that maximi7,es the number of link <lisjoint. paths between a set of source and destination pairs. Having several distinct paths, the blocking prob-ability will eventually decrease because more routes will be available for different source and clcsti1 1atio11 trnffic exdm11gc. The 11etwork will he simulated hy a graph c; = $(V; Ji)$, where V is thf. set. of nodes an<l]:; is the set. of links. Om objective is to maximize P;:$_i$ V l\$_{\cdot i}$,j , where /\:;;; is the maximum number of distinct shortest pc1ths betwee11 node i an<l node j locc1te<l at the edge network under the constn1i11t that the <lcgrce of the new switches added is a given constant.

Keywords: Net.work <lesign an<l planning, prot.cc:t.ion, fault, tolerance, blocking probability, QoS

3 Cluttered Orderings for the Vomplete Bipartite Graph and the Complete Tripartite Graph

Tomoko Adachi, Toho University, .Japan

The desire to speed up secondary storage systems has lead to the development of rcd 1111da 1t arrays of indepenclent disks (RAID) which i1 1corporat.c red1111d,1cy nti-lizing erasure code. To minimaze the access cost in H.AID, Cohen, Colbourn and Froncek (2001) introduced (d, f)-cluttered orderings of various set system for posi-tive integers d, f. Iu cai;e of a graph this a1101 1 1ts to a11 ordering of tlw edge set such that the number of points c:ontainc<l in m1y <l conscc:11tivc edges is bo11nrlcrl by thc number f. For the complete graph, Cohen et 1.1 gave some cyclic co11strnc:ticms of cluttered orderings based on wrapped rho-labellings. t-.f' uller, Adachi and .Jimbo (2005) investig,it.ed cluttered orderings for l.he co111plct.c bipartite graph. RAID nt.ilizing two-rliment.ional parit.y code can be modeled by the compl<'1.<' biparl.it.c grnph. Jvf' uller et al. adapted the concept of wrapped Delta-labellings to the bi-partite case instead of wrapped rho-labellings, and gave t11c explicite constructio11 of several infinite families of wrapped Dclt11-labclli11gs. Herc, we investigate con-structions of more generalized infinite families of wrapped 0f)lta-labcllings leading to cluttered orderings for the corresponding bipartite graphs. '.\foreovcr, we in-vestigate cluttered orderings for the complc1-c t.ripmtit:c grnph. RAID 11ti1i/i11g t.hrce-diment.ional parity code can be modeled by l.he c:mnplct.c t.ripartil.f'. graph. In this U1lk, we will give constructions of wrapped Dclt.a-labelliugs for such cases.

4 On Muitipart.ite Poscts

Ceir Agnarsson, George I\Iilson University

Let P = $(X,)$ be a partially ordered set (or 710sel for short). If the underlying set X of P has a partition $X = X_1 \cup \cdots \cup X_m$ wilh $m \geq 2$, such that P is incluced hy a collectio11 of bipartite po cts $P_i = (X_i, X_i\ t, ;)$ where $i \in \{L \ldots, m-1\}$, then we say P is a *m-parlile posel.* If P is m-pariite for some $m \geq 2$ theu we say it is *mttltipartite.* Such multipartite posets occnr naturally in many situations, in particular wheu combinatorially dllctly;d11g discrete communica.tion 1wtworks.

In this t.alk we rlisc11ss thc order dimension of m11ltipartif<! posct.s and what param-eters can be used to present concrete 11pper and lower hounds for them in genenll. Some open problems will be presented.

5 On Cycle Matrices of Graphs

K. 13r1lr1s1.1bramanir1n. Indian Statistical Institnt<", India; Sahu Alsar<lary*, University of the Scie11ces in Philadelphia

For simple graphs without loops or multiple e<lges, we define four parameters $a(G)$, $A(G)$, $b(G)$, and /J{G} b<lBc<l 011 the cydc sp;icc. We completely dmnictcrizc grnpb for which $a(C) = A(C)$. We also introduce an invariant JJ(G) ahd connect it with $b(G)$.

6 On minimally 3-connected binary matroids

.loP. An<lerson*, Hairlong Wn, The Univ<"rsity of Mississippi

A 3-connected matroid J\f is said to he *minimally 3-connected* if for any clement *e* of /1/, the 111atroi<l *I\I\e* is 11ct :l-co1111cctcd. Dawes *(J. r.ombin. Theory ,',er. R* 40, {Hl8G), l!i!J-168) show<'d that r1ll minimally -connect.eel grnphs c;m be constrnr.t.ed from 1\4 such that every graph in each intermediate step is also minimally 3-c:onnected. In this paper we generalize this result to minimally 3-c:onnectecl binary 11mtroids.

Keywords: binary matroicls, 3-connected matroids

7 Robustness of Property of Being Matchable subject to Vertex Deletion

R.E.L. Aldred, University of Otago, New Zealand; R.P. Anstee*, University of British Columbia, Canada S.C. Locke, Florida Atlantic l:"niversity

\Ve consider cl&;ses of graphs which c1re easily sec11 to have 111c111yperfect 111c1tch-ings. Wf. then consider what. properties to impose on choosiug vertices A for vert.1x deletion in a graph G (from such a class) so that the vertex deleted subgraph $G-A$. has a perfect matching. Certain conditions are easy. Th general, an even number of vertices 11111st he deleted. If the gTaµh is hip;:1rtitc then t.hc deleted vertices 11111;t have equal numbers from both parts of the bipartition. Also one cannot delete all the neighbours of a given vertex. We obtain two results. In one, the deleted ver-tices are co11fi11ed to tlie 'edge' of the graph a11d in the other, the deleted vertices arc reqnirc<I to be far ap;ut.. The motivation w11s a result of Jamison anrl Lockner presented at CGTC 34.

8 DNA Compression using Inversions and Longest Increasing Sequences

Ziya Arnnvut, SU:\"Y Fredonia

Compression of D:\'A sequences is one of the most. challenging tasks in t.hc field of data comprcssiou. A 111011g the gcucral purpose coJcrs, only ciritl1111ctic coder achives compression rate below two hits per symbol. Sta.ndard u11iversal compres-sion tools, such as gzip and \)Zip, usually fail to achieve compression below two bit per sy1ubol 011 DNA data files. DNA compressors ad1icvc co111pressio11 ra1,c below 2 hit.s per symbol. However, most of *DNA* :;pedfic: cmnprcssors r1re ver:v slow and often they use pattern matching techniques. hi this work, we show that using re-cently introduced inversion coding and longest increasing subsequence techniques we can always achieve compression ratio below t,wo hits per sy111hol il.IId u11 some DNA tesl. files we 11chieve bel1 er rnt.e tha11 1h0 most. of l.he DNA compn ssors.

Keywords: DNA compressicm, Inversion Coding, Longest IncrC"asing subse-quence.

9 2-Regular Leaves and Partial Decompositions of the Cornplcte Graph Λ_n

D. J. Ashe*, University of Tennessee at Chattanooga.; C. A. Rodger, Auburn U11ivcrsity; II. L. Fn, N;;tlional Chi,10 T1111g U11iY<'rsity

We fin<l necessary and sufficient conditions for the existence of n fi-c:yclc system of !{,,- E(R) for eYery 2-regular not 1wc:essarily spanning subgraph H of 1_n ·

10 Bibliometrical gorithms for discovering communities in complex networks

Hemant Balakrishnan*, Narsingh Deo, University of Central Florida

Receut studies revel that. most of the real world 11ctworks org,mi,,,c themselves to form commm1it.irs. A c:ommnnit.y is formr<l by subset. of nodes in a graph t.hat are "dosely related". Extracting these communities would lead to a better understc111di11g of such networks. Cu1111111111ity related research hc1; focused 011 two 111a11 problems, communiJ-.y discovery anc! comm1mit.y identificat,ion. From a grnph theoretic perspective community discovery is the problem of classifying nodes of a graph G = (V, E) into subsets C; V, 0 =; i < k, such that nodes belonging to a suhsct. C.i arc all closely related where as co111n11111ity ide11tific,1tio11 is t.hc· problem of identi(ying t.he commmnit.y C; to which a srt. of ncitles S V belong t.o. In this paper we first perform a brief survey of the existing community-discovery-algorithms and then propose a novel approach to discovering communities using hibliogrn.phic: metrics. We l'llso test the propose<! algorithm on rcal-worlcl ndworks and on computer-generated models with known community structures.

11 On the Erdos S'os Conjecture for graphs with no K2,s

Smmin 13ala.c;11hrn.manian*, Edward Dobson, :\,Jississippi Slate University

Let k be a positive integer. Erdos and Sos have conjectured that every gra.ph of average <lrgree gre;i.t.er than k - 1 contains every tree of order k + J. In this paper, we verify Lhat this conjec:Lure is true in the special case of graphs that contain no $K_{2,s}$, wheres 2: 2 and k > 12(s - 1).

12 Being a Unit Triangle Order is a Comparability Invariant

Barry A. Balof*, Whitman College; Kenneth P. Bogart, Dartmouth College

A property P of a partially ordere<l s d is a mmpn.rn.bilily *irwn.ri.rm.1.,* if, given mw two posets X c111d Y that have the same comparability graph, ·then either both X and Y have property P or neither have propert,y P. A theorem of Galla.i's allows us to <lctermi11c the cu111para.hility invariance of a property thrnugh the reversal of order a111.onom011s sct.s within a poset. wil.h th11t; propert,y.

A *uni/. tri.angle order* is a poset X that has a representation by unit triangles, that is, every element in X can be mapped to a triangle, with each triangle having one vertex 011 one of two p,1n1llcl hascli11cs, nm! the other two vertices 011 the other of those two baselines, with all triangles having the same area. In this talk we will show that being a unit triangle order is a comparability invariant, and, with time pen11itti11g, give a su111111cl-ry of tlie know11 results c1Lout c:ompc1raLility i11varic111ce an<l geometric representations of posrts.

13 Some Results Related to Maximal Independent Sets of Vertices in a Graph

Rommel Barbosa, Institute de Informatica, rnivcrsidadc Federal de Goic1s, Brazil

A graph is Z_m-well-covcred if III = IJI (mod 111), for c1ll /, ./ maxiuw.l i11dcpc11dc11t sets in $V(C)$. A graph G is strongly Z_m-well-covered if G is a Zm-wcll-covcred grnph and G\{e} is Z_m-well-covered, Ve E $E(G)$. A graph C is 1-Zm-well-covcred if G is a Z,,.-well-cuvcred graph and G\{1•} is Z,,,-well-covered, \/11 E V(G). We prove some properties for these da.c;ses of g;raphs.

14 Splitters and Barriers in Graphs Having a Perfect Inter·nal Matching

Miklos Bartha, Memorial University of Newfoundland

l\Iatchings with a specified potential defect are introduced, which arc not required to cover a specified set of vertices. These vertices are called external, as opposed to internal vertices which arc expected to be cuvcrccl hy all 11wtc1ings of this nut.me. Snc:h matchings play an import.ant. role in t.hc mnt.hemat.ical <fosC'ript.ion of certain molec:nlar switching devices called soliton autornat,1. A perfect (maxim11111) internal matching is 011e that covers all (respectively, a n1dxim11m nnmber of) interna.l vertices. The not.ion of harriers is adopte<l from cla.c;sic:al ml'l1.r.hing thcory, and splitters are introduced as appropriate counterparts of extreme sets of vertices in graphs having a perfect matching. !Vlaximal splitters are compared -.vith maximal l>ct1Tiers, aud factor-critical g 'dphs are re-iutrnduced in t,hf' new cm1tcxt.. A Tut.te-type charad.erbrnt.ion is given for maximnl splitt.ers in g;rnphs with perfect. int.en ml matchings, and an efficient lgorithm is worked out to locate the maximal barriers of such graphs.

Keywords: gn:i.ph matchings, split.ters, barriers, factor-crit.icnl gniphs

15 On Cycle Extendability

LeH.o_y B. l:k11sley*, David E. Brown, Ut.ah St.at.e Univrrsity

A cycle C of length k iu a graph G is extendctble if there is an iuduc<'d subgraph 011 k+1 vcrtiCf$ of G which rnntains all the vertices of C, mid a cycle of leJJg;t h k+l. A graph, G,is cycle extendible if every cycle of G which is JJOt a Hamilton cycle is extendable. 'Ne investigate cycle ext.e11clible Tfamiltoni,1.11 chordal gn1phs and the bipartite equivalent.

Keywords: Graph, cycle ext.111<lible, Hamiltonian, Chordal

16 On Computing the Number of Topological Orderings of a Direct.ed Acyclic Graph

Wing-Ning Li, Zhichun Xiao, Gordon Beavers*, University of Arkm1sas

C<m the 11mnhcr of topologirnl orderings of a Directed Acyclic Grnμh (DAG) be effic:ient.ly <let.ennine<l? We propose a <livide-a;1<l-co11q11cr rnet.horl that part.itions a DAG into sub-digraphs from which the number of topological orderings is cal-cnlate<l using co111binatorial rnethods. Algorithrns are com;idere<l to identify sub-<ligraphs whose vertices m11st occ11py the s;i.mc specific.: rnngc in any linear onlering. Such sub-digraphs are n_\lled static sub-digTaphs. Transitive closure and transitive reduction are useful in iclentilying the static sub-digraphs. Open issues, such as s11b-dign1phs for which 110 obvious part.itions can he found, ;1re disc11sscd.

Keywords: Direct.cc-l acyclic graph, topological orrlcr, transitive closure, t.rn.nsit.ive reduction

17 Regularity among generalized Schur numbers

Pct.er Illanclwrd, Miami University, OII

We discuss generalized Schur numbers. Let *h(r;* m, *d)* be the le<1st *n* such thnt any imrtit.icm [11] = A, U A2 U · · · U A, hc1s sonic cell A_i containing ;i set {:r:, y, i + ?]} with thr. propert.ies that x, y 2 m, an<l IY - xi 2 *d*. In other worrls, t.hc least n so I.hat no r-coloring of [n] fails to yield a monochromatic.: Schur triple {.x, y, :i: + y} with differences between values at least d and m respectively. We discuss unexpected reg11larity among the 11111nhers l1.(r; m., ,l) for small val11cs of r.

18 New Results on Packing and Covering Designs

Iliya Bluskov*, University of :'\orthern B.C., Canada; 11alcolm Greig, Greig Cornmltiug, Ca1iaclct

Given a set V or size t•, a (v, *k*, >.) covering (packing) design is a collection orb k-subsets (cc11le<l blocks) of *V* such that ec1ch pa.ir of de111e11ts of *V* occurs in at lc;i.,;t. (at most) >. blocks. The covering (packing) 1111111her C(v, /;, >.) (D(v, l..:,>.)) is the minimum (maximum) value of bin any *(v*, k, >.) covering (packing) design. We present some new results on the covering and pa.eking numbers for the parameters (11, 5, ,\), >. > 1. Iu particular, for /. = 5 c1111d 1' even, there circ 24 open cases with >. 21, cach of which is the start. o[an open series for >., >. + 20, >. + 40, ... We solve 22 of these cases with >. 21, leaving open (v,5,>.) = (44,5,13) and (44,5, 17) (an<l the series initiated for the former). In the packing cc1se, we reduce the number of open sets of parameters from 20 to 10.

19 Multicolor Euclidean Gameboard Ramsey Numbers

.kns-P. Boric, Tec:hnische Univcrsit.ii.t Brn1msd1wcig;, Gcnn;i.n_y

For the three Euclidean tessellations of the plane we define IJ_1 to be one cell (tri-angle, square, or hexagon), R_2 to consist of all cells snrronnding one vcrl.ex, and Bn to cousist. of B_{n-1} together with all 11cighbori11g cells. These scquem:cs /J,, ,ire used as host graphs for the 11111lti-color H.amsey 111m1ber r(//1,//2, . . . ,Hc) being the smallest number n such that every coloring of the edges of Bn using the colors 1, . . . , c co11tai11s a graph H_i iu color i for at lc,Jst one i. First result.s o11 the exis-tence of multicolor Enc:lirlcan gamcboarcl Rmnsey numbers ,111rl some cxaC'1. vahlC's are presented.

Common work with Stefan Krause.

20 Linear dependency of sets of independently weighted bina1·y vectors

Kim Bowman, Clernson University

We investigate the following model of random binnry vc•ctors: c.oordinMes nre cho-sen independently: the ith coordinate is chosen to be one with probabilit.y ,L, where Pi is the ith prime. In particular, we study how many vectors need to bc chosen to ohtaiu a linca.rly clcpcu<lcut set with high prohnhility.

Keywords: binary vectors, linear dcpcnrlcncy

21 The Distribution of the Size of the Intersection of a k-Tuple of Intervals

Vlnrlimir Ilm:ovic*, Sh:i.nzen Gao, Heinrich Nkdcrlrn.nsen, Florida Al.lantic University

Let (/1, . . . , h) be a k-tuple of nonempty subintervals of [L . . . , n]. How many of. them intersect. in an interval having l clements (l = 0, . . . , Il)? For /. = 2 we have a bijection of the pairs (/, .J) with I/ n .JI = I to the discrete octahedron. For larger k the results seem to be less familiar; the rcsull.s for· k = 3, ,1, 5 ;\re not in the On-Line Encycloped·ia of Integer Seq1tc11.ccs.

22 Perfect-Matching Preclusion

H.obert C. Ilrighmn*, l:nivernity of Central Flori<la: Frnnk Ifarnry, EJi7,abeth C. Violin, Harvard College; Jay Yellen, Rollins College

The (perfocl-) matching preclusion nmnber,mp(G) of an -vertex graph is the minimum number of edges that must be removed from in order to ensure that the re:mlt.ant. graph does not. have a perfect. matchiug if is eveu, or a 111atchi1]g 011 vertices if is od<I. We establish the value of mp (G) for various classes of graphs.

23 Probe Interval, Interval k-, and Tolerance Graphs

D,wicl E. Brown*, Utah State Uuiversity; Stephen C. Flink, U11iversity of Colornrlo at. Denver

\Ve iutrod11ce a series of ge11endi:1,at.ions of prol,e interv.:\I graphs called t-probe interval graphs, (a probe interval graph is a 1-probe int.erval graph) anrl show, via a method similar to graph homomorphism, that each class, including the class of probe interval graphs, is contained in the class of interval k-graphs. Any probe interval g1-aph is dc,wly a tolcnLIICC gn1ph, but for some f > 1 this relatiom;hiµ f ails. We wish lo rlekrmine !:his t. Also, the interval k-graphs whose complement. describes a poset arc believed to have a nice characteri7,at,ion via forbidden subgraphs, d11d we give the co11ject11re here, d1nd a new description of these intervc1.I k-graphs that is similar to the salient property of fond.ion graphs.

24 Hexagon Decompositions and Packings of the Complete Graph with a Hole

LaKeisha Brown*, Robert (;ardner, East Te11J1essee State University; (;ary Coker, Fri'lnc:is 1-forion University; .Janie Kennerly, Samforrl University

A *decomposd'ion* of a simple graph c; i11to isomorphic copies of a grapl1 g is d set $\{y_1, \cdot\prime 2, \ldots, Jf_n\}$ where ff; .J anrl $V(g_i) \subset V(G)$ for all i, $E(g_i) \cap E(g_j) = \emptyset$ for $i \neq j$, and $\bigcup_{i=1}^{n} E(g_i) = E(G)$, where $V(G)$ is the vertex set of graph G and $E(G)$ is the edge set of grnph G. A mn.ximn.l pn.rkin.1 of 11simple graph G with isomorphic: copies of a graph g is a set $\{g_1, g_2, \ldots, g_n\}$ where g_i g and $V(g_i) \subset V(G)$ for all i, $E(g_i) \cap E(g_j) = 0$ for $i \neq j$, $\bigcup_{n=1}^{n} g_i \subset G$, and $|E(G) \setminus \bigcup_{n=1}^{n} E(g_i)|$ is mini-111.:\l. The complete grapl1 011 v vertices with d hole of si.:e w, $K(v,w)$, hd:; vertex set $V(K(v, w)) = V_{v-w} \cup V_w$, where $|V_{v-w}| = v - w$ and $|V_w| = w$, 11nd edge set $E(K(v,w)) = \{\{a,b\} | a \neq b, \{a,b\} \subset V_{v-w} \cup V_w$ ind $\{a,b\} \not\subset V_w\}$. \Ne give necP.Ssary an<I sufficient. con<lit.ions for 6-cyde decompositions of $K(v, w)$ and give preliminary results concerning 6-cyc:le packings of K(v, w).

Keywords: 6-cycles, graph decompositions, graph packings, complete graph with L hole

25 Alliance Edge- and Vertex-Stability in Graphs

G. Bullington, L. Eroh, .J. Koker, H.l\Joghadam, S. Winters, University of Vlisconsin-Osl1kosh

As defined by S. 1\1. Hedetniemi, S. T. Hedetniemi and P. Kristiansen, a (cldensive) alliance of a graph is a set of vertices satisfying the comlit.ion that every vertex has at 111ost one 111ore 11eigl1bor i11 tha11 in . The cillia.1H:e m1111ber of , , is the s111cdlc;t r:arrlinality of any (nonempty) defensive alliance in . In l.his talk, we give results addressi11g the following question: "\Nhat. graphs keep the same alliance number when a vertex (resp., edge) is deleted?" We clc1ssily dll alliance st.able grnph; having low -valll(.'8 and those within pa.rticular cla ses of graphs (e.g., co111plcte graphs, grirl graphs). We will also presEmt some related honnds m; wP.11 as r<'s11lts for ol.hP.r types of alfainces (e.g., strong alliances, global allia.nc:cs).

Keywords: allia.nces, defensive a.llianccs.

26 Matching Cove1·ed Graphs

Kimberly Jordan Burch*, Montclair State Universily; Earl Glen Whitehead,.Jr., University of Pillslmrgh

Two edges in"' graph G arc in.rl.r.pr.n.dr.n.l if they share no common vertex. A 7ir.i:fr.r.l *matching* of a graph G is a spanning subgraph of G consisting entirely of independent edges. G is said to be *matching covered* if for every edge e in G, there exists a perfcc.;t matching co11tai11i11g r. A matching covered gni.ph is eq11ivalc111. to a totally mat.chablP. grnph. We prove conrlitions 1111rlP.r which several families of graphs arc matching covered. Families presented include meshes, complete tripart.itc grnphs, genenili:1,ed theta graphs, platonic graphs and (k,g)-ca.ges. An m x n mc8h is the pro<h1ct of path graphs h<'l.ving ·111. anrl n vertices. A (/.; ,g)-rn.qr. is a k-r<'gnlar grn.ph of girth g with the fewest possible number of vertices. We also examine suff cient conditions under which a graph will be matching covered.

27 Colouring 4-cycle systems

Andrea C. Dmg-css*, D:wid. A. Pike, :\frmorial l:nivcrsit_v of Ncwfo 1111dla11d

An m-cycle system of order n is a partition of the eclges of the complete graph T<, i11to 111.-cydes. An m.-cyde :sy:stcm of order n is :said to he 1.:-colonrahlc if its vertice:s may be partitioned into k sets (also called colour classes) such that no cycle has all of its vertices the same colour. A cycle system is /.:-chromatic if it is k-colourable, but not (k - 1)-colourable. We focn:s on colomiug:s of 4-cycle :sy:stems:. For ,my L: 2 2, we show that them exists a A:-c:hromatir: 4-r:yde system. In part.irnlar, we construct a 3-chromatic 4-cycle system of order 49.

28 Alphabet Overlap Digraphs

Arthur H. Busch*, J\Jichael S. .Jacobson, University of Colorado at Denver; GuanTao Chen, Georgia State Univcr:sit.y; Ralph .J. F,mdrec, U11iver:sity of Memphis

Michael Ferrara and Ronald .J. Gould, Emory University Nathan Kahl and Charle::; L. Suffel, St.evcu:s fo:situte of Technology The alphabet o-uerlap digmph G = G(d, l:, f) hi:< as its vertices all words of length l; formed from an alplmhet of size d. The arc (W1, w2) is in A(G) exactly when the last t letters of 1111 coincide with the first t letters of w_2. We will discuss various properties of alphabet ovcrl,1p digni.µh:s, and their 1111directed a11alogue including i11depc11dcuce munhcr, diquc nnmbcr, conner:t.ivil.y, panr:ydicit.y, all(! chromat.ir: nmnbcr 11s well as Lhe connection between alphabet overlap digraphs and line digraphs.

29 Some properties of n-dimensional generalized Marlkof equation

Shanzhen Gao, Cafer Caliskan*, Florid;1. Atla.ntic University; Xianglin Liu, Cuang:,;hon Gongye l:niversit.y, China

\Ve discuss some properties of n-dimensional generalized J\,farlkof equations.

30 A Local Method for Community-Mining Based on Clustering Coefficient

Amel Cami*, Nar:;iJJgh Deo, U11ivcr:sity of Celltral f'loricJa

Community mimng in real-world networks has emerged as a problem of great practical importance in the last 2-3 years. :\'lost of the existing algorithms fur solving thi:s proble111 1:1re graph-theoretic in miture: the real-worl<l 11etwork of interest. is modeled as a graph and commnnit.ics arc <let.ermined by analy:,;ing the structure of thi:s graph. At lecJSt two <lh;ti11ct fornmlat.io11s of co11111111111ity 111i11ing have bMn proposed: (1) partition into c:omnmnit.ics refers to part.it.inning a given graph into subsets of nocks, each forming a comm1111ity:. mid (2) seed growth refers to finding the community to which a given 'seed node' belongs. Wldl,! sc,-cral algorithm:; · employing tcd111iq11cs that nrngc fro111 ltien1.n;hic;-1.I d11:stcri11g to spectral partitioning and network flows-have heen put forward for the former, relativdy little attention has heen devoted to the latter. In this paper we introduce a 11ovel algoritl1111 for the :seed g'owth problem. The proprn;ecl algoritl1111 is greedy, and thus very fast. It expands a community by searching the neighborhood of the nodes that already belong to this community and employs dust.ering c:ocffic:ient to determine which nodes to add to the community at a particular step. \Ve presc11t expcri11le11ct re:sults 011 both colllputer-gc11erat.cd and re,il-world 11ctworks.

31 A Generalization of the Erdos-Ko-Rado Theorem

Patricia Carey*, Josh Fair, Anant Godbole, East Tennessee State University

The Erdos-Ko-Ilario Theorem st;:it.cs 1hat ii' 11. > 2r, and A is :i farnily of pairwise intersecting r-subsets of {I, 2, ..., n}, then the nmximum number of sets that. cin be in A is given by

$$|A| \binom{n-1}{r-1}.$$

Furthermore, if A actually has this many sct.s, t,hcre is some element x of {1, 2, ..., n} such that A is the family of all r-size subsets of {I, 2, ..., n} co11t.-;ini11g :1:

We wish 1o gcnernli:,;c this theorem. If A is a family of s11bsel s of {L 2, ..., n}, s11ch that each subset is of sizer, and 'i A, B, C E A we wa11t the condilio11s

IA n n n c i i 0
IA n JJ n c^c 1# 0
IA n B^c n C I f 0
I A" n B n C I i 0

to all hold. An upper bound on the maximum 1111mber of sets t:'111t can be in A n1n be found u:siug the probabilistic 111etl1o<l whe11 the c:i:pcclcd sire of e,,ich set i11 A 'is r. WC also fonnrl an upper hon11d on this nmnbcr using a more gcnc'ral mcU1od tht\t guarantees that IAI = r for each A E A.

32 Some Tricyclic Steiner Triple Systems

Xdl P. Carnes, McNeesc St.ate University

A Steiner triple system of order v, STS(v), is a pair (S; -), where S is a set of v poi11ts am! - is a collection of three de111e11t s11b-sets of S, ca.lied blocks such thr1t any pair of rlistinct points of S is c:ontained in precisely one block of -. An a11tomorphism of a Steiner triple system, (S; -), is a permutation of S which maps - onto ¯. In this paper we give necessary conditions for the exist.ence of a Steiner triple :;;yst.clll of order v admitting an c111tomorphisn1 consisting of three cycles of equal length and 0 or I -xed points.

Keywords: automorphism, tricyc:lic, Steiuer triple system

33 On Friendly Index Sets of Second Power of Paths

Sin-Min Lee, Urian Chan*, Zhou Xin-lin, San Jose State University; Yong-Song IIo, Nan Chiau High School, Singapore

LCt, G he a graph with V<'rtex s d V(G) nm! edge set E(G) am! kt *A* he an ahelir1n group. A labeling *f* : *V(Cl)* _. *A* induces an edge labeling /* : 8(G) -+ *A* defined by *J* * *(xy)* = *J(x)* + *J(y),* for each edge *xy* E E(G). For i E *A,* let *v1(i)* = *#{v* E *V(G)* : *J(v)* = i} and c,(i) = *#{e* E *E(G)* : *f* * (c) = i}. Let *r(J)* = {h(i)-e.r(.j)J : *(i,.i)* E J1 x /1}. A labeling .f of a graph *G* is said to be A-friendly if Jt•1(i) - *v1U)I* :S l for all (i,j) E *A* x *A* If *c(J)* is a (0,1)-matrix for an A-friendly labeling *J,* then *f* is sa.id to be A-cordia.l. When *A* = z,.., the f ie11dly i11<lex set of the grnph *G, Fl(G),* is defined as { k1{0) - *c1(* 1)1 : the vertex labeling *f* is Z2-friendly }. In this paper, we completely determine the friendly index sets of second power of paths.

34 The Queens Separation Problem

R. Douglas ChathamChatham, Gerd H. Fricke, R. Duane Skaggs, Morehead State University

The da<;sic- n-qmens problem rt<ks for r1n arrangement of n qneens on an n x 11 chessboard in which no two queens attack each other. We show that for n>5, we can place n+1 queens that don't a.tta.ck each other on an n x n board, if we are ,tllowcd to ctlso pl,1ce a single pawn 011 the hoard to block a.tt,1cks. We al:;o proof tlmt n+k queens can be separated by k pawns for large enough n.

Keywords: 11-Queens problem, Queen separation

35 On The Construction of Graphs with Large Numbers of Spanning Trees

Andrew Chen*, Abdol-Ilossei11 Esfaha11ia.n, Michigan State University

Let t(G) denote the number of l,ibeled spanning trees of a connected graph (;. Given G, it is k11owll how to co1J1p11tc t(G). However, litllc is

kllown abo11t tlm extremr1l version of the problem, UmI. is, given Lhe nnrnber of vertices n and the number of edges m, find ct connected {n,rn) grnph G such Umt t((:) 2 t(H), where H is any other (n,m) connected graph. S11ch a gniph (; is wiled a t-optimnl graph. Let. t.(n,m) he the m1mber of spanning trees tha1. a t-optiJT};\l (n,m) graph has. We present brute force results (obtained through using a soft-ware called nauty) for determining values of t(n,rn) for n :S 12. These results and· others prnvide motivctio11 for a number of conjed.ures, so11w old and sollle new, with regard to the c:onstrnc:tion oft-optimal graphs. 'fogetlwr, 1.lwse conject.nr<'s suggest a technique for finding many t-optimal (n,rn) graphs when 2m :S 3u.

Keywords: spanning trees, graphs, t-optirnal

36 Stable Multisets

Eddie Cheng, Oakland University

A stable multiset is a generalization of stable set (or independent set) such tha.t a vert.cx can be included more thau 011cc up to some upper houll[]:; iuclncc<l hy the vertices and edges. This concept was introduced recently hy Koster and Zymolk1.l. In this talk, we report some of their results as well ns our resn]t.s (joint \\ork with Sven de Vries). The talk will include a result. 011 a polynomial ti1ue algorithm for this problem on a special da<;s of graphs.

Keywords: stable set, independent set, polynomial time algorithm

37 Tiling with Triominocs

Patrick Callahan, Univcrnit.y of California; Phyllis Chinn*, Ilnmboldt. State l}niversit,y; Silvia Heubach, California State University

Solomon Golomb. in a Hl!i:l talk at the Harvnrrl l\fat.hemrit.ks Clnh rldinerl a da.c;s of geometric figures called polyominoes, namely, connected figures formed of con- grll(mt squares placed so that ca.ch square shares one side with at least one other :sqnare. Do1ni11oe:s 11sc 2 :sqm1!'cs; Tctris piece:; (or tctrominoc:s) 11sc 4 :,;quarc:s. Poly- ominocs wen, popnlarize<l by !\fart.in Gardner in his Sci<,nt.inc American c:ohmms. Many of the initial questions asked about polyominoes concern how many can be funned u:sing n-:squa.'es. In this pa.per we con:sider tiling:; of rectangle:; u:sing t,he :l-sqnnre fig11rc.c;, or triominoes. Since t.here ar<' only two s11d1 shnpes wit.h 3-squa.res, we count. instead how many ways they ca.n be used to tile 2 by n and 3 by n rectangles and how many of each shape are used among all the tilings of a µmticulcir :sie rectangle.

Keywords: tilings of rectangles, t.riominoes

38 Cages of degree k are k-cdge-conncctcd

Michael H. Moriarty, Peter R. Christopher*, \Vorcester Polytechnic Institute

\Ve deten11ine the edge-com1ectivity of cage:s, reguhtr g'a.phs of mi11im1111 order having specified girth. We show that cages of degree arc k-edge-connected.

39 A Heuristic Algorithm for Computing Optimum Core-Based Multicast Tree

Ping-Tsai Chnng, Long Island University

\Ve present a heuristic algorithm to compute Optimum Core-Based Multicast Tree (OCB1..IT). An OCB:V1T is defined as the shortest-path multicast tree with the minimum value of the avenige group ·:shctrc delay iu c\given uet.work with H distin- guish,xl m11ltica.c;ting norlc set.. ThP. OC131IT prnblem ha.c; beP.n st.nrliP.rl by Clrnng int.his conference (33CGTC) in 2001, where Chung sturlied two algorithms to com- pute 1:1.pproximations to OCBl'dTs. Both algorithms achieve approximation ratio of 2, 1hat is, they genera.tee! t.he average grn11p-share<l delay for an OCI31VIT is g11aranleecl to he within or better than two limes an optimum group-shared deln.y for any weighted graph.

I11 thi:s work, we µreseut a 11cw approximation algorithm whid1 achieve:; approxi- mnl.ion rnt.io .of :\/2 to an OCB.MT for any wdght.r.d grnph. We analyze the time complexity and address lhc possible applications of this new algorithm.

40 Using Domination to Analyze RNA Structures

Travis R. Corik<\ Dr.bra Knisky, Teresa vV. lfaynes, Enst Tennesse<i State .Universily

Unclerst.ancling RSA mokc:nlcs is import.aut. to gcnomics research. Rcccr:111.lv re- searchers at the Courant Institute of Mathematical Sciences used graph theory to model RNA molecules and provided a database of trees representing possible ::;econ<lary RNA :structure:;. They a.bu u:scd cigcuvaluc:, of these trees to help liud novel- RNA. In t.his paper we nse <lominat.ion para.nmt ers Lo predict whid1 l.recs arc more likely to exist in nature as RNA structures. This approach appears to h,we µronii:,e in grapl1 tl,eory application:, in ge110111ic; re:sca.r<'h.

41 Moore-Grieg Designs II

.Tarred T. Collius*, Norman .l. Finizio, University of Hhodc Jslmtd

l\Joore-Greig De:sig1i:s, a new dc1ss of l,lock desigm;, are ,·e:solvable BIBD:s that prn;- sr.ss a 1111mber of fa.c;cinating fr.at.nres. In this sr.conrl se mcnt of 011r inv< st.igation of these designs we discuss the designs in cornplet.e gencrnlily. \Ve also demonstrate the presence of infinite classes of generalized whist tournament designs having f'actio1ial frequc11cy.

42 **Ternary complementa1·y pairs modulo 3**

Robert. Craigen, University of l\Janit.oha

Tr.rnary complementary pairs arc sc(.111<'11c.<'s wi1.h zero a11tocorrd,-1.tion ,rnd cnt.rics 0, ±1. They appear in the construction of Hacla.ma.rd matrices, weighing matrices, orthogonal designs, radar, GPS, signal synchronization and range fi11ding nppli- cation:s i11 e11ginceri11g. They may 1i.lso l,e treated a::; two poly110111iab J, g :such that all x's in the expression J(x)J(x-^{1}) + g(x)g(x-^{1}) ca11c.d. For r.xampl<-, taking J(x) = 1 + x^2, g(x) = l + x - x^2, we h,we

f(:i:)J(:t:-1) + 9(:i:)y(:1:-1) = (I + t:^{2})(I + :i:-^{2}) + (1 + :; - :;^{2})(1 + :,,-1 - .,,-^{2}) = !i.

Constructing a complete theory of their structure has been problcmatic--·thcy ap- pear too sporadic.

It has recently been show11 that ignoring the sig11 by regarding the sequence:; (or polynomials) morlnlo 2 gives a t.rnct.ablc theory of strnc:t.mc, c:oarsly 011t-li11in lhe structure of the general case. Ju this t.alk we explore the corrcspo11di11g approach modulo 3. ln this case we not only obtain a liner approximation to the desirr.d :st.rnct.me, hut we abo get nicthods that. c.u1 con:,;trnct. (ordinary) ternary co111ple- mcntary pairs directly, something not yet found in the 1110d 2 cnsc.

43 On the Non-Existence of Planar DSS

Larry Cummingfi, Univerfiit.y of Waterloo

A collection of no11-trivial <lisjoi1 ıt suhsets of Z_n with the property that all 1101H1ero elements of Z_n ca11 be represented as differences of elements from distinct sets is called a difference system of sets (DSS). General DSS were first introduced by V. I. Levcm;htei11 in the context. of systematic co1111mi-f'ee codes. The ca8e for two fiCtfi had been fitu<lie<l by D.T. Clague. For arbitrary finite alphabets we prove that if the union of sets in a DSS forms a (v, k, >.)-difference set and they differ in size by at most one then >. > 1.

Keywords: Difference Systen1s of Sets, com met-free codes, (-u, k, ,\)-difference sets

44 Average Distance and Eulerian Gra hs

Pct.er Dankclmann*, David Erwin, Ilcnda C. Swart., Univernity of KwaL';ulu-NataL South Africa; Refael Hassin, Tel Aviv University, Israel

The average distance of a connected graph C J = *(V, J@)* is defined as the avcra.ge of the distances between all pairs of vertices.

In this paper we determine lower bounds on the average distance of an eulcrian graph of given or<ler n and fiizc n + k, where 0 S k S (n - 3)/2. For given k and large n, our bounds are best possible up to a small additive constant.

As an application we consider the problem of a<lding k edges, 0 S k S G) - n, to a cycle of le11gth n to obtain a graph of s111allest possible average distance.

45 On the Total Influence Number of a Graph

Sean Daugherty*, Jeremy Lyle, Renu Laska.r, Clemson University

On a graph G = *(V, E)* we introduce a parameter called the *total influence number,* 17-,-(G). This is a 11atmal extension of the graph para111eter known as the ·injfoence *mimher, ry(G).* The influence number of a set *S* ⊆ *V* is *ry(S)* = L_{u} S 1/2d(a,s) where *d(11., S)* is the distance from tt to the closest member of *S*. The influence number of a graph is 17(G) = maxscv *17(S).* The total influence number of a set considers all possible di:;tct11cc:;: itr(S) = LuES I:..,11 8 1/2d(u,v). The total influence number of a graph is TJ'r(G) = maxs ;y TJ'r(S). In this pa.per, we explore general properties of and theorems related to the total influence number. We also show how to fi11d a 111axi11111111 total i.11flue11ce set on vcirions classes of grapl1s i11-d11<ling c:ompkt.e graphs, complete bipartite graphs, an<l paths. Thr concepts of i11fl11c11ce a11d total infhwncc get their mm1e f'om •.pplin1tions in psychology deeding with the r:omm11llication and powrr/influcnr:c in social net.workfi. Ot.l1rr npplirn-tions include facility location problems where t,he <1uality of service provided decays expommtially with respect to distci.nce.

Keywords: distance in graphs, influence rnunhcr, vertex i11dcpc1H.lcucc

46 Even-Balanced Bipartite Graphs and Intersections of Bipartite Star Designs

Kathryn L. DcLmnar*, LaGnmge College; D.G. IIoffrnnn, Auburn Univcrsit,y

In thifi talk we give nec:cfifiary and fillfficient. r:ondit,ionfi for !he existence or rw11-bala.nced bipartite graphs an<l show how these graphs can be 11sed to solve the intersection problem for certci.in bipartite star designs.

47 Desarguesian nets wit.bout, ovals

David A. Drake, University of Floridn

Let TT = *TI(D)* be the Desa.rguesian affine pla.ne coordina.t.ized by a division ri11g D. An r-11d. I: hr.lrl by lJ ifi t.hc union of.,. parallel dri srs of lines or 11. A set *8* of r points of }; is called an *oval* or 1: if each two but no three points of *S* ,ire collinear in I:. Necessary and sufficient conditions for Il to hold an r-nct with oval are know11 for r 5 7. Assume t.hat r = 6 or 7 an<l, i11 l.he G8C r = 7, thcit. D # 2; nn<ler these ri.<;finmptions, we prove that IT hoklfi an r-nc1. **without** an ovril if an<l only if IDI 9.

48 Planar Ramsey Numbers for Small Graphs

Andrzej Dudek, Emory University

The planar Ramsey number $PR(G_1, G_2)$ is the smallest integer n such that every planar graph on n vertices contains either a copy of G_1 or its complement contains a copy of G_2. So far, the planar Ra1m;cy nurnlien; lw.vc been dctenni11cd for t:0111-plete graphs and cycles. By usi11g computer search and many lheoretkal results we found most of the planar Ramsey numbers P *R(G'₁*, G_2), where G1 and G_2 belo11g to the set Un{K,., *K*,. - c, C,.}. Furthermore, lmsecl 011 the program µl:1.11tri de-vclope<l by G. ilrinkmmm an<l Il. McKay, we implemcnt.e<l n tool 1hat enables one to compute planar Ramsey numbers for any pair (G_1, G_2) of 2-conucded graphs with at most 64 vertices.

49 Five or six properties of the numbers 5 and 6

l\,lat.t.hieu D11fom*, UQAl\I; .lean l\I. Tmgeon, University of Mont.real

If we m11lt.iply a series of intP.gers all ending wit.h 0, or all ending with 1, or all with 5, or all with 6, we get an integer ending with that same digit. i\ow the numbers 25 and 76 h,1.ve the same property, and so do 625, 376, and so 011. We shall explain how the :;cqucnccs Cll{lillg with G or with 6 can he cxt.c11dcd i11dcfinilcly, so that we get all solutions or the equation $x^n = x$, for every integer n, where x is an integer -.vith infinitely many digits. We generalize to bases other than 10.

50 A Network Topology With Efficient Balanced Routing

Diuny:;io:; Kuuntani:;, V,itsal Shanidbhai G,mdhi, \Va:;im El-Hajj*, Ghm;:;en Ben J3r,1him, ·western Michigan University

In this paper a special network topology is considered in terms of how nodes should be interconnected. The considered network will be speci-ed by a graph G = (V;E), where Vi:; the :;et of nudes and E i:; tire :;et of links. We as:;mne that tire :;et V ha:; cardin,1lity j V j = k(k j l) + l, where k i s " power of a prime nnmber. We de-ne a function f: V ! U Jt V such that jUj = k j l. For each v 2 V we -nd f(v). Following this approach, E is de-ned by f(v; f(v))jv 2 V g. According to om scheme, any two :;itc:; can co111nm11icc1te hy t.rnversing ex- actly 2 nodes regardless of the network si7.e. Contrarily to l.he existing ront.ing approaches where r011ting decisions are based on a large set of information du-

plicated at each site, the routing scheme we propose greatly reduces the si7.e of the i11for111;1tio11 set tlmt should be maintained ,1t each site.

Keywords: :'\et.work Topology, 13alance<l Ro11ting, Network Congestion, Vir-t.nal Top lo logy

51 Principles and Preliminary Results of Force-Directed Floorplanning.

.Jomrna Ellis-Monagha11, .Tmnry Lewis, Greta Pangborn, St.. Michaels Collegr:
Paul Gut.win, Cadence Design Systems.

A major component of computer chip design is generating an optimal netlist lay-out, i.e. determining where to place the gates (functional elements) and how to route the wires (cum1cction:; between gates) when mau11facturi11g a chip. Floor-planning, an early step in this process, <let.ermines a rough high-level grouping and locating of related gates within the chip area. The floorplan components are ge11ernlly nictangular of fixed area but not aspe<.:t ratio. They are al:;o higlily i11-terc:onnected, but may not overlap in the layont. area. Thus, floorplanning involves Gut.I, geometric and graph theoretical cu11sicle1·atiom;. Floorplc11mi11g i c11n,11tly oft.en donf by h,1.nd, bnt <lne to the highly competitive natmr of the microelc'r:-tronics industry, there is strong interest in heuristics that may shorten the chip design cycle by automating this process. \Ve apply force-direc:Led graph drawing ted111iques to the llourplm111i11g problem, 111odifyi11g lhe111 hy developing" pli_y:-;ic,1l model Lhat allows components to pass through each other and ndjm;t aspect ratios as needed while approaching a solntion.

52 Simultaneous Flows in Multiple Networks

Alexander Engau*, Uorst v,r Hamacher, University of Kaiserslaut.crn

The development of network flow programming was originally motivated f'om clas-sical operations re:;earch ta:;k:; such ,is comnnu1iec1tion, t,ra11spurt.nt.io11, prod11ctio11 or sche<l11ling. However, it has also been fmmd that a);1rg< nnmbr.r of oUwr com-binatorial problems can frequently be formulat.ed in terms of network flows. While :;uch problems ca11 generally be embedded into t1le theory of linea.r progra111111ing, a nnmher of benefits arises from a separat.e trc11tme11t and by making nse of the special network struc:Lure. In particular, many solution algorithms allow for a significant improvement with respect to complexity, running time a11d required co111puti11g re:;011rces.

We st.ndy an integer program whose constraint. mat.rix can be partit.io1wd into a collection of submatric:es that are consecutive one i11 rows. Based on linear pro-gramming relaxation and duality techniques, this integer program is transfonncd into ,1 ;i11111lta11euu; flow pruhle111 i11 several u1H.ledyi11g 11etworks thctt. n1-c rdntcd through a bijection on snbset.s of t.hf'ir rcspe<:t.ive arcs. Similar l.o sim11llaneo11s Hows that have identical values on corresponding arcs in different. networks, 011e can study simultaneous tree problems, matching problems, etc. This new a.rea minrd "simultaneons graph t.heory" will he suhjec:1- of fort.hcoming l111ivrrsif_y of I<aiserslautern working papers.

53 Path and Cycle Decomposition Numbers

Grady l3111lington, Linda Eroh*, Kevin l\lcD011g-al, IIosien l\fogha.dmn, St.even .l., Winters University of Wisconsin Oshkosh

For a fixed graph /f without isolated vertices, the H-der:omposition number $d_{11}(G)$ of a grapl1 G is miu(IV(I<)I, IV(G)I) where I< is al1 f/-<lccmnposahle gnipl1 wil11 i11di11cedi s11bgraph G. Eq11ivalently, it is the minimum number of vertices 1h/1t must be added to G, along with any number of edges incident with the new vertices, to produce an !/-decomposable graph. This parameter was previously studied by Kdl<'r, Vanci<'ll, anci \Vintc,rs. In this t.alk, we prc,sc,nt c,xact form11las for rl,,(G) in the cases where /-/ is a path or a cycle and G is a path or a cycle. We prove a general lower bound which is useful in these cases.

Keywords: edge det:0111po,;itio11, H-deco111posi.1.ble, det:0111positio11 11u111ber

54 Latin Squares Based on Direct Products of Elementary Abelian Groups: a Progress Report

Anthony B. Evans, \Vright State University

Tt is well known that we can construct sets or pairwise orthogonal Latin squ<1res from the Caylcy table\ of a gro11p G, w,ing sets of pairwise acijaccnt orthomorphisms of G. Restricting ourselves to g.roups of the form GF(q1)+ x GF(q2)+, we find that many classes of orthomorphisms of this group can be obtained by solving systems of clilforcnce equation:; i11 the ring of f1111ction:; G F(q1) -> GF(q1.). We will cxami11P. some of these new d11sses of ort.homorphisms an<l their or1:hogonalilies.°

55 Sum Coloring on certain classes of Graphs

Gilbert Eyahi*, RCJm Laskar, Clclllson Univcn,ity

An J,(2, 1) r.nlnring of a grnph C = (V, E) is a vcrt.c,x coloring .f : $V(G) \to \{0, 1, 2, \ldots, k\}$ such thal IJ(u)-J(v)I 2 for all uv E E(G) and IJ(u)-f(v)I 1 if d(1t,1•) = 2. We refer to an *L(2,1) coloring* as a coloring. The *span* >.(G) is the s111allest k for which G has a coloring. A .pan coloring is a coloring whose great.est color is >.(C). An f,{2,1)-r.nlnri.n_q f is a full-c:oloring if $f: V(G) \to \{0,1,2,\ldots,\lambda(G)\}$ is onto a11d f is an irreducible no-hole coloring (inh-coloring) if $J: V(G) \to \{0, 1, 2, \ldots, k\}$ is onto for some k and there do not exists 1:1colori11g ff such that !J('IL) f(n) for ,ill n E V(C) ,rnd y(-1•) < J('u) for some v E $V(G)$. The Assignment sum o f f on G is the sum of ,ill the labels assigned to the vertices or G by the coloring J. The *Sum coloring nmnber* of *G,* 1) G), is the 111ri111u111 assignment sum over all the possible colorings of - f is a *Sum* coloring on G, if its assignment smn equals the *Snm* r.nlnri.n_q *rmmhr.r.* In this paper, we invcstigc1te the Sum colo1·ing nm11bc,-s of certain classes of graphs. It is shown tlic,L, $L(P_n) = 2(n - 1)$ and $L(C_n) = 2n$ for all n. We also give, ,111 c,xad. vain< for the *Sum coloring number* of a star and conjecture a hound for the *Smn coloring number* of an arbitrary tree *T,* not a star with max degree $\Delta \geq 3$.

56 Characterization of Digraphs with Equal Dominati n Graphs and Underlying Graphs

Kim A. S. Factor*, Marquette University: Larry .J. Langley, U11iversity or the Pacific

A domination graph of a digraph D, do111(D), is created using the vertex set of D and edge whenever or for any other vertex z. The underlying graph of D, lJG(D), i:; the grnph for which D is a hioric11tatio11. Using results ol>tai11c<l by Drigha111 and Dutt.on on neighborhood graphs, we c:harad.eri?.e symnwt ric cligrnphs whr.rr. dom(D)=UG(D). Building upon the case of sym.met.ry hy introducing bioricnt.atio11,; of underlying graphs, we co111pletdy clmrac:tcrize digni.phs whose u11dcrlyi11g graphs arc idcnt.ical to thr.ir domination gr:iphs.

Keywords: <lomination graph, tm<lerlying graph, grnph c>q11ality

57 Defining a Class of Computational Curves based on a Recursive Structure Graph

.James D. Fac:t.or, l\larq1wt.t.c, lJnivf\l'Sit.y

Given a path of length n, a recursive algorithm based on the subdivision of each edge i11 the path will be 11se<l to <lefi11e a structure graph. This structure graph will capture the combin:it.orial, <:onnf'ct.ivit.y, and topological propert.ics of a well-defined framework into which it is embedded. Edges and vertices being mapped to links and joints, respectively, in space construct this framework. The l1:1st vertex placed by t.hc algorithm is mappcci to a disti11g11ishc<l joint. As the frnincwork moves, ii is shown that the disting11ished joint sweeps onl a Bezier curve or degree n.

58 Counting Even Partitions and Selmer Group Elements

n. Fa11lkner*, K ..lames, Clemson University

A positive integer n is called a congruent number if there exist a right triangle with ratioual length ::;i<le:; a11d a.rea n. It nu1 I>e ;how11 that the elliptic curve <lefinr:d by, $E_n : 1/ = :1:^3 -$ 11.".l.:1:.has infinit:dy many rational points if and only if n is a congruent number. One common way of bounding the number of rational points on such a curve is to study its corresponding "Selmer group". \Ve will give a de::;criptio11 of all of the Selmer group::;, S.,., i11 term:; of certain graphs. Suppose $n = P_1 \cdots P_t$ p; a prime for 1 $ i $ l, cfofine a graph $G(n)$ in the following way. Let the vertex and edge sets of $G(n)$ be defined as $V = \{P_i, \cdots, p_t\}$ and E(G(n)) = {p I(;;) = - 1

1 $ i ,f j $ l} where ( ) is the Icngendre symbol. A partition of $G(n)$ is an ordered pair (V_1, V_2) where $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = 0$. A partition (V_1, V_i) is 8c1id to lie even provided thctt for any·,, E Vi, U{v -+ V_2} i; even, cmd for any // E Vi, ti{v -+ Vi} is even. In I.his 1:alk a fonnmlr1 for 1he size of the Selmer group is fo11nd by finding the dimension of certain subspaces of the null space of the Laplace matrix, defined by f,(G(n)) = dia.g($d_1, \cdots, d_t$) - A(G(n)) where d; = L = 1 Oij (J $ i $ /) and (n;j) = A(G(n)), the adjacency matrix of G(n).

Keywords: Elliptic C11rve, Selmer Gro11p, Congruent. N11mber

59 Two generalizations of deBruijn digraphs

Miclmcl S..facoh::;011, Arthur IL Busch, U11iven;ity of Colorado at Denver; Gua11tao Chen, Georgia State University; Ralph J. Faudree, University of Memphis; Michael Ferrara., Ronald .J. Gould, Emory University Nathan Kahl, Charle:; Suffcl, Stevens lm;t.itute of Technology; Ewa Kubicka, (;rezgon: Kubicki, l:nivcrsity of Louisville; Allan Schwenk, Western l\·Iichignn University

V/e give a broad definition of a class of digraphs motivated by the well known de Rruijn digra.pl1::;. We u:;e two examples to <le111011:;trate that the <le Rruij11 digraphs can be c:onsid0.rc<l a. a spec:ial ca.se of this dass rlefine<l here an<l wr: consider two applications of other special cases of this class of generalized de Bruijn digraphs. First, we show how this class can be utilized to find all possible k-subsets of an 11-:;et. Next, we show tlrnt t.his dci.:;s of digraph::; can he m;cd to rcprc:;cut a dus::; known a.lphabcl.-ovP.rlap grnphs an<l show that I.hey are hamilt.onian.

Keywords: de Bruijn digraphs, line digra.phs, hamiltonian digraphs

60 Designing Fire Resistant Graphs·

St.nart Crosby, A. Finbow*, n. IIartnell, Hm1i1- l\.Io11ssi, I<M.c Pnl frrson, l);mia Wattar, Saint Mary's University, Canada

We consider the following scenario: Let f c1nd d be positive integers. 'Fi,·es' bre,tk out at a set S of .f vertices in ,t co1111ectc<l ::;imple gntph G (i.e., t1e vertices of S are coloured red). Then the following set of events occnrs repcat.edly uni.ii all the vertices are coloured:

The 'defender' 'fireproofs' (colours green) d non-colo11rcd vertices (all of them if there arc less than ti) after which the fire sprcMls to all no11-colo11rc<l vcrl.icTs which are adjacent to any red vertex.

Let r be the final number of red vertices. For each set S of J vertices in G, m(S) is the 111ini111um vc1ine of r ta.ken over all defen::;e::;. For fixed f a11d d, we wish, for each 11. to design a connectccl graph with n vertices snch that. the avrrng-c value of m.(S) (taken over all subsets S of cardim11ity J) is 111ini11111m. Parti;.1.l progress on this problem will be presented.

61 Moore-Grieg Designs III

.farred T. Collins, Stephanie Costa, Rhode Island College; Norman .J. Finizio*, UniYcrsily of Rhode fsland

l\foorc-Cn::ig Designs, a new ch1ss of block de::;igus, ,ire resolvable I3II3Ds t.liat possess a number of fascinating fea.tures. In this third segment of onr investigation of these designs we emphasize the presence of "nested" resolvable relative difference fa111ilies and 11e;ted fn-tme::;.

Keywords: RI3II3Ds, frames, rr:solvahlc relative difference families

62 Wiener Polynomials for Recursively Defined Rooted Trees

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The Wiener polynomial of a comHx:t.cxl graph G is W(G; q) = P{u,v}qci(n,v), where the sum is over all unordered pnirs {u, v} of distinct vertices in G, and d(u, v) is the distance between u and v in G. Thus, \V(G; q) is the generating function for the di:;t.am:c <listrilmtiu11 d<l(G) = (D1,D2, ...,Dt) where Dk i; the 11umhcr of nnordcreci pairs of dist.ind vertices at ciist.ancc k from each other and t is the diameter of G.The derivative W0(G; 1) is the well-known Wiener index of G. For a specified vertex 11of a co1111ectcd grapl1 (;, the Wie11er µulynurnial of G relative tu 11is the polynomial W11(G; q) = Pv nd(u,v), where the snm is over all vertices v of G, including v = u. We discuss the Wiener polynomials for recursively defined trees, paying special attention to Fibonacci trees and complete dendrimers.

Keywords: Wiener i11dex, Vliener polynomia.l, <lista.nce, tree, Fil>o11acci tree, ciencirimer.

63 Edge Colored Complete Bipartite Graphs with Trivial Automorphism Groups

:'viike Fisher, California Staie University, Fresno; Garth Isaak, Lehigh University

Our work genera.li;„es results obta.ine<l by l-farary ,v. .facul>se11 a.ml by Harary & Ranjan. Ilarary nnd .Jacobson examineci the minirnnm number of edges 1ha1. neerl to be oriented so that the resulting mixed graph has the trivial automorphism group and determined some values of s and t for which this number exists for the complete hipartit.c graph K.„t- Ju a follow up paper, Harary and H.anjau dct.cr-minP.ci fmther bo1111ds on when some of the edges of K.,„. arP. 11blc to bP. oriented so that the graph admits only the identity automorphism. Since we may think of such partia.1 or.ieutations as 3-edge colori11gs when s f l, it is natural to consider this prublc111 fur <:-edge coluri11gs where < 2: 2. In this paper, we dctcrmiuc the values of s and l for which there is an edge coloring of the complete bipartite graph I<s,t which admits only the identity automorphism.

Keywords: edge colorings, c1utornurµliis111 groups

64 Fullcrcnes and nut graphs

Pnt-rick Fowler*, Cnivei-sit.y of Exeter, UK; Irene Scir.ilrn, University of l\lalta

Fullerenes are all-carbon molecules with trivalent poly}iedral skeletous, haviug 12 faces pentagonal and all others hexagonal. :'„[any questions abo11t their chemistry can he cast in graph-ihcorctind form. This talk <le,tls wit.Ii follcrcncs whose skele-tons are nut graphs: a nut-graph has exactly one zero eigenval11e in its adjacency spectrum and no zero entries in the corresponding eigenvector. In chemistry, this specia.l eigenvector currespon<ls 1u a- uu11-l>undiJ1g orl>ital .:iml has implications For ele<:tron dist.ribnt.ion m1ci reac:tivit_y. Some propert.ies of n11t-fllllcrN1cs nm! con-struciions for the graphs will be discnssed.

65 Self-assembly graphs from pat.hs

G. Franc:o*, :\. .lonoska, Univ<'rsily of Sont.h Florie-la

In DNA nanotechnology it has been shown that 3D DNA structures can be self-a:;se111bled experimentally; fur exa111ple, the cul>c, the idrahcdruu, cUH.l even 11011-regnlar graph st.rnct.nres have heen ohtainc\cl. This work proposes a 1.lworc1.ical model to study possible graph structures obtained by self assembly from a given set of single-stranded DNA molecules.

Given a collection of <lircctcd paths an<l cycles with vertices lal>eled fu111 7, = {a, t, r, f/}k-, where k is a fixed positive integer, we acid n matching set of 1111d-rec:ted edges such that two vertices are incident wit11 l.he same edge only if they have complementary labels. In order to obtain a graph which rcprcscnt.s a self as-scrnhlcd DNA struct.nrc, the umt.ching set mu:;t respect. certain con:,;1.raint.s ddincd by mP.ans of n set of forbiciden s11bgrnphs.

We present a general model and simple examples for building snch graph stnwt11r<'s from a collection of directed paths and cycles, while respect.ing the constraints of furbi<ldcu suhgntphs. We conclu<le with some open problems.

Key words: DNA Compnting, SeH Aiisembly, Forbiclding-Enlorc:ing S_yst.cms.

66 Orthogonal double covers of complete graphs by caterpillars of diameter 5

Dalibor Froncek, University of Minnesota Duluth

An *orthogonal double cover* of the complete graph K_n by a graph G is the set of n subgraphs $G_1, G_2, \dots, G_n$ of K_n with the followillg properties:

(1) G hac; ·11. - 1 eclges nnd C_i ::::' G for every i = l, 2, . . . , ·11;

(2) every edge of K_{ll} appears in exactly two copies of G (double cover property);

(3) every two· distinct copies G_i, G_j of G intersect in exactly one edge (orthogo-nc1lity property).

Gro11a11, l\!lulli11, c111d Rosa conjectured that. for every tree T with n vertices except for J_4 there exists an ODC of K_n by $1'$. They also proved the conjecture for all caterpillars of diameter 3. Later, Leck and Leck proved it for all caterpillars of dic1.meter 4 a.ml cll trees with up to 14 vertices. We prove tl1e conjecture for 1111 c11.rcrpillars of rlimnctcr li and order n 2-1; for orders l!i $\lesssim$ 11. 23 we; prove it with several exceptions, which we believe are only temporary.

The method we use is a common generalization of methods developed for ODCs by Grona11, l\lulli11, and Rosa and by Leck and Leck and for complet,e graph factori-:a-tions by Tcre,m l<ovnrov11, who present.eel them here n year ago. If time permits, we also mention further generalization that is useful for caterpillars of small orders. We believe that this will help 11s to settle the missing cases.

67 Constructions for anti-mitre and 5-sparse Steiner triple systems

Yuichiro Fujiwara, Keio University

A *Steiner triple system* of *order* 11, briefly STS(v), is an ordered pair *(V,B)*, where V is a finite set of ll eltirnents called *points*, and 8 is a set of 3-elellle11t subsets of V c1ll1ed hlor.ks, snch thilt. c11ch unordered pnir of clistinct clements of V is cont11incd in ex11.ctly one block of !3. A *(k, l)-conflgnralion* in an STS is a set of l blocks whose union contains precisely l; points. The uniqne (6, 4)-configuration is called the *Pasch config·umlion.* The m:ilrc is 011c of two (7, 5)-coufigurat.iom; which con-tains no l'm,r.h confignrnt.ion ac; its snbsl.rnct.nrc. An STS is said to be 11n1.i-mifre if it contains no mitre configuration; and it is 5-sparse if it contains neither Pasch nor 111itre co11fig11ratio11.

l11 this tnlk we prcseut 11ew coustructiolls for ,111ti-111itre STSs aud 5-sparse 011es. By virtue of the constructions for anti-mitre STSs and known results, we can con-struct anti-mitre STSs for over 13/14 of the admissible orders. For 5-sparse STSs, we give a co11strudio11 which extends substantially the spcctrnrn of known such syst.cms.

68 On the Extension of an m-sct Family

.Jnnirhiro F'Hlrnyama, I11rliana State Uuivcrsif.y

Let n, m and l be positive integers such that m < $l \lesssim n$, and U be a family of m- ets, each element of which is chosen frolll [n], *i.e.,* U E (f;,:l). The l-e:tl,:n.,-i.on *Ext(U, l) of U* is ddinc<l by

$$E:rt:(U, l) = \{s \; E \binom{7}{} \; /11 \; E U, IC \; s\}-$$

It h11s been pointed ont. that. the, extension is closely r<'latcd 1o the well-known open problem called the *Isometric Problem for* m-sels.

In this paper, we will show that

$$F;xt(U, l) I^2 \binom{n}{l}\left(1 - \exp\left(- \frac{(l - m)llll)}{(;,)}\right)\right).$$

This bonnd is nsefnl for small m snch as 2, allll illlplies the followi11g dairn: Let G be a11 n.-vertex gniph whose edge si:1,c is n(,-t>- - k. Then, there are ,1t 1110;t $\binom{!}{l}$ exp (- ;,V. l) J many I-cliques contained -in C.

Keywords: Extremal Set Theory, lsomclric Problem for m.-sets, Han11ning Space, Hamming Distance, Shadow

69 Minimizing the Number of Constraints in an ILP Model for Tournament Feedback Arc Sets

Ryan Fuller*, Darren A. Nc1.n1y,1n, Rocl1cstcr lrn;tit.nt.e of T<·!chnology

We consider the following question: Given a set of n players in a round robin tour-nament, what is the smallest sized tournament for which there exists ttn opl.irnal nmking where each of f11c origin,il 11. plciycr:s ,u·c pci.irwise m11ke<l w1-011g! We i11vcs-tig11te this probl<>m 11sing met.hods from grnph theory ancl inf.r.ger progn11nmi11g. Given an acyclic: digraph D we seek a smallest si7.ed to11mame11t T that has D as c1 minimum feedbc1ck arc set. The reversing- number of a dign1ph, -r(n) cquals IV(T)-V(D)I- lsa11k nnd Nam.ym1 formulat.c,d 1111integer linear progrnni, *ILP(n),* whose optimal value gives the reversing number or a tournnment. It turns out that. in many cases, several of the constraints can be n•moved with no effect on the objective value of *ILP(n).* We investigate various subsd.s of co11strai11ts when• t.he objcctivr. valnr. is the smne ac; if it were n1.lc:nlatt>d over the foll set of c:onst.rainl.s.

Keywords: feedback arc set, tournament, i11tr.gcr linear program

70 (0, 1)-matrices with Constant Row and Column Sums

Slrn.n7.hen Gao*. Ildnrich Nicxkrlrn.11se11, Floricla Al'lant.ic Univn-sit.y: Zhongh1rn Tan, Guangzhou Gongye Cnivcrsity, China

L d $f_{s,t}(111, 11)$ he I.he nnmher of (0, 1) - mat.rices of si;,,e m x n snch that each row has exactly s ones and each column has exactly l ones (sm. = nl). How to determine f.,,t(m, n.)? As R. P. Stanley observes (Em1men;1tive Combinatorics I (1997), Example 1.1.:{) the <letennination of f_s,t(m,, n) is an unsolved problem, except. for v<>ry small s, t. Iu this pap<>r we give mth<>r involved c.losed formn-las for $f_{2,2}$(n, n), $/_{3,2}$(m, n.), f,_,(n,n), $J_{,-2}$(m, n.). We discnss recursion formulas, genen,1ting functions c1nd present several instructive reformulations of the problem.

71 Domination Cover Pebbling

James G,1rdner, ETSU

Given a configuration of pebbles on the vertices of a graph, a *pebbling move* is defined by removing two pebbles from some vertex and placing one pebble on an adjacent vertex. We introduce domination co-uer pebblin$_g$. The <lolllinatio11 cover pebbling numbm-, lj;(G), of a graph G is the minimum number of pebbles 1.m<lcr any configuration such that after a sequence of pebbling moves, the set of vertices with pebbles forms a dominating set of G A brief overview of pebbling and basic rc::ult:; of dorniHation cover pehhliug will he givc11.

72 On *P(a)Q(b)-Super* Vertex-graceful Tree

Sin-Min Lee, Anupam Geng*, San Jose State University

Given integers a,b > I, a graph G with vertex set V(G) and edge set E(G), p = IV(G)I au<l q = JT..:(G)I, is said to he P(a)Q(b)-:mper vertex-graceful (i11 short P(a)Q(l,)-SVG) if them exists a fnnct.ion pair $(f, j+)$ which assigns integer labels to the vertices and edges, i.e.,

f: $V(G)$-, $P(a)$ and $j+$: E(G) _, Q(l) are onto, $j+(u, v)$ = $J(u) + f(v)$ for any (u,v) E $E(G)$, and

Q(b)={±b, ±(b+l) ..., ±(b-l+ci/2)}, if ci is even,

{O, ±b, ..., ±(b-l+(ci-1)/2)}, if ci is od<l,

P(a)={±a, ±(a+l) ..., ±(a-l+p/2)}, if pis even,

{O, ±a, ±(a.+l) ..., ±(a-l+(p-1)/2)}, if pis odd.

\Ve clcicrmine here da.s:;e:; of tree:; that are P(a)Q(b)-super vertex-grc1.ceful for $a = 2$.

73 Hamilton paths in graphs whose vertices arc graphs

Krystyna T. I3ali11slm, Kr7.ys;,;t.of T. Zwier;,;ynski, Technical University of l'o;,;nai'1, Poland; Michael L. Gargano*, Louis V. Quint.as, Pace Cnivcrsit.y

Let U(n, f) denote the gni.ph with vertex :;ct. the set of mdi.tl>ekd gn1µhs of order n and having no vertex of degree greater than f. Two vertices H and G of U(n, f) are adjacent if i.lnd only if H and G differ (up to isomo1·phism) by exactly one edge. The pro>le111 of <leten11.i11i11g the values of ll c111d f for which 0(11, f) cont,Li11:; a Hamilton path is invest.igat.ecl. There arc only a fow known non-trivial cases for which a Hamilton path exists, namely, for U(5, 3), 1.-(6, 3), and U(7, 3). On the other hand there are many cases for which it is shown thal no Ilantilion path exists. The complete ;ol11tio11 of this prohlc111 i:; uarcsolvcd.

74 Stratified Domination in Digraphs

H.alncca Gern*, Ping Zhm1g, West.cm l\lic:higan University

A digraph i:; 2-:;t.rat.ified if its vertex set is part.it.ioned into two cla scs, where the vertices in one class ;ire colorc<l reel and those in the other class arc c:olorecl hhH'. Let F be a 2-strntified digraph rooted at some blue vertex v. An F-coloring of a digraph D is a red-blue coloring of the vertices of D in which every blue vertex 1! belong:; to d copy of F root.c<l dt v. The F-dolllinat.ion 11u111>cr i:; I.lie 111i11i11111111 nmnbtr of red vertices in 1111F-coloring of I.). We present. some results i11 t.his area.

75 Bounds on the Domination Number of a Graph

fly Ge1111 (;. Chappell, .John (;imbel*, Chris Ha.rtman, University of Alc1ska

Let G be a graph with au ordered set of vertices and maximum degree 6. The dominc1tion number -y(G) of G is the minimum order of a set S of vertices having the property that each vertex uot in Si:; adjacent to ;0111c vcrt.cx iu S. Eq11iv,1h:11tly, we can label the vcrt.ic:es from {0,1} so I.hat. the snm over ea.ch dosecl neighhorhood is c1t least one. The minimum value of the sum of all labels, with this rest.rictiott, i:; the do111h1c1tio11 nt.mtbcr. Tlie f1-c1.ctio11al <lo111inatio11 1111111hr 1*((;) i:; cldincd i11 the same way except. t.hat the vertex labels arc chosen from [0,J]. Let g;(G) be the approximation of the domination number by the stamlnrd greedy algorit:hrn. Using techniques from the theory of hypergraphs, we obtain for !:;, 2, -y(G) :; -y(G) S 1g (G) c(log 6) 1*(C). Herc, c is some constant. Wc di:;cuss t.hcsc ho1111ds and sharpness.

76 A Physicist looks at Graph Isomorphism

I3ryant Gipson, Ilmnboldt. St.ate University

Complexity theory lws shown tlmt the µrol,le111 of <lcten11i11i11g graph isomorphis111 falls bctwer.n P 1md NP-Complr-,f;e. A1111Jy;;ing degree seci11ences, dim11etcr, nmnber of components an<l other relational invariants of a graph reduces the si7,e of the cla:ss of graphs for which au O(N!) search 11eed be done. Co111p11ti11g the eigc11val-11es for 11gr11ph m1 trix gcmernte<l by 11specific: vertex l l.iheling nm-rows the problem further. Cmrently the computationally worst case scenario is that of the relatively rare class of cospectral graphs (non-isomorphic graphs with the same eigenvalues) wit.li i<lcnticctl degree sequence,;. Drawi11g f rom the theory ofQmmt11111 Computing, 11polynomial time 11n11ry operaf.or 1.ermed the "Level opcrnf.or" for gr11phs is introduced and its various properties are illustrated -specifically with regard to its use in distinguishing cospectral g raphs a11d further reducing t1le set of graphs for which isomorphism must be exh,lnstivdy compnte<l.

Keywords: cospect.rnl, eigenvalues, gr1ph operators, (f1Ja11tmn physics, graph isomorphism

77 On the nonexistence of a (I76, 50, 14) difference set

Oliver Gjoncski*, Ent.es College; Ken W. Smith, Cent.ml l'l'lichig11n University

The Iligm11n-Sims symmetric. design with p11rmneters (171>, !iO, 1'1) is 11n importa.nt combinatorial structure of interest to mathematicians because of its large sporadic automorphism group, in addition to the recently discovered rich tight subdesign strnctme. The existence of the Higman-Sims design raises the question as to the existen('.e of a <lifforence se1. with these. p11rameters. The sear('.h for 11<lifference set with these parameters historically has focused on the five abelian groups of order J 76, and even then the results have been difficult. The connection of a nonabelia.n simple grollp with these parnmet.ers s1Jggesf:s th11t one shoHl<l look more c.arefnlly at the remaining 37 nonabclian groups of order 176. We will use a wide arnly of techniques to eliminate the possibility of a difference set in all the groups of order 176,

78 Probabilistic Aspects of Graph Pebbling and Cover Pebbling

Anant Godbole, East Tennessee State University

There has been a recent spurt of research activity in the area of graph pebbling and graph cover pebbling. h1 this ta.lk, we focus on a new probabilistic dcvelopn1ents: What is the cov<r pebbling threshold fort.he complete grnph? A snrprisingly sharp a11swer is obtai11ed botl1 for l\laxwell-Bolbmm1111 and Bose-Ei11st.dn pebbling, with the _gol<len ratio playing a key role. All 1lw t.f'rms used in t1le al>ovf' a.bstrnct. will be defined as pnrt of the t..ilk. This is joint work with Nathaniel \-Vntson um! Carl Yerger.

79 A Non-Unit F ree Tetrahedron Order·

Ashifi Gogo*, Barry Balof, Whit.man College

A free tetrahedron or<ler is a parti,tlly ordered :;ct for which each ele111e11t can be ident.ifie<l with a tet.rnhe<lron snch that. all tctrahedrri have one vert.C'x on each of three parallel baselines and a fourth f ree vertex between tlu: three lmsclines. Two tetrahedra i11tersect if and only if the.ir corresponding clements are i11compan1.-hlc an<l the tetrahedra preserve the or<lcr ol' clclllc11ts t.lmt arc cornpan1hlc. Free tef.rnhe.dron orders an 11generali:;11f.ion of interval and trape?:oicl orclcrs and are 11 special class of (n, i, !)-tube orders. A unit free te.trahedron order is one in whiCh all tetrahedra have the sa11e volume. A proper free tetrahedn111 order is one ill whir.h no tet.r11hcdron complctdy c.ont.ains anot.her t.drahc,dron. We settle t.hc 11nil versus proper question for these orders by finding a proper free tetrahedron order thnt docs not have a unit free tetrahedron represcntnlion.

80 Maximum Size Antichains in COLEX

John Goldwasser*, Yongbin On, \Vest Virginia University J\tt.ila S,1.li, Jlungurian Academy of Sciences

We define the order COLEX o11 the :;et P(Z+) of all [iuitc subset:; of the prn;it.ive integers by A < B if A is a proper subset of B or if the largest. element in A but not in B is less than the largest element in B hul not in A.So {2,3,6,8} < {2,7,8}. We <lc11otc the first 111sets in COLEX oil P(Z+) by C(111). A collection T of::mb:scts of 11set. is 11n 11ntichain if no set in T is 11snbsct. of any ot.her. W<- find a formnla for the maximum size of an antichain in C(m). The fonnul11 is in terms of a sum of binomial coefficients related to the cascade fonn used lo cak:ulatc the size of the shadow oft.he first m sets of :;iilc k in the COLEX order. Tl1c special ca8c when m is C(flllll to a powr.r of is Sperncr's theorem.

Keywords: COLEX, antichain, Sperner's theorem

81 Binary Strings and the .Jacobsthal Numbers

Ralph P. Grinrnldi, Rose-Ilnlman Instit.nte of Technology

Starting with the alphabet {0, 1} and then ihc la.ngnage A = {0, 01,11} over this alμhabet, we fiJ1d that the 11m11bcr of string,; of length II i11 A* is given by the n-th .Jac:obsthal nmnber .J(n), where .1(0) = 1, .1(1)= l,an<l .J(n) = .l(n-1) + 2*.l(n-2), for $n > 1$. In this presentation various properties of these strings are examined an<l enumerated. These include (1) the total number of 0's and 1's that occur among all the strings of length 11;(2) the m1111bcr of r11m; that occur among 1111the string:; of lcngf.h n; an<l() f.he n11mber of l1wels (0 followe<l by 0, or 1 followed by 1), rises (0 followed by 1), and descents (1 followed by 0) that occur among the string:; of length 11.

82 Super-simple 2 - (v, fi, 2)-dcsigns

Hans-Dietrich Gronau, "Cniversity of Rostock, Germany

A 2 - (-,, J:;, >.)-dc;ig11 is ,1 μ,,ir (V 13), where V is a ·1.'-clcmcnt :;ct of point,; ,ind IJ is a collection of k-element subsets of V ca.lied blocks such that every pair of points is in exactly>. blocks. A (v,k,>.)-dcsign (V,B) is super-simple if any two blocks intersect i11 at tnost 2 points. The concept of :,mper-simμle designs wa introdm:c<l by Mnllin an<l Gronan in I'I1I10. In the talk we study f.hc spect.rnm of super-simple $(v, 5, 2)$-designs. We show that a super-simple $(v, 5, 2)$-design exists if and only if v = 1 or 5 mod 10, except definitely when v = 5, 15 and possibly whe11 v = 7:i, !)5, 115, 133, l!J5, 21G, 2:n, 285, 3G5, 385, 51 :i, what b joint work with Kreher an<l Ling. We add rcsnlf.s by Hartmann on f.he asympf.of.ic exisf.cnc:e ol' super-simple designs and new results by Abel and Ling, who excluded a few cases i11 doubt.

83 Construction of a family of uniform central graphs with small diameters

Sul-yo1.111g Choi, Le l\tloyne College; Puhua. Guan*, University of Puerto Rico

A graph is ca.lied a uniform cen1.m1 graph if i1s cen1-ral vertices have a same s<1. of eccentric vertices. We show that the conjecture *'if a graph wilh radius r is un-ifoml. cenlral, then ils diameter is at lcasl* $r + [(r+1)/2]$' is not true by co11-sf.rncting a family of 11niform central grnphs with radius r (2:) and diamct.cr $r + m$ (1 m [r/2]). This can be generalized to a construction of a uniform central graph which has a given graph as its center.

Keywords: eccenf.ricit.y, u11ifom1 central graph

84 Extensions of Rado Numbers to the real line

Caitlin Brady, H.nt.h IIaas*, Smit.h College'

Given an equation L, its *Rado number*, $L(n)$, is the least integer such that in every coloring of 1, 2, ..., $L(n)$ with n colo,·s there exists ,1 mo11ochn>mc1.lic solution Lo the eqnatiou L. These m11nhers have been si11dicd for n1c111y equation:; by 111a11y authors. Here we extend this idea io coloring f.he rea1 line. In parlic:11lar, we prove that $t = y(m^2 - m - 1) + (m+1)c$ is the least. real number such tlrnt in every 2-coloriug of the real mnubers [y, t], where y is a positive n,11 1111111bc; there exist,; a monoc:hronrntic solntion tor·+ $x_1 + x_2 + \ldots + x_{m-1} = x_m$ where -r: < y(m - 2).

85 Weak Independence Numbers for Grid Graphs

Tfeiko Harborth, T1! Rnrn11:;cl1weig, (:cnwrny

Whal is l.he maximnm mtmber of mnrk0<l sqnares of a d1C'ssboard snch thal. each marked square has common edges with at most k other marked squares (k=0, 1,2,:{,4)". The <ase k=:i rernains opm1 since it requires the nnknl>Wn do111-inat.ion nnmbcr for grid grnphs. (Common work wi1l1 l!C'iko .Dicf.ric:h)

86 Trees with equal domination and restrained domination numbers

.J. H. T-fo.tti11gh*, Georgia State l:niver:;ity; P. Da11kel111;11111, i\l.A. lle1111i11g. lf.C'. Swart, UK:\'Z

Let G = (V,];') be a graph. The set S is a dominating set (DS) if every vertex in V - 8 is a.dja.cent to a vertex in S. Further, if every vertex in V - 8 is also adjacent to a vertex i11 $V - S$, then Si:; a restrained dorninntiug sci (H.DS). The do111i1_1,tio11 number of G, denoted by 7(G), is the minimum c;:,.rclinality of a DS of G, while the restrained domination number of G, denoted by $1'_r(G)$, is the rnininnm1 car-dinality of a RDS of G. The graph G is 7-excellenf. if every vertex of G belongs to some minimum DS of G. A c:onstrnc:t.ive c:harnc:tcri,1ation of trC'cs with <'qnal domination and restrained domination numbers is presented. A a (onseq11e11<'c of this cha.ra.cteriza.tion we show that if T is a tree, then 7(T) = ~r (T) .if T is a 7-cxcdlcnt. tree.

Keywords: resf.rn.inerl, domination, excellent.

87 Counting rises, levels and drops in composit.ions with parts in a set A

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A r.omposition of n E }' is n or<lered collection of one or more posit.ivc integers whose sum is n. A palindromic composition of n is a composition in which the summands are the same in the given and in reverse order. The number of summc111d:; i:; called the uurnber of part:;. We derive the generating function for the number of parts, rises (snmman<l followed by a lmger snmman<l), levels (a summand followed by itself) and drops (a summand followed by a smaller summand) for a general set *A*, and are able to derive a11 previously known results as special ca.:;c:;. We abo derive uew rcs11lt; for Carlit:t cornposition:; (no adja<;ent :;unumutd; can be the same) and for partitions.

Keywords: Composition, Palindromic compositions, Carlitz compositions, partitions, generating functions.

88 Semiregular Factorizations of Graphs

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A (d,d+1)-graph is a grnph in which t1le degree of each vertex lie:; in t1le set {r,r+l}. Such a graph is sometimes c.llcd scrnircgnlm. An (r,r+l)-fa.c:t.ori;r,ation of a graph G is a decomposition of G into edge-disjoint (r,r+l)-factors.

Let r and s be given positive integers. We show that there is a number D(r,s) so thc1t if G i:; a simple graph with 111i11imum degree d and maximum degree d+s, an<l if <l > D(r,s) t.hcn G has an (r,r+l)-factori;r,ation. We also obt.ain bonnds for D(r,s).

89 Gregarious 4-cycle decompositions of some complete multipartite graphs

Eli;r,ahct.h .l. Ilillingt.on, The Univ0rsity of Q11ccnsland; D.G. Iloffman*, A11h11m University

A 1-cyclc in H c:omplct.0 m11ltipartit.c grnph is said to he gregarious if its fom v0r-Lices lie in different partite sets. Determining which complete multipartit.e graphs admit a 4-cycle decomposition is relatively easy; but if we insist each 4-cycle in the deco111po:;itio11 be gregc1rio11s,ihe problem become:; :;11rpri:;i11gly thorny. Here we set.tie the cHse where at most one part, is of a <liffere11t si1.e from the n st.

90 On the Shields-Harary Numbers of a Tree

.J. Jiolliclay, S. !Iolliday*, University of Tennessee ,11 Martin; P. D. .loh11son, .Jr. Aub11rn Univcrsily

The Shiclds-Uarary graph pan1mct.crs are measnrcs oft.he rohnst.ness or integrity of a graph. These parameters arose from a problem of the late Allen Shields, re-construed in a a graph theory setting by Shields ,uid Frank Harary in 1!)72. In this paper, we will give :;0111cresults about. the Shiclds-Ifon:1ry number:; of trees.

91 Broadcast Covers in Graphs

J(an H. S. 13lir, St.eve Hort.on*, Unit.NI St.at.cs .\Jilit.ary Aca,Jr,my

A bi'oadcasl covc1· i:; a integer valued fu11ctio11 .f o11 the vertices of a graph s11d1 that. ev0ry r:<lg0 *uv* is <list.ance at most *J(v)* from some vert.<x *v* E *V*. WP. dHn regard the vertices *v* with *f(v)* > 0 as broadcast stations, each having a transmission power that might be different f'oJH the powers of other stc1tio11:;. \Vheu f (+) s; 1 this is the stanclard vert0x cov<'r problem. The opt.imal broadc:a.c;t cover problem seeks a broadc::ist cover tlrnt minimizes the sum of the costs of the bro::idcasts assigned to the vertices of the graph. We present a theorem about the nature of broadcast cover:; t.hat c:;tabli:;hes d poly11omic1.I ti111e algorithm for the prohlc111 011 arbit.rn.ry grnphs. We also <lisc11ss the broadcast. <lornination problem and some int.cresting relationships between it and broadcast cover.

Keywords: vertex cover, algorithms, broadcasts

92 Locating and Total Dominating Sets in Trees

Teresa W. Haynes, East Te1111cs;ee State 1;11iversity; Midwcl A. Ilcnning, University of Nata.I, South Africa; Jamie !\I. Howard*, lndiau River C'omnrmnity College

A set *S* of vertices in :1 graph G = (*V*, J;') is a total dominnting set of G if every vertex of *V* is adjacent to a vertex in *S*. Total dominating sets of minimum car-<linality which l1ave t1le ad<litiom1.I property that disti11ct subset.:; of *V* are tol;illy <lominate<l by dist.inc:t. s11bscts of the tot.al <lomilrnting sci' arc consirl<rcd in this talk . The concepts of a locating set and a total dominating set arc merged to define two new para.meters. In addition, bouuds on these parameters in a tree are pre cntcd ,1u<l the rntio of the:;c parn111etcrs i11 tree:; i:; investigated.

Keywords: differcntiating tot.al <lominating set., locHt.ing-to(:al domiimt ing sd.

93 On c-Bhaskar Rao Designs and Tight Embeddings of Path Designs

Sp<'nc<'r P. I111r<l*, Dincsh G. Smvat.C'

Under the right conditions it is possible for 1.lie ordered Llocb of a path design Path(v, k,) to be considered as nnonlen ri blocks m1<l t.hereby crcat.e a BIBD(v, k,). We call this a tight embedding. We show that for any triple system TS(v, 3) there is always shuch an embedding and that the problem is equivalent to the existence of a (-1)-IlH.D(v, 3, a), i.e., a c-Ilha:;kcu- nc10 design. That is, we also prove the incidence matrix of any TS(v, 3) can be suitably signed, and, moreover, the signing determines a natural partition of each block making the triple system a 11ested design.

94 List-coloring triangulated polygons

.J. P. Hntchinson*, :\facalester College; R. Ramamurt.hi, Cc1lifornia Stc1le University at San l'vlarcos

A triangulated polygon (tp) is a 2-connectcd, outerpla.na.r,nea.r-triangula.tion. We prove cases when a tp can be list-colored when degree-2 (resp., degree-3) vertices an given 2-list.s (resp., 3-list.s) and all others -I-lists. We conjectme that the limiting case is the presence of at least four separating triangles (with all edges interior) due to a non-list-colorable example of A. Kostochka.

95 Tree Traversals and Permutations

Tod<l Feil, Kevin I-Int.son*, R. Matthc-,w Kret.c:hnrnr, Dcnison l:nivcrsity

In this tcdk, we discn::;:; how prcorder, iuorder, and postorder traversal:; of bimiry trcc-,s can bc; nse<l to establish multiple bijections betwcen binary trees and stack and stnck-sortable words. We show that these operators satis(y a sort of multiplicative ca.nc1illation. As a result of viewing t1le::;e words as tree traversals we s11ow a simpl<' argument to c:onnt the number of stack wor<ls which r1rc also stack-sortable. Finally, we show these operators help to define a natural equivalence relation on binary trees and stack words. Some properties of the resulting equivalence classes arc discussed.

96 Real Number Radio Channel Assignment for the Lattices

.TC'rrold R. Griggs, Xiaolma Terr sa .Tin*, University of S0111h Carolinr1

The channel assignment problem is to assign radio frecp1ency channels to transmitters in a network, using a sma11 span of channels and sa.lisfying some frequency seinu·atio11s to avoid interfereuce. Griggs (l!J!)2) for11111h1lted the corresponding integer graph $L(2, 1)$-labeling problem, which has been the objec:t of n considerable number of papers. We extend it and propose the real nmnber graph labeling prol.Jlem here, which allow the labels c1ml the constraint.::; k; to b1: 11olll1eg;itive real 1111mbers. An $L(k_1, k_2, \ldots, k_n)$-labeling of iraph C is an nssignmC'nt or non-negative real numbers to the vertices of G with x E $V(G)$ la.bele<l $J(.r.)$. such t.lwt. $|J(n) - f(v)| \geq$ L; if n and v are at distance i apart, where k; E [0, oo). We denote by ,\(G; k1, 12, $\cdots$, k,,) the miuinnun span over such Ia.bdiug f E L(/;:1. k2, $\cdots$, k_p)(C). We show >.(CJ; k_1, k_2) is a conti1111011s mid piecewisie-linear function or 1:1, k_2, and >..(G;k_1, k_2) = k_2>.(G ; k, I) for real numbers $k_1 \geq 0, k2 > 0$ and $k = 1$-:J/kz. In a ntdio 111obile network, la.rge service a.rea.s are often covered by a network of nearly <:ongrnC'nt. polygonal c-clls, wit.h eac-h t.rnnsmit.t<'r at. t.hc c-enkr of a cell that. it covers. All transmitters may be pla.cecl in the triangular lat.tic-e Γ_6, the SC]Uarc lattice fo, or thc-, hexagonal lattice $\Gamma_{,,}$. We determi)I(vahws of the minimum sp,tn ,\(fc ; k, 1) for all k $\geq$ 4/G, have honnds for U < k < 4/G, nml det.cn11i11c >..(f0;k, 1) and >..(r ; k, I) for all k $\geq$ 0.

97 Intermediate Distance-dependent Subgraphs

Garry Johu *, Saginmv V,t!lcy State University; T'uu] llrowu, Raytheon Corp.

In an effort to model optima.I locations for emergency fa.cilities in a cit.y, the cent.er and median for a graph were studied. The center is the subgraph whose vertices have the smallest. ecce11tricity (distance to a farthest vertex) and t.he 1ncdi1:11 is the :rnbgrnph with the smallest. stat.ns, or distance (snm of dist.ann s to all ol her vt!rtices). The structure, properties and connections between the cent.er and median have been known for some time. Next, their counterparts, t.hc periphery and margi11 of ,t graph, were i11troduce<l. The vertices of t.hcse :s11l,gni.ph,; hcwe the lnrgcst: eccentricity and largest distance, respedivcly. Aga.in, 11111ch is k11own about these subgraphs of graphs and trees. l'vlost recently, some of the s11bgraphs consisli11g of the remai11ing, or i11tern1ediate, vertices have bee11 studied. For insl.clllCl·!, the int.erior is the subgraph whose vcrt.kes an' not. in tJw p<'ripher_v and l.he an1111!11 includes the vertices in neither the center nor !:he pciriphery. In this paper wP. investigate four other subgraphs: the exterior consisting of the vertices not. in t.he center, aucl t.he core, the 1m1.11tle a11d the crust A i11tcr-;11t:di,ttc subgraphs related lo the mP.r!ian and margin.

198 Some Generalized Graph Partitioning Problems Wit.h Restrictions

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This pa.per considers problems of the following type: given a graph G = (V, H), vertex sets U; C V for l =; i =; r, partition V into I: different parts Vi, ... , Vk with so111e restrictiom;. There are two specific restrictions u.mler consi<lera.tion in this tnlk: (1) c-,ich V: contnins at most one- vc-rt.c-x from U; for 1 S i S r; (2) ca('h U; belongs to just one part v; for some l =; i =; r. The objective function to optimize is I:7= 1 a;e[Vi] according to (1) or (2). Some heuristic algorithms are proposed.

199 Dccycling of Fibonacci Cubes

.lommn A. Ellis-l\lona.ghan, Saint Midmrl's College-; David A. Pike-, Y11ho Zo11*, J\Icmorial University of NewfoundlaHd

The decycling number 'v(G) of a graph C is the snrnllest number of vertices Llwt can be delcted from G so that the resultant graph coHtains no cycle. A Fibo1w.cci string of order n is a binary st:riug of length n with 110 two co11sccut.ive ones. The Fibo1Mc:ci ('.Ube of order n is the graph whose vc-rtic.es are thc- Fibona.n:i st.rings of length n such that two vertices are adjacent if they differ i11 just oHe position. The ramily of Fibonacci cubes has applications in interconnection topologies.

Iu this ta.lk, we will study the decydiug n11111bcr of I.he Fibo11c1cci cnlie8. Lower and 11pper bounds or th0. dP-c-ycling number for thP. Fibcm:i.c:ci c-11hes will be prPsenl<'d, as well as the exact v,1lue of the clecycling number for n < 8.

Keywords: clecycling number, pa.th number, Fibonacci cubes

160 Query Time Algorithm for All Pairs Shortest Distances on Perrnutation G1·aphs

Alan P. Sprague, University of Alabama at Birmingham

V/e present an algorithm for All Pairs Shortest Distances on a permutation graph on n vcrt.ir.cs t.hat., aft.er 0(11.) preprocessing time, can <lcliver an answer 1o a distance query iu 0(1) time. The method involves a reduction to bipartite permutation graphs, a further reduction to unit interval graphs, and finally a coordinati-:atio11 for unit i11terval grnµhs.

Keywords: Perm11t:it.ion gr;iphs, al$_{\mathrm{p}}$orithm, APSP.

161 Regular Graphs on Mobius Strip

Shan7,hen Gao, Michal Sramka*, Florida Ailantic University; Zhonghua Tan, Guangzhou Gongye University, China

A connected graph is embedded in the smface S, then the complements of its image are a family of faces (or regions). If every face of the embedding is topolgically ho111co111orphic to ,\11 open <lisk of $\mathbb{R}^2$, t.hcn the c111hc<li11g is called a 2-ccll embedding. A k-reg11lar graph th;i1. 2-cell emb<ds into a imrface S, in which 1he boundary of every region has the same number of edges, say m, is called a m\#-regular graph on S. A k-regular grnph is called a (k, m)-reg11lc1.r graph of S if it is a 111.\#-reg11lar g:raph on *S*. We <lisc11sc; (1:, m)-reg11lar graphs on t.he l\Iobi11s Strip.

162 Expectations for Graph Self-Assembly

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I\Iolecular self-assembly is a process of creating complex structures from simpler ones through physico-chemical properties without any hnma11 mediation. lJn-<lcrst,u1di11g how na11ostructu1·c:; arc :;clf-a::;se111hled i11to more complex ones is a crucial component of nanotechnology that may lead towards understanding other processes and structures in nature. We present a model of self-assembly, inspired 1,y DNA na11otech11ology an<l DNA coH1puti11g, aHd describe how this rno<lel call be Hsed for predict.ion oft.he ont.comes in 1he graph self-assembly. Using probabilistic methods, we show the expectation aud the variance of the number of self-assembled cycles, J<$_{\mathrm{J}}$, ,incl genera.lize these 1·csults for Kn.Open questions will be discussed as well.

Keywords: Self-Assembly, Exped',1t.ion of I<,,, Vari11ncP. of K$_{,,}$.

163 Mixed Radix deBruijn Sequences

A. Grcp;ory St.nrling*, Goi-don [kavcrs, l."11ivcrsit.y of .i\rlmnsa.c;

We introduce mixed radix deI3rnijn sequences, a generaJi7,atio11 of the well-kuown fixed radix deR$_{\mathrm{ru}}$ij n sequences also known as 'f leprinter sequences (for radix two). Let { 1110 , ml , ... , mk-1 }he a set of radices for 111ixed radix repn c11tat.iuu of the integers modulo n = mi , i = 0, 1, ... ,k-1. 0 di mi , elk-] , dk-2 , ... , d1 , dO a representation and (dk-1 , dk-2 , ... , <ll , d()) = dO + di *mj , i = 1, ... ,k-1 , j = U, 1, ... , i-1 its valuation. A pernmtat.ion of I.he set of k radices gives another represent.at.ion system fort.he same s d of int'.eg<'rs mod11lo 11, along wit li its attendant valuation function.

A mixed radix deI3rnijn sequence on this set of mdic:es is a circular sequence oft.he mixed radix <ligit.s :such thc1.t any cuntiguons s11bstri11g of k of the digits co11tai11s exac:t.ly one digit. for eM:h of t.he k radices, ;i,nd moreover. t.he val11at,io11s of tlwse substrings yield each of the integers modulo 11 exactly 011ce.

½'e nse a genenili7,ation of the deBruijn digr;lphs to pro<lnce mixed radix deBrnijn sequences.

Keywords: !\fixed rn.dkc-s, <leOrnijn scx111ence, dc-Ornijn digrnph

164 Mutually Independent Hamiltonian Paths in The (n, k)-Star Graph

Eddie Cheng, Dan Steffy*, Oaklancl Uniwrsit.y

The (n, k)-star graph, denoted S$_{n,}$k, is a generali7.ation of the stnr gn:i.ph. a pop11lar and well st.udied interr.onnection network. We say that two ha111ilto11ian 1n1ths P$_1$ =< ·1t1, 112, ... 11... > an<l !'$_2$ =< 111, 1.:t, . . . ,.. > a.re inrlr.pr.ndr.nl. if 11., = ·111, 11n = vn and 11i ≠ tt; for 1 < i < n. We say that a seL of hamiltoHinn paths is m11lttall$_y$ *independent* if they arc pairwise independent. \Ve will give preliminary res11lts involviHg the 1nH11uer of 11111tmdy i11dependeHt hc1111ilu11ic1u pcit.hs u1-•\ee11 pairs of vertices in S$_{n,}$..

Keywords: haniiltoninn, interconnection networks, nmt.1w.lly indcpc11dent hmniltonian paths

165 Some graphs for which even size is sufficient for splittability

E7.ekicl Miller, Gary E. Stevens*, FTICA

A graph is sa.id to be splittable (2-splittahle) if its edge set can be partilioncd into two subsets so that the two induced subgraphs are isomorphic. Having an even 11umber of edge::; is obviously ,t 11cccssc1ry condit.iou for split.t.,Ll>ility ,tnd iu this pn.per we look at some basic dn.<;ses of graphs for which it is also s11ffide11t. Then two classes of cnterpillars are shown to have this property. Finally, similar results for k-splittability arc considered.

166 A Construction For Singular Tournament Matrices with Full Boolean Rank

J. Richard Lundgren, Dustin J. Stewart*, University of Colorado at De11ver

A to1m1c1mc11t 111atrix is the cidjacency matrix of a tourna111cnt. There exist several examples of to11rnament nrntric:es in which the real rank of the ma.f.rix is greater than the Boolean rank of the matrix. This has le,id some to ask if there exists a tournament matrix in which the Boolean rank is greater than the real n111k. In this talk we µresent a method for constructing tournan1cnt 1m:1triccs in which the Hoolea.n rn.nk is larger than the r0al rnnk. V,lf'. do =o by const.rm:l.ing a class of tournament matrices with full Boolean rank, and then solving a particular network .flows problem in order *to* find an infinite class of singular tournament matrices wit.bin this dm,s.

Keywords: To11rnament,, To11rnnnwnt. matrix, Rank, Boolean rnnk, Net.work flows

167 Characterizing Iliclique-Helly Graphs

l\frtrina Groshans, Universi<lad <le l:htenos Aires, Argentina; .Jayme L Szwarcfiter*, L'niversidade Federal do Rio de Janeiro, Brasil

A family :F of subsets of a set is *intersecting* when every pair its subsets has a 11011 empty iutersectio11. Say t.hat :F is Hell$_y$ when cv)ry iutersecting sub^{f}a mily of it ha<; a non !"mpty inters0.<:1:ion. Hf'.!ly families of s11bsets have been st.mlie<l iii different contexts. In the scope of grnph theory, this study has motivated the introduction of some classes of graphs, cls clique-I-Icily graphs, disk-Helly graphs anq ll(;ighhorhood-Ilclly graphs. These classes correspond to the Cct8CS where l.he fal!lilies subject to the Belly Properly are (maximal) cliques, disks and neighborhoods, respectively. On the other hand, define a bicliqtLe of a graph as a maximal subset of its vertices i11ducing a co111plete bipartite graph. Bicliq11es in graph theory h,we been also consi<lered in <lifferent contexts and form a st.rnctmc with int.cresting propertie::;. Ju this work, we consider t.hc gn1.µhs whose fa111ily of bidiques i:; a Helly fn.inily, the bir.liqur-:-Hr.ll$_y$ qrnphs. We <lesnibc st.rndnrnl c:harn<:1.eri:.mt:ions of it. The characterizations lead to polynomial time algorithms for recognizi11g biclique-Helly graphs. 'Ne reoc1ll that a graph might have an expo11c11t ial uuu1lwr of hidiq11cs. Therefore the algorit.lnu by Berge for rccog1ii:.li11g !Icily families of ;111> sets could not be applied directly to recogni.r.c bicliq11c-Hclly grnphs in polynomial time.

Keywords: Hicliques, Hiclique-Helly graphs, Cliques, Cliq11e-Hclly graphs, I-Telly T'ropcrt.y

168 Authentication Codes based on Affine Transformations

N. Gutierrez, I-I. 'lhpia-Ilecillas*, L'niversidnd J\utonorna :t\Ictropolit.a11a i\lexico

In HJ92 G.J. Simmons introduced the concept of (11nco11dit.ional) authent.ical.ion code (A-code) for a receiver to authe11tic,1te i11fon11atio11 sent by a sender Ly 111ca11s of n public chr1nnC'l. In recent yC'ars a number of anl hors have lweu ini.<'rest.ed in combining aspects of several areas including linear trausfornintious aud error-correcting codes to produce A-codes. In this talk some A-codes arc described by 1lla11s of aflinc tra11sfonm1tious over ,1 tinite field wit.li 'I = JJ$^{/}$. (JI cl prime and *r* d posil.ivP. inl.f'.ger) with probabilit.ies of s11ccessful impersonation att.n.c-k and sm-c:essful substitution att.ack equal to $1/q$.

169 A Sierpinski graph and some of its properties

Alberto J\lokak Teguia*, Auaut. P.Godl>ole, East, Te1111cssec S!.c1t.e Uuivcrsit.y

The S-ic17J·i1iskifrnclal or Sicrp·i1isk·i gasket E is a fan1ili,1r ohjcd studied l,y specialists in dynamical systems and probability. In this pa.per, we consider d grnph 8,, derived from the first n iterations of the process that lecids lo 1:. and sludy some of its properties, i11dudi11g t1le cycle structure, do111i11utio11 1111111bcr and pebbling number. Various OJWII qncst.ions arc posC'd.

170 Double domination edge critical graphs

Derrick Thacker*, Teresa \\[1]. Haynes, East Tennessee State Universit.y

In a graph G = (V, E), a subset S V is a double <lontinnting set if every vertex in V is dominated ,it le,t::;l. twice. The minimum rnrdiuality of a double clornimiting set of G is the <lonble clominat.ion nnmber **Yx** 2(G). A grnph G is donhlc <lomination edge critical if for any edge tlV **E** 1:,'(C), the **'Yx2(G** + uv) < **Yx** 2(G). We investigc1te properties of double domina.tion edge criticc1.l graphs. Th particular, we characterize the d011ble domination edge crit.irnl grnphs G wil.h **/x:z(G) E** p. ··!}.

171 Partitions of difference sets and code synchronization

Vladimir D. Tonc:hev, :.'lic:higan Tcchnolo;,?;ic.al University

Difference systems of sr.ts (DSS) are combinatorial arn:rngemcnts that arise in con- neciio11 wit.li code 8Jnchrunization arnl avoidi11g conflicts iu ,1sy11dhru11om; 111ultiple- a<:cess channels. Some combinatorial and algebrn.ic constrnctions of DSS oblriincd as partitions of cyclic difference sets are discussed.

172 Transitive Closure of a Lattice Fuzzy Matrix

Zengxiang Tong, Otterbein College

This is the contimm1.ion of my two papc\rS ent.itkd Connededn0.ss of an f1177,y Grn.ph and An Algorithm for Fi11ding the Colll1ecLedncss l\la.trix of a Fuzzy Graph, which were pnblished in the journal Congressus Numerrant.ium (1995 and 1996). In this paper, the author i11truduces the concepts of an L-fuzzy graph :\lid its c:onnect:edness, ,me! nses Lat.I.ice Fit7.7.y matrix to denote m1 L-ftt7,7,y graph, and the transitive closure of the matrix to denote the connectedness of the graph. The properties of the connectedness of a11 L-fuzzy graph arc studied, and two algorithms for finding the connect.e<lness matrix of an L-flt7,z.v grnph, i.e., 1he transitive clo- sure of a LaLt.ice Fuzzy matrix, arc presented. **Keywords:** fuzzy graph, lattice, connectedness, matrix.

173 Expected value and dice games

Lorenzo Traldi, L!\fayett.e College

A generalized die is simply a finite list $X = (x_1, \ldots, x_m)$ of integers, mid the ex- pected value of the die is the mean Σx_i. If $X = (x_1, \ldots, x_m)$ and $Y = (y_1, \ldots, y_n)$ are two dice then we say X is stronger, or X wins the contest, if there :ire more µ,1irs (i, j) with $x_i > y_j$ thcm there :ire pairs (i, j) wit.li $x_i < y_j$. Colllmon sense s11ggesls t.h;it the rdat.ivc streng1.h of X !\nd Y should be relat.ed to t.heir expecled values. If $X_1, \ldots, X_m$, $Y_1, \ldots, Y_n$ are restricted to two values then this suggestion is valid, but otherwise it is not. Two striking results :ire:

1. If the integcrn which aµpe,1.r 011 the dice in q11e:;tion arc rc:;tricted to three values then 1.here is a nmnerical meflst.ire which <let.ermines 1he relative. strengths oft.he dice. However the expected value is 11ot such a measure.

2. Among the 462 six-sided dice involving integers $1 \leq x_i \leq 6$, there are only :;cve11 whose contests with the re:st :1re detenni11ed hy expected values. Four of the seven arc obvious: the iwo weakest. dice are (1, 1, 1, 1, 1, 1) and (1, 1, 1, 1, 1,2) and the two strongest dice are (5, 6, 6, 6, 6, 6) and (6, 6, 6, 6, 6, 6). Another one of the seven is the familiar (1, 2, 3, 4, 5, 6). Try to find the other two before yon come io the talk.

174 Ruin problems in Stochastic Risk Computing

IIoa Tran, New York 1:nivcrsity & Fordh:un UnivC'rsit_v

As the tool to predict the collapse in terms of finance of n company, tile probability of ruin plays a crucial role. The interest nit0, init.ial compo1.1ndi11g a.sset.s, togdlicr with ruin time, ruin function will he cfo;cus:,;cd for the 11ew directions of observ- ing the c1_mnce of being collnpsed of the company. As the interest rnte becomes larger, the observation is the probability of ruin will be smaller. Random walk, Brownian motion a11d the co111wctio11 with Capital Asset .Prici11g l\,fodd also will be ad<lrrsserl. The moclcls nm assist decision makers or investors to make <lCcisim1 io choose bet.ween insurance and investment risk.

175 A new formula for computing Frohcnius numbers in three variables

Jcrnct Trimm, Ovcrtou11 :v1. G. Jcndn, Auburn University

Given a set of rel.:i.tively prime prn;itive integers, after sollle point ,di posil:ive inl.c- gers are represe11t;1hle as a linear cornbin;ition of th,\ set with 11omH'.!?;ntivc, int.eger coefficients. Which integer is the last one not so representable is the 1"robcni11s problem, or the Frobcuius stcimp problem, and the number in question the Frobe- 11ins 11u111her of the set. While the two-variable solntio11 is widely k11ow11, a11<i the gcnC'ral solution is NP-hard, then lmvc been s>veral n.lgorithllli<: so11t ions of the three-variable problem. In this paper we present a fornmlnic solnt.ion for the Frobcnius number of most rel.:1tively prime triples.

Keywords: Frobc11iw; 111u11bo;- conductor, n11111eric sc111igro11ps, dioµlmnt.iue equations

176 Periodicity of subtraction games with subtraction sets {l, b, c}

.Jean l\I. Turgeon*, University of Montreal; Daniel A11det, :\1atthie11 D11fmr, U.Q.A.J\l

\We consider games defined by subtraction sets of the form {l, b, c}, i.e. a game where two players have a stack of chips i11 com111011 a.lld tc1.kein t11r11 either 1 or b or c chips, where l < h < <: The winnm· is the one who takes the la.t chip. Given a particular set {l, b, c:}, computing the losing positions ns a function of the number ll of chips (a position f'om which you call only put your opponent in a winning positio11; f'o111 a winning position, there is a possibility of µlcicing your opponent i11 a losing position) presents no problem. This function always becomes eventually periodical. The interesting problem is to find a general relation between the set {1, b, c}, c111Cl the dm.rc1.ctcr of that periodicity. We shall presellt a cornplete solutiou, including the Grundy v11lues of et<h position. The more general c11se {a, b, c} is still open.

177 A Hybrid Model for Classification Rule Discovery

Michael L. Gcirgano, Gokhora Uran, Pcice University

A genetic algorithm, swarm intelligence, and hill climbing hybrid heuristic; is applied to the data mining task of developing classification rules and comparisons are made with other methods.

178 Bounds for Representation Numbers of Hypercubes

.lames Urick, Rochester l11stit11te of Technology

A graph G has a representation modulo n if there exists an injective mc1p f: $V(G) \to \{0, 1, \ldots, n\}$ such t1c1t vertices u and v are adj,1cent if and only if $|f(u) - f(v)|$ is relatively prime to n. The representation 1mmber n:p(G) is the smallest n such that C has a representation modulo n. We ge11eratc new bounds for representation numbers of hypercubes.

Keywords: vertex lc1bcling, repuese11tatio11 111odulo 11, product dirnension, l1yper-c11bes

179 The Forcing Connected Domination Number of a Graph

Robert Vandell, Indiana Univer::-i1.y - P1ın11e Univcrsit.y

In .JCl\JCC 25 (1D97), Harary ct al defined the forcing dornina(.ion 11mnber f(G, γ) of a graph (;_ Tn this paper we extend this definition to connected domi11c1.tio11, and cv;1luate the parameter for ccrtclin grnphs, 111ost 11ot.1hly grids. For a com1ectcd grc1ph G the connected domim:i.Lion number γ_c(G) is the minimum cardinality of a connected dominating set of the graph. For a connected dorninati.ug set S of cardi11a.lity γ_c(G), a subset T is called a forcing set if S is the unique 111i11i11111111 conncc:t.ed dominating set containing T. The forcing nnmbcr f(S, γ_c) of S is the minimum cardinality of a forcing subset of S. The forcing co11ncctcd domination mnnber f(G,γ_c) of a graph G is the minimum forcing number ,unong the miiii111um cu1111ected domi11ati11g sets of G.

180 The pebbling number of graph

.Jessia :\Iuntz, Sivaram Narayan, Noah Streib, Kelly VanOcht.en*, Central l\Iid1igan Uuiversit.y

To make a *(p, k) pebbling move, p* pebbles are removed from a vertex. Then, $p - k$ pebbles arc tossed out and the remaining k pebbles arc ph1ced 011 an adjarnnt vertex. The *(p, k) pcbbliny number, N,* is the smallest 1111111br of pebbles needed s1ıch tlmt for every distribution of N pebbles it is possible to move k pebbles lo any desired vertex by a sequence of *(p, k)* pebbling moves. The (p, k) pebbling number of a gra.ph G is denoted $f_{p,k}$(G). The most commonly used pebbling niove is the (2, 1) pebbling rnove, and the (2. 1) pebbling nnmher of a graph C is ,kno1.cd *J(U).*

The *optimal pebbling rmmber* of G, denoted f_{opt}(C), is the smallcsl n11mbcr of pebbles neede<l such t1c1t every vertex i1l G is µeLblca.ble by a scq11c11ce of (2, 1) pebbling moves for a p11rtic1ılar distrih11ıion of tha1· n111nbcr of pehhlcs.

We present resnlt.s 011 (p, *k)* and optim11I pbbling rn1ml)('rs of graphs of dimnc-tcr three, including results of a sharp upper bound for (2, l) pebbling numbers of grnphs of dic1111ter three.

181 Noncooperative Bottleneck Flow Control *in* Two User Networks

Ping-Tsai Chnng, Long Islarni Universit,_v: n.ichard V;in Slyke*, Polyt.echnic University

\Ve :;tudy a11 adaptive, di::;t.ributed algorith111, the bot.tleueck flow coutrnl algorithm wlwrc each 11scr a<lj11st s its rate b,ised on a sat.urntion measnrn for the t:hrnnghpnt. versus delay tradeoff at the bottleneck link. Bach user iteratively updates its flow to meet. its individual sat11ration measure. Our work focuses on individual (or 11;cr) opti111izatio11 as opposed to sy:;tc111 opti111izatio11. Convcrgc11cc a11alyscs arc based 011 a noncooperative game theoretical formulation. Under this formulation, the convergence to a :\'ash equilibrium point of the bottleneck flow control for an arbitrary two user 11eiwork is :;hown.

182 Planarity and colorability: a survey

V. Voloshin, Troy University

l'vlixed hyper.graph is a triple H=(X,C,D) with vertex set X and two families of subsets, C and D, called C-edges and D-edges respectively. Proper k-coloring of H is a mapping from X into a set of k colors in such a way thd.t every C-edge has two vertices of a Common color and every D-cdge lws two vertkcs of Different colors. Mixed hypergraph is called colorable if ii admits at least one proper coloring and uncolorable otherwise. In a colornble mixed hypergraph, the maximum and minimum number of colors over all proper colorings which use all k color:; is r.alled the upper ::ind lower chromatic nnmbers respect.ivdy. Mixed hypergrnph has a continuous chromatic spectrum if proper colorings exist using all numbers of colors between the lower and upper chromatic numbers. !\-fixed hypergraph is ca.lied planar if it ca.n he emhccldccl in the plane in :;uch a way that. edge:; intersec:l. only at the respe<:l,ive neighborhoods of common vert.ice:-. Planm- mixed hypergrnphs generalize pl<tnar graphs and hypergraphs. We survey results <1nd for111ulate :;orne open problc111s 011 colorability, lower and upper chro111atic urnnbers, and the chrmnati<: spectrum of planar mixed hyp< rgraphs.

183 Triad Designs

W. D. Wallis, Southern lllinois l.Jniversiiy Carbondale

\Ve sha11 discuss a family of tournaments in which each match has size 3 and the order of players is important.

184 Connected Domination in Grids

Peter Ilambnrger. Chip Vandell, I\Jatt. \Va):::;h*, Indiana Univnsity - P11rd11e University

The connected domination 1111111ber of a graph was defined by Sa.mpa.thkumc1.r and Walikar in 1D7!J: !.r (G) is defined as the mi11i111111 c,.i.nliuality of a do111im1ti11g ::;ct. which induces a connected graph in G. We consider this p<'l.rameier and some of its close relatives in the context of transportcition networks, concent.rating particularly 011 (finite an<l i11finite) grid gra.ph::;.

185 Binary trees with the largest number o'rsubtrees with at least one leaf

L.A. Szekely, Hua. Wang*, University of South Carolim1.

We charn<:1,crize binary trees with n leave:-, which h,ive 1he great.c::::;f, 1111111ber of subtrees with at least one leaf. These binary trees coincide with t:hosc which were :;hown by Fischen11a1111 et al., Jele11 and Triesch to mi11i111i;e the Wie11er i11dex. Knndsen provided a m11ltiple parsimony ali,gnment. with affine ,gap cost nsin;; a phylogenetic tree. In bounding the time complexity of his algorithm, a factor was the number of so-called "acceptable residue configurations". In our terms, it is the nm11bcr of suht.rce:; contc1,i11ing at least 011c leaf vertex. K111tdse11 cst.inmtccl the maximum number of acceptrtble residue config11rations over all binary trc•t'S. \•Ve determine this maximum exactly.

186 On the Edge-Graceful Spectra of the Double Cycles and Their Coronae

Sin-Min Lee, Ho Kuen Ng, San .Jose State University; Tao-Ming \Vang*, Tung-Ha.i University, Tai·a.11

Let G br. a (p, q)-grnph and I; > 0. A graph G is :-ai<l to hr. /,:-c,dge-gnwcful if 1he edges can be labeled by k, k + 1, ..., k + q - 1 so that the induced vertex smns (mod p) are distinct. 'We call the set of all such k the edge-graceful spectrum of G, am! denote it hy t'._ql(G). Iu this paper the cdgc-grnccful spt•d.m111 of the do11i>lc cycles and their coro1111e are del-ennined.

1 8 7 On P(a)Q(h)-Super Vertex-graceful I-regular and 2-regular Graphcs

Sin-Min Lee, Ho Kuen Ng, San Jose State University; Yung-Chin \Vang*, Tzu-Hui Institute of Technology, Taiwan

(;iven i11tegen; a1> > 1, a gri:lμh G with Yertex set V(G) i:llld edge set E(G), p=IV(G)I ll.nd <J=IE((;)I, is said to be P(a)Q(b)-s11pcr vcrt.ex-graccfnl (in short P(a)Q(b)-SVG) if there exists a function pair (f,f+) which assigns integer labels to the vertices and edges, i.e., f: V(G) -+ P(a) and f+: E(G) -+ Q(b) are onto, f+ (u,v) = f(n)+f(v) for cttiy (u,v) hdongs to E(G), Mid
Q(b)={b,(b+l),...,(b+q/2),-h,-(b+l),..., -(h+q/2)}, if q is even,
Q(l..>)={0,b,...,(b+(q-1)/2),-b,-(b+(q-1)/2) }, if q is odd,
P(a)={a,(i:t+ 1),...,(a+p/2),-a,-(a+J),...,-(a+p/2)}, if p is even,
P(n)={O,a,(a+l),(a+(p-1)/2),-a.-(ll--!-l),-(a+(r-1)/2)}, if pis odd.
We determine here classes of 2-regular graphs that are P(a)Q(b)-super vertex-graceful for different a,b.

188 Using randomized sampling against NTRU

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The public key encryption method NTRU is very promising because of its simplicity and its speed. Coppersmith and Shamir transferred the problem of finding the μrivate key in NTH.U int.o il "short v<ct.or prol.>lem" in a lattice. Due to some attacks the pll.rmnet.ers of NTRU were c:lmngerl in 200:l. This lms conse<J11ences on lattice attacks on NTRU. Here, we transfer the problem of finding the private key of the new NTRU scheme into a "closest vector problem" in a lattice. Further, a 11ew proln1hilistic algorithm by Schuorr called random samμliug wcis implement.c<l and nscrl ngninst. the new V(·\rsion of NTH.U. It w/ls possible 1o break instances of length up to 97 on a standard personal computer.

Keywords: NTRU, public: key cryptography, randomized sampling, lattice basis reduct.ion.

1 8 9 A Proof of Petersen's Theorem

John .J. Watki11s, Colontdo College

In 1891 Julius Petersen published a paper that c:ont.aincd his now famous theorem: any hri<lgclcss cubic graμh has a]-factor. These days Pctcrscu 's theorem is always proven inrlirect.ly 11sing either llnll's 1heorem from 19:ll'i or T11t.l.e's 1.heorem on !-factors from 1947. We ,viii disrnss a number of attempts that have been made over the years, including Petersen's own attempt, at a direct proof of this result.

1 9 0 On the super vertex-gracefulness of cartesian product of graphs

Sin-Min Lee, San .Jose St.at.e Universi1y; \Vnmli Wei*, florirla .At.la.nt.ic Univerf;il.y

For any positive integers p and q, we denote P={l, 2,..., p/2 }u{-1,-2,...-p/2}, if pis even, and P ={0 }u{l, ..., (p-1)/2 }U{-1,-2,...-(p-l)/2 }, if pis ode!. Q ={I, ..., q/2 }u{-1,-2,...-q/2}, if q is even, and q ={0 }u{l, ..., (q-1)/2 }u{-:I ,-2,...-**(q-1)/2**} if Q is odd. **A** (p,q)-grnph **G** is **called** snper vert·ex-grnc:cfnl if Ihere exisl.s a function pair (f, f+) which assigns integer labels to the vertices and edg<)S; that is, f: V (G) ---+P, and f+:. E (G) ---+Q. s11ch that f is onto P i:lnd f* is onto Q, and f+(n, v) = f (n)+f (v) where (11, v) E E (G). In [l!lj the first: :111thor init.iat.e 1h< investigation of the super vertex-graceful grnphs. We consider here graphs which are <:artesian product of graphs that are super Yertex-grnccful. In particulur, we show t.hat all torus grn.μl1s ,ire not :;nper vert.ex-gn,u:dul.

191 Variations on Discrete Renyi Parking Problems

1\-lichad L. Gargano, .Joseph F. Malerha, Arthm Wcise11sed*, Pace University

Consirler a pat.h with x crlges. At t.imc l = l a cnr randomly pl'trks on an edge reducing the available parking spaces. At each lime period m1other car arrives and parks l"i:tlldornly in a feasible parking space (i.e., so that its not. L>locki11g any other pnrkerl c:nr). The process enrls when t.hcre are no more fca.c;ihlc spaces. \Vhat. percent of the spaces do you expect to be utilized?

192 Percolation Threshold Bounds for Archimedean and Laves Lattices via the Containment Principle

.John C. Wierman*, .Johns Hopkins University; Robert Pcirvictnc11, lJniversit:y of l\fclhomne

Percolation models are infinite random graph models for phase Lra11sil ions and critical phenomena. The percolation threshold corrnsponds to the. c:rit.ical temperature or phase transition point. The containment. priut:iplc sta1.t:s that if o11c graph is isomorphic- to n snbgrnph of /\not.her, its perr:olation thrP.<;holrl is grcaf.('r t.han or equal to that of the other graph. We c011sider two classes of planar infinite lattice graphs which arc studied in the physical science literature. We find all subgraph relationships among ;i cl,1ss of 21 lattice grnphs, proving i111pos:sihili1.y of a s11hgrnμh relationship iu all other cases. Using bounds determined by other methods, we use the containment principle to improve percolat.ion threshold bounds for some of Lhe lattice graphs.

Keywords: perr.olat.ion, random hrrnph, s11hgrnph

193 Lattice Paths and Subgroups of Riordan Matrices

Wcn-jin Woan*, Davirl Ilo11gh, Ilowarrl Univcri-it.y

\Ve com;icler those la.Wee pcit.hs that. use steps selected from: $U = (1, 1)$, $L = (1,0)$, $D_1 = (1,-1)$, $D_2 = (1,-2)$, $Da = (!,-)$, ... with assigncrl wcights l, wo, w1, w2, w:i,... .. We define a weight polynomial $w(x) = 1 + w0x + u:1x^2 + w_2x^3 + w_3x^4 + \ldots$ The lattice paths generate a lower triangular *Riordan* matri:r; /1/, A lower triangular matrix is i;aid to he a Riordall matrix, if the gc11crati11g f11nct.ion of l.he k-t.h mlumn of M is $g f^k$, whcr<' $g = g(x) = 1 + a1x + a2x^2 + \ldots$ and $J = f(x) = x + b_2x^2 + b_31^\ddagger + \ldots$ where $J = x(w(J))$. The set Rafail Riordan ma.trices is called the Riordan group. Here we study a list of subgroups a11d its relation wit.h latt.ir:e paths.

Keywords: Lattice Paths, H.iord,1n !\:la.trices and St.ieltjes Mat.ric:cs.

194 On P(a)Q(l)-Super Vertex-graceful Unicyclic Graphs

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For ally integer a ;;?ll, a graph G with vertex set V(G) and edge set E(G), p=IV(G)I and q=IE(G)I, i:; said to be P(a)Q(l)-super vert,(x-graceful (i11 short P(a)Q(l)-SVG) if there cxii-t.s a fnnct.ion pair (f, f+) which ac,cigns integer labels to the vertices and edges, i.e., f: V (G)-> P(a) and r^+ : E(G) _. Q(l) are onto, f+(u, v) = f (u)+f (v) for any (u, v) E E(G), and

q(l)={±l, ..., ±q/2}, if q is even,

{O, ±1, ..., ±(CJ-l)/2}, if CJ is orlrl,

P(a)= {±a, ±(a+l) ..., ±(a-l+p/:l)}, if pis even,

{O, ±a, ±(a+l) ..., ±(a-l+(p-l)/2)}, ifp is odd.

We determine here classes of unicyclic graphs that are P(a)Q(l)super vertcx-grac:cful for a =2. Moreover, some conjectures arc proposed.

195 Embedding Graphs on the Torus

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A lortLs is a surface shaped like a doughnut. *A lopolo_qical obslmclio11* for the toms is a graph G with minimum degree three that is not embeddable on the torus but for all edges c, r, - c c111bed:; 011 the torus. *A* 111i1101· orde1· obslntcl'io11 ha:; the acl-rlit.ional property that. for all erlg:cs r:, G contrn<:t. c ember!:=; on t.hc t.orns. The aim of our research i:; tu find <111the obstructions to the 1.orus. *A* secJ.n:h for d co111pldc set. of toms obst.rnct.ions is fm:ilit,at.erl by dct.ennining t,hc smnll obstr11ctio11s 11si11 the computer. Polynomial time algorithms have been proposed for this problem, but they arc complex and potentially have a high consta11t overhead t.h,\t could make thc1JJ less desirable for s11Ja11 graphs. In t.hi:; talk, we desci·ilic ııı „ltcrn<1te approach based on Demouc:ron's planarity test.ing algorithm which works in expo-nential worst case time yet is very effective for small graphs (t.he potcnl.ial l.orns obstructiuus).

Keywords: t.opologir.al graph thcory, emh<•rlrling graphs 011 n1c 1:nms, algorit.Juns for graph embedding.

196 On Super Edge-graceful Eulerian Graphs

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Let C be a (p,q) graph in which the edges arc lnbelcd 1,2,3, ... q so that the ver-tex sums are distinct, mod p, then G is called edge-graceful. J. l\Iitchem and A. Simoson i11trod11ce<l the concept of super c<lge-gn1.cd11l graphs which is a stro11gcr concept. t.han e<lge-graccfnl for some da.c;scs of grnphs. We show here some c11kria11 graphs are super edge-graceful, but not edge-graceful; and some arc cdgc-grnceful but not super edge-graceful. We :;how that Rosa's type co11<litio11 for c11leria11 :;11µcr cdge-grnceful gmph:=; rloes not. cxi:;1:. Moreover, some c-onjed.nres arc proposer!.

197 Detectable Colorings of Graphs

(;ary Chartra11d, Remy Esc11<1<lro, F11tab;i Oka1noto, Pi11g Zl1ang*, \Vcsf,crn l\Iichigan Uuivcrnit.y

Let G be a connected graph and let c : J:;(G) -+ {1. 2, ..., k} be a coloring of l.he edges of r, (where adjacent edges may be colored the sa me}. For each vertex ı· of C, the color cor!c ofv is the k-t.nplc c(11) = (a_1, n, , •, •,ak), whcrc n; is Lti(' nmnhcr of edges incident with tt that arc colored i (1 i k). The coloring c is called detectable if distinct vertices have distinct color coclcs. \Ve present. some results in this area.

198 Some Generalized Graph Partitioning Problems Wit.h Restrictions

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This pa.per considers problems of the following type: given a graph G = (V, H), vertex sets U; C V for l =; i =; r, partition V into I: different parts Vi, ... , Vk with so111e restrictiom;. There are two specific restrictions u.mler consi<lera.tion in this tnlk: (1) c-,ich V: contnins at most one- vc-rt.c-x from U; for 1 S i S r; (2) ca('h U; belongs to just one part v; for some l =; i =; r. The objective function to optimize is I:7= 1 a;e[Vi] according to (1) or (2). Some heuristic algorithms are proposed.

199 Dccycling of Fibonacci Cubes

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The decycling number 'v(G) of a graph C is the snrnllest number of vertices Llwt can be delcted from G so that the resultant graph coHtains no cycle. A Fibo1w.cci string of order n is a binary st:riug of length n with 110 two co11sccutive ones. The Fibo1Mc:ci ('Ube of order n is the graph whose vc-rtices are thc- Fibona.n:i st.rings of length n such that two vertices are adjacent if they differ i11 just oHe position. The ramily of Fibonacci cubes has applications in interconnection topologies.

Iu this ta.lk, we will study the decydiug n11111br of l.he Fibo11c1cci cnlie8. Lower and 11pper bounds or th0. dP-c-ycling number for thP. Fibcm:i.c:ci c-11hes will be prPsenl<'d, as well as the exact v,1lue of the clecycling number for n < 8.

Keywords: clecycling number, pa.th number, Fibonacci cubes