## Thirty-Fourth Southeastern International Conference on


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## Combinatorics, Graph Theory <br> \& Computing

Florida Atlantic University
March 3-7, 2003
Program and Abstracts

## Invited Talks

Monday, March 3, 2003
Spyros Magliveras, Something Euler Would Would Have Liked (9:20AM)

Tuesday, March 4, 2003
Joan Hutchinson, Extending Precolorings of Graphs (9:30AM)
Joan Hutchinson, On Visibility Graphs (2:00PM)
Wednesday, March 5, 2003

Richard A. Brualdi, My Favorite Classes of Matrices: Some Recent Developments (10:30AM) Alfred Menezes, Curves and Cryptography (2:00PM)

Thursday, March 6, 2003
Alfred Menezes, Curves and Cryptography (9:30AM)

Monday

## Something Euler Would Would Have Liked

Spyros Magliveras
Florida Atlantic University

Tuesday

## Extending Precolorings of Graphs

Joan Hutchinson
Macalester College
Suppose part of a graph is (properly) precolored. When does The precoloring extend to the entire graph? More precisely, if a subgraph of G is $s$-colored with $<=$ chi $(\mathrm{G})$, does this extend to a chi $(\mathrm{G})$-coloring of G ? If the colo,ing docs not extern.I. docs it extend to a (chi(G)+l)-coloring? And suppose the components of the subgraph are s-colorable and each is colored withs of chi(G) colors, but not necessarily the sames colors. Then what extension theorems are pos.sible? Most of this talk is joint work with M. 0 . Albertson. We consider these questions in general, for planar graphs, for graphs embedded on non planar surfaces, and for embeddings with all noncontractible cycles long. We obtain in most cases a best-possible result for the number of colors needed for these extensions. The talk will emphasis three ideas: coloring using Kempe chains with one extra color. finding nice noncontractible cycles in embedded graphs, and introducing the concept of relative width of an embedded graph.

## On Visibility Graphs

We consider a variety of visibility graphs, related theorems and algorithms for their characterization and layout. We represent graphs in the plane by horizontal line segments (bars) with vertical visibility (bar-visibility graphs), by rectangles in the plane with horizontal and vertical visibility(rectangle-visibility graphs), and by arcs from concentric circles with radial visibility (polar visibility graphs). We also venture into3-dimensions using parallel rectangles with upward visibility, onto the cylinder, the Mobius band, and the torus with parallel line segments and orthogonal visibility. In some of these depictions a graph characterization is given, for others the problem is NP-complete. Finally we introduce the"visibility number" of a graph, the minimum $t$ with which vcllices can be represented each by tor fewer bars in the plane and edges by verticalvisibility; we study how this parameter performs on familiar graphs. These results include joint work with P. Bose, Y-W. Chang, A. Dean, M. Jacobson,
J. Lehel, T. Shermer, A. Vince, and D. West.

Wednesday

## My Favorite Classes of Matrices: Some Recent Developments

Richard A. Brualdi

University of Wisconsin- Madison
My favorite classes of matrices are the classes of $(0,1)$ - matrices with a prescribed row sum vector Rand a prescribed column sum vector $S$ (also known as the classes of bipartite graphs with prescribed degree bi-sequences Rand $S$ ). The investigation of these classes was started almost 50 years ago by Gale, Ryser, and Fulkerson. Some problems in areas such as ecology and discrete tomography can be advantageously formulated and conceptualized in terms of them. In this talk, I will review some classical aspects of classes of matrices of O's and l's, discuss some recent work, and highlight some of the applications.

## Curves and Cryptography I

## Alfred Menezes

University of Waterloo
Algebraic curves over finite fields can be used to design public-key cryptographic protocols. We survey recent work on the security and efficiency of such protocols that use elliptic and hyperelliptic curves.

## Thursday

## Curves and Cryptography II

## Alfred Menezes

University of Waterloo
Algebraic curves over finite fields can be used to design public-key cryptographic protocols. We survey recent work on the security and efficiency of such protocols that use elliptic and hyperelliptic curves.

Monday, March 3, 2003

| 18:00 am | Rq;istration until 5:00, entrance area of Grand Palm Room, where coffee will be selTed. |  |  |
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| 19:20 am | Spyros Magliveras, Grand Paln1 Room |  |  |
| 110:20 am! | Coffee |  |  |
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| $16: 00 \mathrm{pm} 11 \quad$ Reception, Visual Arts Patio |  |  |  |
| Transportation back to motels (TBA) |  |  |  |

## Tuesday, March 4, 2003



## Wednesday, March 5, 2003



Thursday, March 6, 2003


## Friday, March 7, 2003



> Retracts of Cartesian products of $(2 k+1)$-angulated graphs and Construction of Cores

## Zhongyuan Che* and (laril LColli11s, Wrsleyan University

A connected graph $G$ with odd girth $2 \mathrm{k}+\mathrm{I}$ is weakly ( $2 \mathrm{k}+1$ )-angulated if every edge of $G$ is in a $(2 \mathrm{k}+1)$-cycle, and strongly $(2 \mathrm{k}+1)$-angulated - if any two vertices of $G$ is connected by a sequence of ( $2 \mathrm{k}+\mathrm{I}$ )-cycles with consecutive ones sharing at least one edge. We characterize the retracts of the Cartesian product $G M H$ of two connected graphs with one factor being wakly or strongly $(2 \mathrm{k}+1)$-angulated. The result is then applied to construct cores using vertex-transitive $(2 k+1)$-angulated graphs, in particular, using Kneser graphs $K(n, 2 n+I)$.

Keywords: retract, cartcsian product, core, strongly ( $2 k+1$ )-angulatccl, weakly $(2 k+1)$-angulaLccl

## 2 <br> Divisibility Exponents and the Zimin Recursion Larry Cummings, University of Waterloo

E. N. Gilbert first observed in $1!558$ that the "change digits" of the binary reflected Gray code of order n could be obtained if $\mathbf{1}$ is added to the ordered sequence of non-negative integers which are the highest powers of 2 dividing the integers $1,2, \ldots, 2^{\prime \prime}-1$ We prove that the recursive Zimin construction for square-free strings also generates these sequences and give a generalization for arbitrary positive integers greater than 2

Keywords: Gray codes, Zimill recursion, divisibility sequences

## 3 skolem Labelling of Generalized Windmills Catharine Baker*, Mount Allison University; and Ben Seamone, Mount Allison University

In 1991, Mendelsohn and Shalaby defined a (weak) d-Skolem labelling of a graph $\mathrm{Q}=\boldsymbol{(}, \boldsymbol{E})$, to be a vertex-labelling of Q using labels $\{\mathrm{d}, \ldots, \boldsymbol{d}+$ 1 1屰1\} such that each label $i$ is used twice, on vertices that are distance i apart. This labelling is strong if every edge is essential (i.e., the graph is no longer d-Skolem-labelled if an edge is omitted). They then determined precisely when paths, cycles [1991] and k-windmi11s [1999] could be Skolemlabelled. Such labellings can be used to design a schedule for testing a communications network for node, link and distance reliability. We extend these results to generalized windmills with vanes of unequal lengths.

4 Some Properties of the Directed Path Graph Operator Daniela Ferrero, Southwest Texas State University

For a given graph $G$ and a positive integer $k$ the $\mathrm{P}_{\mathrm{k}}$-path graph, $\mathrm{A}(\mathrm{G})$, has for vertices the set of all paths of length k in $G$ Two vertices are connected when the intersection of the corresponding paths forms a path of length $k-1$ in $G$, and their union forms either a cycle or a path of length $k+1$ in G Path graphs were proposed as an extension of line graphs. Indeed, $P_{l} G$ coincides with $L(G)$, the line graph of $G$. Analogously, for a given digraph D and a positive integer k we can consider the $\overrightarrow{\mathrm{P}_{\mathrm{k}}}$-path digraph of $D,{ }^{\circ} /(D)$. In this case, $A(D)$ coincides with the line digraph $L(D)$ and also $1->$; $(\mathrm{D})$ equals $L(L(D))$. In this talk we present an extension of some known results for the directed line graph operator to the directed path graph.

Keywords: digraphs, line digraph, directed path graphs

## 5 The Twisted Torus, the Tangled Torus and Toughness

 Ward Heilman, Bridgewater State CollegeSeveral people (M. Abreu, and G. Davis, G. Domke and C. Garner Jr. and $W$. Heilman) have independently identified a category of graphs, a member of which had been called a "twisted tower". We now call such a graph a "twisted torus" because the simplest example is a torus. This graph can be viewed as a 2 -circulant, and also viewed as a product of a path and a cycle (together with a set of twisted edges). We extend this work by describing a related set of graphs eah of which we call a "tangled torus". The toughness of a graph $G$ is defined as $t(\mathrm{G})=\min \overline{T w}(\mathrm{lfi})]^{\prime}$ where $S$ is any set of vertices which disconnects the graph $G$ and ( $G-\mathrm{S}$ ) is the number of components of $G-S$ We determine the toughness of some of these graphs and provide bounds on the toughness of others.

Keywords: circulant, torus, toughness

## 6 some binary bent functions arising from bent functions over Z4

H. Tapia-Recillas, Universidad Aut6noma Metropolitana-I

Since its introduction in the mid 70 's, the notion of binary bent functions has received the attention of several researchers. It plays an important role in Coding Theory, Cryptography, Designs and other areas of Discrete Mathematics. This concept has been generalized to the case of functions defined over the ring, $Z_{9}$, of integers modulo $q$. On the other hand, the Gray map defined over the ring $\mathrm{Z}_{4}$, has been of considerable importance in the study of several properties of codes including linearity and cyclicity. In this talk it is shown that by means of this map certain $\mathrm{Z}_{4}$-bent functions induce binary bent functions.

7 Weighted Steiner tree on the rectilinear space Dionysios Kountanis and Satyapurnadevi Padala*, Westem Michigan University

Given is a set of $N$ points on the rectilinear space and a set of expected communication loads l ; j between any two points i , j with i $\mathrm{j}^{j}$ The points are to be connected with a rectilinear Steiner tree which minimizes the quantity $\mathrm{I}: 7=1 \mathrm{I}: ;=\mathrm{I}(\# \mathrm{i}) \mathrm{l} ; \mathrm{jdij}$ where $\mathrm{d} ; \mathrm{j}$ is the rectilinear distance between the points $\mathrm{i}, \mathrm{j}$. The problem is NP-Complete. A polynomial process obtains solutions which closely approximate the minimum weighted Steiner tree. Bounds of the approximation process are computed.

Keywords: Steiner tree, communication, rectilinear space

## 8 Properties of 2 -Prinititive Tournament Digraphs

Leroy B. Beasley*, Utah State University; and Cora L. Neal, University of Alaska.

A tournament digraph is an orientation of an undirected loopless complete graph. A 2-edge-colored digraph is 2-primitive if there are integers $h$ and $k$ such that between any two vertices there is a walk in the digraph which uses exactly h arcs which are colored the first color, and k arcs which are colored the second color. We investigate the 2-colorings of primitive tournaments which yield a 2-primitive digraph.

Keywords: tournament digraph, primitive, edge coloring

9 Bounds on the Metric and Partition Dimension of a Graph Glenn Chappell, John Gimbel* and Chris Hartman, University of Alaska

We will let $d(\mathrm{u}, \mathrm{v})$ represent the distance between vertices u and v . Given a graph $\boldsymbol{G}$ and $\boldsymbol{S}$, and nonempty subset of $\boldsymbol{V}(\boldsymbol{G})$, we say $\boldsymbol{S}$ is resolving if for each pair of vertices u and v in $G$ there is a vertex x in S where $\boldsymbol{d}(\boldsymbol{u}, \boldsymbol{x}) \nexists \boldsymbol{d}(\boldsymbol{v}, \boldsymbol{x})$. The minimum cardinality of all resolving sets is the metric dimension of $G$ Given w, a vertex of $G$, the distance from w to $S$, denoted $\boldsymbol{d}(\boldsymbol{w}, \boldsymbol{S})$, is the minimum distance between wand the vertices of $\boldsymbol{S}$ Further, given $\boldsymbol{P}=\{\boldsymbol{P i}, \boldsymbol{P} \mathbf{2}, \ldots, \boldsymbol{P} \boldsymbol{k}\}$ an ordered partition of $\boldsymbol{V}(\boldsymbol{G})$, we say $P$ is resolving if for each pair of distinct vertices $u$ and $v$ in $G$ there is a part $\boldsymbol{P}$, where $\boldsymbol{d}(\boldsymbol{u}, \boldsymbol{P}, \boldsymbol{)}) \neq \boldsymbol{d}(\boldsymbol{v}, \boldsymbol{P} ;)$. The partition dimension is the minimum order of all resolving partitions. The metric dimension and partition dimension of $\boldsymbol{G}$ will be denoted by $\operatorname{dimM}(\boldsymbol{G})$ and $\operatorname{dimp}(\boldsymbol{G})$ respectively. We present several bounds on these parameters. Specifically, if a and $b$ are integers where 3 a $b+1$ then there exists a graph $G$ where $\operatorname{dimp}(\boldsymbol{G})=\boldsymbol{a}$ and $\operatorname{dimM}(\boldsymbol{G})=\boldsymbol{b}$ answering a question of Chartrand, et al.

Keywords: Metric and Partition Dimension

10On a-Type Matrices Lyndsey Van Wormer* and Aklilu Zeleke, Alma College

For a positive integer a, we define a Fibonacci type sequence by the recurrence $\boldsymbol{P}_{\boldsymbol{r}}+\mathbf{2}=\boldsymbol{a} \boldsymbol{P},:-1+\boldsymbol{P},: \mathbf{2}$, for all $\boldsymbol{n} \quad 2$ with $\boldsymbol{P} \boldsymbol{t}=0, \boldsymbol{P} \boldsymbol{f}=1$. We say a 2 by 2 matrix $A$ is a - basic if all entries of $A$ are in the set $\{0,1$, a $\}$ and all entries of $\mathrm{A}^{\mathrm{n}}$ are in $\{\boldsymbol{P}, \mathbf{0}, \boldsymbol{1}, \boldsymbol{P} ; \boldsymbol{P}+\mathrm{P}+\mathrm{i}\}$. In this talk we characterize all 2 by 2 a basic matrices. We will also discuss some elementary properties of a basic matrices including detrminants and characteristc polynomials. Finally we will discuss convergence properties of sequences related to the eigen values of a basic matrices.

Keywords: a - basic matrix, detrminants, characteristc polynomials, eigen values

11
Reducing Congestion Probability Using Deviation Index as a Metric
Dionysios Kountanis and Sathya Priya Durairaju*, Western Michigan University
Network Congestion is said to occur in a network either when packets arrive faster than they can be forwarded or when the resource demands exceed the buffer capacity and the packets are lost. Buffer queue lengths increase if the packet arrival rate exceeds the transmission rate. Given a topology and a communication load, the congestion depends on the routing scheme. A routing strategy which balances thP. buffer loads with smaller values reduces the probability of congestion. If a routing minimizes $\left(o+M_{c}\right)$, where a is the standard deviation of the buffer loads and $\mathrm{M}_{\mathrm{c}}$ is the maximum buffer load, then congestion is minimized. To establish this, the deviation index of the load is used as a metric of congestion.

Keywords: Networks, communication, routing scheme, deviation index

12
A Database of Visualizations of Graphs Reinhard Laue, Universitaet Bayreuth

Special visualizations of graphs are stored in a database prototype and are accessible in a client server version using any modern browser via the internet. A request may specify a graph or an SQL where-statement concerning some other properties. The set of properties presently is small but will be extended. A canonical form is computed to identify the isomorphism type of a graph. The resulting graphs are displayed by a Java program. An XML-format is used for loading and saving graphs for further processing. The database includes solids derived from the Platonic solids, chemical graphs, and some lattices. It presently has about 100000 entries. We present an application to the visualization of some t-designs.

Keywords: Graphdrawings, Database

On Resolvable Decompositions of Complete Multipartite Graphs Minus a One-Factor into Uniform Cycles D. G Hoffman and S. H Holliday*, Aubum University

In this talk we shall present a resolvable decomposition of complete multipartite graphs minus a one-factor into even-length cycles.

14 On Roots of Generalized Fibonacci Polynomials Dan Schwegler* and Aklilu Zeleke, Alma College

For a positive integer $n$, we define a sequence of Fibonacci polynomials by the recurrence realtion $\mathrm{G}_{\mathrm{n}}+2(\mathrm{x})=\mathrm{xG}_{\mathrm{n}}+\mathrm{l}(\mathrm{x})+\mathrm{G}_{\mathrm{n}}(\mathrm{x}), \mathrm{Gl}(\mathrm{x})=$ $x-c, G 0(x)=-c, c$ a fixed positive integer. Let $a_{e}=e_{t e-}$ and denote the maximal root of $G_{n}(x)$ by $a_{n}$. In this talk we prove that the sequence $a 2_{n}+1$ converges to $d_{e}$ from below and $a 2_{n}$ converges to $d_{e}$ from above. We will also derive several recurrence relations related to the Fibonacci and Lucas numbers.

Keywords: Fibonacci type sequence, Fibonacci polynomials, maximal roots

5 Load Balancing and Congestion Avoidance Routing Dionysios Kountanis* and Konstantinos Kokkinos, Western Michigan University

Today's high-speed backbone networks are expected to support a wide range of communication-intensive applications. One of the most important issues in Quality of Service (QoS) is efficient routing. Many QoS routing solutions have been published lately for different criteria of QoS requirements and resource constraints. In this paper we focus on the derivation efficient routing schemes to reduce the probability of hot spot creation in the network. Furthermore, we provide a detection of congestion mechanism that re-routes traffic to maintain balancing with small communication cost. Several theoretical results relatively to network traffic balancing have been derived as well as, heuristic algorithms for the case of static rerouting without bandwidth guarantees. A network simulator has been developed for the project experiments. The experimental results verify our theoretical model.

16
Stitching Images Back Together
Frithjof Lutscher, University of Alberta; Jenny McNulty, University of Montana; Joy Morris*, University of Lethbridge; and Karen Seyffarth, University of Calgary

Sometimes in order to store or print out an image, it must be broken down into smaller pieces, and the technology often produces distortions in this process that render it impossible to realign the sub-images precisely. This problem is presented in a graph-theoretic framework, and possible "optimal" solutions for reconstructing the original image are discussed. One formulation of the problem is equivalent to that of finding an optimal tree spanner for a grid graph.
Keywords: image processing, tree spanners

17 Coloring with no 2 -colored $\mathrm{P}_{4}$ 's
Michael. Albertson•, Smith C., Glenn Chappell, University of Alaska; H A. Kierstead, Arizona State University; Andre Ktindgen,and Radhika Ramamurthi, California State, San Marcos

A proper coloring of the vertices of a graph is said to be apathic if at least 3 colors are used on every 4 -vertex path. Apathic colorings, also known as "star colorings", are a strengthening of acyclic colorings, i.e., colorings in which at least 3 colors are used on every cycle. We show that every acyclic k -coloring can be refined to an apathic $\left(2 \mathrm{k}^{2}-\mathrm{k}\right)$-coloring. We prove that planar graphs can be apathically 20 -colored and construct a planar graph which requires 10 colors. We prove other structural and topological results: cubic graphs are apathically 7 -colorable, graphs of maximum degree 6. are apathically ( $6 .{ }^{2}-6 .+2$ )-colorable and graphs embedded on $S_{9}$ are apathically $20+5 \mathrm{~g}$-colorable. We provide a short proof of the result of Fertin, Raspaud, and Reed that graphs with tree-width $t$ are apathically $\left(c(t+2)^{2}\right)$-colorable, and show that this is best possible.

18
All 2-Regular Leaves of Partial 6-Cycle Systems D.J. Ashe*, University of Tennessee at Chattanooga; H.L. Fu, National Chiao Tung Univeristy; and C.A. Rodger, Auburn University

We find necessary and sufficient conditions for the existence of a 6-cycle system of $K_{n}-E(R)$ for every 2-regular not necessarily spanning subgraph $R$ of $\mathrm{K}_{\mathrm{n}}{ }^{-}$

The Analysis of Experiments on Heuristic Algorithms: Improving the State of the Art Gagan Jain* and Carla Purdy, University of Cincinnati

Experimental design, data collectiou and analysis of heuristic algorithms to validate claims in such applied research areas as VLSI design and computer system performance pose several challenges for a researcher. In many cases it is difficult or impossible to determine the efficacy of one algorithm over another due to the lack of common analysis tools and/or insufficient data collection. To remedy this situation, we have designed a statistical analysis tool capable of guiding the researcher through all phases of a heuristic algorithm experiment, includiug experimental design, data collection, data analysis, and report generation. Om system uses an expert system approach to answer questions based on the quality and size of the datasets that are available at a given time. The expert system engine has been designed to be modifiable and easily extended to accommodate a growing set of possible user queries. The system will support multiple users and facilitate collaboration in a distributed experimental environment.

20Graph theoretical problems in next generation chip design Joanna A Ellis-Monaghan*, St. Michaels College; and Paul Gutwin, Principal Technical Account Manal?,cr, Cadence

A major component of computer chip design is generating an optimal netlist layout, i.e. determining where to lay wires (connections between functional elements) when manufacturing a chip. Because of its basic structure (nodes with edges between them!), the overall problem of netlist $l_{a y}$ out contains many sub-questions that lend themselves to graph theoretical modeling and analysis. We will describe the basic principles of netlist layout, and present several open questions inherent in the problem. Possible approaches to these questions include concepts from hypergraphs, graph partitioning, graph drawing, graph and geometric thickness, tree width, grid graphs, bipartite graphs, planar embeddings, and geometric graph theory. Due to the highly competitive nature of the microelectronics industry, there is strong interest. in graph theoretical results that $\mathrm{m}_{\mathrm{a}}$ shorten the chip design cycle.

Keywords: Netlist layout, hypcrgrnphs, graph partitioning, graph drawing, graph and geometric thickuess, tree width, grid graphs, bipartite graphs, planar embeddings, and geometric graph theory. communication. networks, broadcast, algorithms

A k-trestle of a graph $G$ is a 2 -connected spanning subgraph of $G$ of maximum degree at most $k$ A graph is said to be chordal, if each cycle different from a 3-cycle has a chord. The toughness of a non-complete graph $G$ is $T(G)=\operatorname{minC}(-\mathrm{s}))$, where the minimum is to be taken over all nonempty vertex sets $S$, for which the number of components $w(G \backslash S)$ of $\boldsymbol{G} \backslash \boldsymbol{S}$ is at least 2 . For a cor:1plete graph $\boldsymbol{K}_{\boldsymbol{n}}$ let $\boldsymbol{T}\left(\boldsymbol{K}_{\boldsymbol{n}}\right)=$ oo. Gao proved: Each chordal polyhedral graph has a 6 -tresle. By Bohme, Harant, and Tkac we know that each chordal polyhedral graph of toughness 1 has a 2-trestle (Hamiltonian cycle). We have shown: Each chordal polyhedral graph $G$ of toughness greater than ' or if, or ( has a 3-trestle, or a 4 -trestle, or a 5--trestle, respectively. There are chordal polyhedral graphs of toughness $\quad>0,72$, or $\quad>0,593$, or $\quad>0,536$, without a 3-trestle, or a 4-trestle, or a 5-trestle, respectively.

Keywords:plane graphs, chordal polyhedral graphs, Non-Hamiltonian, $k$ trestle

## 22 small Forbidden Configurations

Richard Anstee ${ }^{\star}$,University of British Columbia; and Attila Sali, Alfred Renyi Institute of Mathematics

We continues the work begun by Anstee, Ferguson, Griggs and Sali. We define a matrix to be simple if it is a $(0,1)$-matrix with no repeated columns. Let $F$ be a $k \times l(0,1)$-matrix (the forbidden configuration). Assume $A$ is an $\boldsymbol{m} \times \boldsymbol{n}$ simple matrix which has no submatrix which is a row and column permutation of $\boldsymbol{F}$. We define forb $(\boldsymbol{m}, \boldsymbol{F})$ as the best possible upper bound on $n$ which depends on $m$ and F . We complete the classification for all 3-rowed $(0,1)$-matrices of forb $(\mathrm{m}, F)$ as either $0(\mathrm{~m}), 0\left(\mathrm{~m}^{2}\right)$ or $0\left(\mathrm{~m}^{3}\right)$ (with constants depending on $\boldsymbol{F}$ ). We also present a new conjecture that essentially says that we know the optimal constructions in an asymptotic sense.
0. Favaron, Universite Paris-Sud; G.H. Fricke*, D. Skaggs, Morehead State University; W. Goddard, University of Natal, Durban; S.M. Hedetniemi, S.T. Hedetniemi, R.C. Laskar, Clemson University; and R. Kristiansen, University of Bergen

A set of vertices $S$ of a graph G is an offensive alliance if for every vertex $v$ in its boundary $\boldsymbol{N}(\boldsymbol{S})-\boldsymbol{S}$ it holds that the majority of vertices in the closed neighborhood of v are in $S$ The offensive alliance number is the minimum cardin2lity of an offensive alliance. In this paper we explore the bounds on the offensive alliance and the strong offensive alliance numbers (where a strict majority is required). In particular, we show that the offensive alliance number is at most $2 / 3$ the order and the strong offensive alliance number is at most $5 / 6$ the order of $G$.

24 Routing in Unidirectional Alternating Group Graphs and
Yuyin Chen*, Cranboork-Kingswood Upper School; Eddie Cheng and Serge G. Kruk, Oakland University

Distributed processor architectures offer the advantage of improved fault tolerance and reliability. Many application required unidirectional interconnection networks. In this talk, we report the results of computational experiments of known near-optimal routing algorithms for alternating group graphs and split-stars.

Keywords: Routing, interconnection networks, diameter

## 25 A short proof of a characterization of strongly chordal graphs <br> Michael J. Pelsmajer*, Illinois Institute of Technology; and Douglas B. West,

 University of Illinois at Urbana-ChampaignA graph $G$ is chordal if every induced subgraph of $G$ contains a simplicial vertex (a vertex whose neighborhood is a clique).
A graph $G$ is strongly chordal if every induced subgraph of $G$ contains a simple vertex, where a vertex is simple if the closed neighborhoods of its neighbors form a chain under inclusion.
In 1983, Farber gave several characterizations of strongly chordal graphs, including the following: A graph is strongly chordal if it is chordal and contains no trampoline as an induced subgraph, where a trampoline (or a sun) is a graph on 2 k vertices (for some $k 2^{2} .3$ ) consisting of a 2 k -cycle and a clique on the even vertices of the cycle (the odd-indexed vertices form a stable set). We present a short proof of this theorem.

> 26 (t 12) GWhD(12n +1$)-$ Existence Results for $t=\mathbf{2}, 3,4$ S. Costa, N. J. Finizio* and B. J. Travers, University of Rhode Island

For each $t=2,3,4$ it is shown that ( $\boldsymbol{t} 12$ ) $\mathrm{GWhD}(12 \mathrm{n}+1)$ exist for all n except, possibly, for a certain set of values of $\boldsymbol{n}$ The exceptional set, whose order does not exceed 85, is the same for each $t$

## 27 Approximation Algorithms for the Student Scheduling Problem

Eddie Cheng, Serge Kruk*, Marc Lipman, Oakland University
We discuss the student scheduling problem as it generally applies to highschools in North-America. We show that the problem is NP-hard. We discuss various combinatorial formulations and show how a number of practical objectives can be accommodated by the models.

## 28 cops-and-Robbers on Cyber Graphs

Narsingh Deo and Zoran Nikoloski*, University of Central Florida
The problem of cyber attacks with worms is becoming a part of life with the Internet. A worm propagates by replicating itself and sending copies along the links to other computers on the network. The network can be modeled as an undirected graph in which a node represents a computer and an edge represents a (physical or virtual) link. The copies of the worm can be viewed as robbers traversing the graph. To control robbers' propagation, software agents (cops) can be deployed at some nodes and moved along the edges of the graph. Containing the propagation of a network worm can thus be represented by a cops-and-robbers game on large graphs, where the number of robbers increases with time. Cops and Robbers is a classical two-player game on an undirected graph $G$, a set of cops versus a single robber. (Note that in our case, the number of robbers grows.) The cops start the game by choosing nodes (not necessarily distinct) where they are deployed, after which the robber chooses a node. Given a set of allowed moves $\Phi$, the two players take turns in making moves, beginning with the cops. The objective is to capture the robber by placing a cop on the robber's node. The minimum number of cops sufficient to capture the robber on $G$ is called the cop-number for $\Phi$, denoted by $c(G, \Phi)$. In this paper we survey various results for three classes of cops-and-robbers games and present some new results on trapping the robbers.

Keywords: cops-and-robbers game, cop-number, network worm, cyber attacks Thor Whalen, Emory University

Several results deal with relating graphs parameters with the minimum and maximum length of a path between any two vertices. We extend some of these results to path-systems and linkages between any disjoint k -sets of vertices. If $\boldsymbol{G}$ is a graph, we let $\mathrm{a} 2(\boldsymbol{G})=\min \{\mathrm{d}(\mathrm{u})+\boldsymbol{d}(\boldsymbol{v}): \boldsymbol{w} \boldsymbol{v}$ ı. $\boldsymbol{E}(\boldsymbol{G})\}$. We relate this famous graph parameter to the existence and size of pathsystems and linkages. Our results are shown to be best possible.

## 30 On Partial Linear Spaces with a pseudo-geometric <br> GQ(s+1,s-1) point Graph Mikhail Klin and Sven Reichard* ${ }^{*}$, University of Delaware

A partial linear space $p / s(s, t)$ is an incidence structure of points and lines, such that mo two lines intersect in more than one point, each line contains $s+1$ points, and each point lies on $t+1$ lines. Collinearity of points defines a graph, the point graph of the pls.
We are interested in $p / s(s, s)$ such that the point graph is strongly regular and pseudo-geometric, with parameters corresponding to a $\mathrm{GQ}(s+1, s-1)$. Recently, Brouwer, Koolen, and Klin used a computer to construct auch a $p / s(4,4)$ on 96 points. We investigate possible structures for $2:: ;$ ::; 4 and give a computer free description of the $p / s(4,4)$, showing that its automorphism group is an extension of $E_{16}$ by $S_{6}$.
Keywords: generalized quadrangles, strongly regular graphs, partially balanced designs

Evolving Efficient Security Systems Under Budget Constraints Using Genetic Algorithms
William Edelson*, Michael L Gargano, Paul Meisinger and Paul Benjamin, Pace University

The EASI model (estimate of adversary sequence interruption) is a dynamic, analytic method widely used by security professionals to evaluate a physical security system. Our method involves using genetic algorithms to evolve such systems when budget constraints must be considered.

Keywords: physical system security, genetic algorithms, EASI
32 Deciding the deterministic property of soliton graphs in linear time
M Bartha, Memorial University of Newfoundland
A soliton graph is a finite undirected graph having a perfect internal matching, that is, a matching which covers all vertices except possibly the ones with degree one. Such matchings serve as the states of an appropriate finite state automaton, the transitions of which are defined by switching along alternating walks between two vertices of degree one. A soliton graph is deterministic if the corresponding automaton is such in the usual sense.
A natural reduction can be defined on soliton graphs, which eliminates vertices of degree two provided that their neighbors also have a degree at least two. It has been proved earlier that an elementary soliton graph is deterministic iff it reduces to a graph containing odd-length cycles only, using this reduction procedure. By this result it is straightforward to decide the deterministic property for elementary soliton graphs.
The general decision algorithm relies on the proof that every deterministic soliton graph can be decomposed into a number of elementary ones plus graphs having a unique perfect matching. The decision if a graph has a unique perfect matching can still be done in linear time, once an arbitrary perfect matching has been found for the graph previously.

Key words: graph matchings, soliton automata

33 Congruent decompositions of complete graphs Roger B. Eggleton*, Illinois State University; and Harry Calkins, Wolfram Research, Inc.

Graph decompositions of complete graphs have been widely studied. Given a simple ${ }_{\mathrm{gr}}$ aph G, the complete graph Kn has a G-decomposition if Kn is the union of a family of pairwise edge disjoint $\operatorname{sub}_{\mathrm{g}_{\mathrm{r}}}$ aphs that are each isomorphic to G. If Kn has a G-decomposition, does Kn have a representation in the Euclidean plane that can be decomposed into a family of edge-disjoint geometrically congruent copies of G? More generally, does Kn have a representation in cl-dimensional Euclidean space Ed (for some $\mathrm{d}=$ 2) that can be decomposed into a family of edge-disjoint isometric copies of G ? We discuss such questions, with particular attention to the case in which $G$ is self-complementary.
Keywords: graph decompositions, isometric copies

34
Metering Schemes Based on Polynomials over Finite Fields Hiroaki Uehara, Keio University

A metering scheme is a protocol for counting the number of hits a server receives from clients on a network or for investigating whether this number is greater than a certain value. For example, a metering scheme can be used in order to decide whether a company puts an advertisement on a web server by observing if the number of clients (monitors) visited the server is greater than a certain threshold value.
Naor and Pinkas (1998) introduced the first cryptographically secure metering scheme. In a ( $k, n$ )-metering scheme, a server can compute a proof if and only if k or more clients out of n clients visit the server during a certain period. There are some security problems which should be taken into consideration in a metering scheme.
We propose a "perfect" metering scheme based on a power operation of polynomials over finite fields and show that our proposed metering scheme is securer against an attack of clients than the modified Ogata-Kurosawa scheme (2002).

Analysis of the Sensitivity of a Time Dependent Minimal Node Base Directed Communication
Joseph DeCicco*, Michael Gargano and William Edelson, Pace University
Models using Genetic Algorithms to find optimal matroid bases where the matroid element costs are time dependent have been studied theoretically. Here, we do empirical experiments finding minimal node bases for satellite communication applications focusing on sensitivity analysis concerning cost functions and network structures. These will be compared to benchmark studies.

Keywords: Genetic Algorithms, Matroids, Sensitivity Analysis

## 36 Generalized Matchings in Graphs

W. Goddard, S.M. Hedetniemi, S.T. Hedetniemi and Renu C. Laskar*, Clemson University

For a graph $\boldsymbol{G}=(\boldsymbol{V}, \boldsymbol{E})$, a matching $\boldsymbol{M} \quad \boldsymbol{E}$ is a set of independent edges. Matchings in graphs have been studied extensively for many years. Matchings with additional properties are also studied to some extent, such as strong or induced matchings, uniquely restricted matchings, etc. In this paper we introduce several other variants of matchings and study their properties. We include some open problems.

Keywords: matching

# 38 

 An asymptotic existence theorem of a BIB design with nested rows and columnsYukiyasu Mutoh, Keio University
A balanced incomplete block design with nested rows and columns, denoted by $\operatorname{BIBRC}(\mathrm{v}, \mathrm{k} 1, \mathrm{k} 2, . \mathrm{X})$, is a block design in which elements of blocks are arranged in arrays of size $\mathrm{k}_{1} \times 12$ such that $\left[\mathrm{X}=\mathrm{k}_{1} \mathrm{XR}(\mathrm{i}, \mathrm{j})+\mathrm{k} 2 . \mathrm{Xc}(\mathrm{i}, \mathrm{j})\right.$ $. \mathrm{Xs}(\mathrm{i}, \mathrm{j})]$ is a constant not depending on the choice of points $\mathrm{i}, \dot{j}$ where $\mathrm{X}_{\mathrm{R}}(\mathrm{i}, \mathrm{j}\}, . \mathrm{Xc}(\mathrm{i}, \mathrm{j})$ and. $\mathrm{Xs}(\mathrm{i}, \mathrm{j})$ are the numbers of times that the pair i , $j$ occurs in the same row, in the same column and in the same blocks, respectively. In this talk we will give a necessary condition of X for the existence of a $\operatorname{BIBRC}\left(\mathrm{v}, \mathrm{k}_{1}, \mathrm{k} 2, \mathrm{X}\right\}$ when $\mathrm{k}_{1}$ and k 2 are given. Moreover, we will give the existence of BIBRCs for a sufficiently large $v$ satisfying some necessary condition.

Can the vertices of a graph $G$ be partitioned into $A U B$, so that $G[A]$ is a line-graph and $G[B]$ is a forest? Can $G$ be partitioned into a planar graph and a perfect graph? The NP-completeness of these problems are just special cases of our result: if $\boldsymbol{P}$ and $\boldsymbol{Q}$ are additive induced-hereditary graph properties, then $(P, Q)$-colouring is NP-hard, with the sole exception of graph 2-colouring (the case where both $\boldsymbol{P}$ and $\boldsymbol{Q}$ are the set O of finite edgeless graphs). Moreover, $(P, \mathrm{Q})$-colouring is NP-complete iff $P$ recognition and Q-recognition are both in NP. This proves a conjecture of Kratochvil and Schiermeyer.
Note: Pis an additive induced-hereditary property iff, whenever X and Y are graphs with property $\boldsymbol{P}$, all their induced-subgraphs have property $\boldsymbol{P}$, and their vertex-disjoint union also has property P .
We use uniquely $(P, \mathrm{Q}\}$-colourable graphs to provethe result.

40Some Properties of Triangle Graphs
Jay Bagga*, Ball State University; R. Balakrishnan, Bharathidasan University;
R. Sampathkumar and N. Thillaigovindan, Annamalai University

In their paper "A Survey of Clique and Biclique Coverings and Factorizations of (0,1)-Matrices in Bull. of the ICA, vol. 14(1995\}, the authors S.D. Munson, N.J. Pullman, and R. Rees defined the triangle graph of a graph and asked for a characterization. Given a graph $\boldsymbol{G}$, its triangle graph $T(G)$ has the triangles of $G$ for vertices, and two are adjacent if they share an edge in $G$. The triangle graph is thus a generalization of the line graph concept. We present several properties of triangle graphs, give some necessary conditions, and pose some open problems.

Keywords: Line graph, Triangle graph

## 43 application of graph theory in fommal leaming theory Andrew C. Lee, University of Louisiana at Lafayette

## 42 <br> Construction for BIBRC not having completely balanced property <br> Kazuhiro Ozawa, Gifu College of Nursing

As an extension of Theorem 4 in Uddin and Morgan (1990), a direct construction for BIBRC which is not completely balanced is presented. Although resultant ciesigns would not have fewer replications than the designs which have been available previously, the construction is significant in the sense that those can provide a $\operatorname{BIBRC}(\mathrm{v}, \mathrm{b}, \mathrm{r}, k, k, A)$ or a $B \operatorname{IBRC}\left(v, b^{\prime}, r^{\prime}, k+1, \mathrm{k}+1, \mathrm{~A}^{\prime}\right)$ such that $v$ - I is not divisible by $k$ (or $k-1)$.

Consider an abstract model of learning machine, where the machine (a Turing machine) can observe data (modelled as function values $J(O), \ldots$, $f(n), \ldots$ ) and attempts to find a program to reproduce the data. Assume further that the machine is allowed to post questions to a teacher, who will provide correct answers whenever questions are posted. The study of learning phenomena with respect to this model and its variants are often referred as query inference in formal leaning theory.
In such models, the questions that are allowed to be posted roughly represent the additional knowledge needed to learn a given concept. In this talk we will consider the case where the questions posted by the machine are boolean questions formulated in some query languages. Our focus will be on query languages that have close connections with w languages. We will demonstrate that how some notions in graph theory (e.g. De Bruijn graphs) can be applied in our study of these query languages.

Keywords: Theory of Computing, Artificial Intelligence, De Bruijn Graphs

44 Cycle Decomposition Numbers of Graphs<br>Steven J. Winters, University of Wisconsin Oshkosh

One of the most fundamental problems in graph theory concerns whether a graph $G$ can be decomposed into subgraphs, each of which is isomorphic to a given graph H. This includes such well-known problems as to whether a given graph is 1 -factorable, or perhaps has a hamiltonian decomposition. Not all graphs $G$ are $H$-decomposable, so a natural question is: "How close to being H-decomposable is G?" This talk will explore one possible response to this question. For a graph H without isolated vertices, the H decomposition number of $G$ is the minimum number of vertices that must be added (along with the appropriate edges incident with these vertices) to G to produce a graph $F$, containing $G$ as an induced subgraph, such that F is H -decomposable. Results will be presented where H is a cycle.
Keywords: decomposition, cycle

We consider matrices with entries from finite fields $\boldsymbol{\operatorname { G F }}(\boldsymbol{q})$ with k-wise independent columns, where each column contains at most $r$ nonzero entries. In general, $\boldsymbol{k} \mathrm{r}, \boldsymbol{q}$ are fixed positive integers and m is large. Given a number m of rows, let $\mathrm{N}_{\mathrm{q}}(\mathrm{m}, \mathrm{k}, \mathrm{r})$ denote the maximum number of columns such a matrix can have.
For the case $\mathrm{q}=2$ and $\mathrm{r}=2$ the values of $\mathrm{N}_{2}(\mathrm{~m}, \boldsymbol{k} 2)$ are asymptotically equal to the maximum number of edges in a graph on $m$ vertices, which does not contain any cycle of length at most $\boldsymbol{k}$. The growth of $\mathrm{N}_{2}(\mathrm{~m}, \boldsymbol{k}, 2)$ has been studied a lot in the past, however not that much is known on the exact asymptotic growth rate. For $q=2$ and arbitrary $\mathrm{r} 2: 1$, the following bounds on $\mathrm{N}_{2}(\mathrm{~m}, \boldsymbol{k} r)$ were given by Pudlak, Savicky and this author for $k \quad 4$ even and $\mathrm{r} \quad 2: \mathrm{N} 2(\mathrm{~m}, k, \mathrm{r})=\mathrm{n}\left(\mathrm{m}^{\mathrm{k}} \mathrm{r} /\left({ }^{2}\left({ }^{\mathrm{k}}-\mathrm{I}\right) \mathrm{l}\right)\right.$ and for $k=2^{\mathrm{i}}$ it is $\left.N_{2}(\boldsymbol{m}, \boldsymbol{k}, \boldsymbol{r})=\mathrm{O}\left(\mathrm{mf}^{\mathrm{k}} \boldsymbol{r} / \mathrm{k}^{\mathrm{k}}-\mathrm{I}\right) /^{2}\right)$. Fork even and $\operatorname{gcd}(\mathrm{k}-1, \boldsymbol{r})=1$, the lower bound was improved by Bertram-Kretzberg, Hofmeister and this author to $\mathrm{N}_{2}(\mathrm{~m}, \boldsymbol{k} \mathrm{r})=\mathrm{f} 2\left(\mathrm{~m}^{\mathrm{kr}} /\left({ }^{2}\left(^{\mathrm{k}}-1\right)\right) \cdot(\mathrm{In} \mathrm{m}) 1 /\left({ }^{\mathrm{k}}-1\right)\right)$.
Here we generalize and extend some of these earlier results to the case of arbitrary prime powers $q$ for even $\boldsymbol{k}$ we obtain the lower bounds $\mathrm{Nq}(\mathrm{m}, \mathrm{k}, \mathrm{r})=\mathrm{fl}\left(\mathrm{m}^{\mathrm{kr}} /\left({ }^{2}(\mathrm{k}-\mathrm{l})\right)\right)$, and $\mathrm{N}_{9}(\mathrm{~m}, \mathrm{k}, \mathrm{r})=\mathrm{fl}\left(\mathrm{m}(\mathrm{k}-\mathrm{l}) \mathrm{r} /\left({ }^{2}\left(\mathrm{k}-{ }^{2}\right) \mathrm{l}\right)\right.$ for odd k . Moreover, fork=2i we show that $\mathrm{N}_{\mathrm{q}}(\mathrm{m}, \mathrm{k}, \mathrm{r})=0\left(\mathrm{~m}^{\mathrm{kr}} /(2(\mathrm{k}-\mathrm{J}))\right)$ if $\operatorname{gcd}(\mathrm{k}-1, r)=\boldsymbol{k}-1$, while for arbitrary even $\boldsymbol{k} 2: 4$ with $\operatorname{gcd}(\mathrm{k}-1, r)=1$ we have $\left.\mathrm{N}_{\mathrm{q}}(\mathrm{m}, \mathrm{k}, \mathrm{r})=\mathrm{fl}\left(\mathrm{m}^{\mathrm{kr}} /\left(2^{(\mathrm{k}}-\mathrm{J}\right)\right) \cdot(\operatorname{logm})^{1 \mathrm{f}}(\mathrm{k}-1)\right)$. Also for char $(\boldsymbol{G F}(\boldsymbol{q}))^{\mathrm{q}}>2$ we have $\mathrm{N}_{\mathrm{q}}(\mathrm{m}, 4, \mathrm{r})=0\left(\mathrm{mf}^{4 \mathrm{r}} / 3{ }^{3} /^{2}\right)$, while for char $(\boldsymbol{G F}(\boldsymbol{q}))=2$ we can only show $\mathrm{N}_{\mathrm{q}}(\mathrm{m}, 4, \mathrm{r})=\mathrm{O}\left(\mathrm{mf}^{4} \mathrm{r} /^{3} 1 /^{2}\right)$. Matrices, which fulfill these lower bounds, can be found in polynomial time.

Keywords: forbidden subgraphs and subhypergraphs, approximation algorithms

An increasing sequence of integers can be thought of as a set of numbers. The complement of this set when viewed as a sequence in increasing order is called the complementary sequence of the original. Some interesting properties of these sequences and more specifically of arithmetic sequences will be explored.

48
Discrete Isoperimetric Inequalities: a survey
Henry Liu*, The University of Memphis
This talk is a brief survey of some celehrn.ted theorems in discrete isoperimetric inequalities. We will focus on the results that were proved by combinatorial methods only.
The very first of these results, dating back to the 1960s, was the Kruskal Katona theorem. We then mention several subsequent key results, involving both the vertex and edge boundaries, that were proved since then. Next, we put more emphasis on the results and ideas that were considered much more recently. Interestingly, many implications between very strong results have been suggested. For example, the solution to a shadow minimization problem may be applied to obtain a solution for an edge isoperimetric problem.
Finally, some open problems will he rnent.ioncd.
Keywords: Vertex/edge isoperimetric inequality, Kruskal -Katona theorem,discrete cube/grid/torus, vertex/edge boundary, ranked poset, ideal (or down set),shadow minimization problem, Macaulay poset

## 50 Designs from Discrete Log Tables Malcolm Greig, Greig Consulting

Several interesting designs can be obtained simply from the table of discrete logarithms in $\mathrm{GF}\left(\mathrm{p}^{\mathrm{r}}+\mathrm{c}\right)$. As an important example, if $\mathrm{n}=\operatorname{gcd}(\mathrm{r}, \mathrm{c})$ and we use the extension over $G F\left(p^{n}\right)$, then laying out the table in a natural way as a $\mathrm{p}^{\mathrm{r}}$ by $\mathrm{p}^{\mathrm{c}}$ array (with $\log (0)=\mathrm{oo}$ ), then treating the rows as the blocks of a difference family in $Z_{p} r+c_{-} 1 U\{o o\}$ gives us a $\left(p^{r}+^{c}, P^{c},\left(p^{c}-1\right) /\left(p^{n}-1\right\}\right)$ RBIBD. This is the best possible $>$ for these ( $v, \boldsymbol{k},>)$ ) parameters. Rather suprisingly, this result was unknown until 2000, when Philipp Woffel established the existence of an optimal family of hash functions in his Dortmund thesis, whence the result follows by an equivalence of Stinson's. However, no direct proof was available for the RBIBDs. We will give some other examples in Baker's Uniform Base Factorizations, which leads to a bound of $v>90192$ for the sufficiency of the necessary conditions for a $(v, 16,15)$ RBIBD, and examples in nested designs, including a new class of Z-cyclic triple whist designs.

Keywords: Discrete logs, Optimal hash families, Triple whist designs, RBIBDs

51 Influence Digraphs Induced by Time-Stamped Graphs Eddie Cheng ${ }^{*}$, and J.W. Grossman, Oakland University; and M.J. Lipman, Indiana University-Perdue University at Fort Wayne

A time-stamped graph is an undirected graph with a real number on each edge. Vertex $u$ influences vertex $v$ if there is an increasing path from $u$ to $v$. The induced influence digraph of a time-stamped graph is the directed graph that records the influences. In this talk, we discuss the realizability problem: Given a parameter value, does there exist a time-stamped graph whose induced ifluence digraph has the given parameter value?

Keywords: Time-stamped graph, influence, collaboration

53 A Miscellany of Chessboard Problems John J. Watkins, Colorado College

All of the standard old chessboard problems from recreational mathematics, The Knight's Tour Problem, The Covering Problem, The 8-Queens Problem, that we know so well have long since evolved into central problems in their own right within Graph Theory in the study of Hamiltonian graphs, domination, and independence. But these problems have also shown a remarkable ability to shake off their dusty past and spring once again to life in new surrounding::. So, in this talk, I am interested in taking these problems onto new territory and surveying the scene there. Onto boxes: how many independent kings can you place on a $2 \times 2 \times 2$ cube? Onto a surface where the knights domination number is not monotonic. Onto the Klein bottle where something amazing happens to bishops domination. And even onto a logo for See's Candies to look for knight's tours.

54
Some Extremal Subfamilies of some Extremal Families of Nearly Strongly Regular Graphs
K. J. Roblee*, Troy State University; and T. D. Smotzer, Youngstown State University

We build on a previous result concerning regular simple graphs for which there is some $t>0$ such that any two adjacent vertices have exactly $t$ common neighbors, and the union of their neighborhoods includes all but $\mu=2$ vertices. We are concerned here with graphs satisfying such requirements with $\mu>2$. We give an upper bound on the order of the graph in terms of $t$ and $\mu$. This bound turns out to be sharp if $\mu / 2$ divides t . Further, for such restricted values, the extremal graphs for our inequality turn out to be unique for $t$ sufficiently large.

Keywords: Extremal, Strongly Regular

55 Discrete Antimatroid Topology John L. Pfalz, University of Virginia

The elements of this topology are sets of atoms, $\mathrm{A}(\mathrm{o}), \mathrm{A}(\mathrm{l}), \ldots, \mathrm{A}(\mathrm{n})$, of dimension O through $n$ The topology of the space is determined by an adjacency relation a specifying which atoms are adjacent. This relation is subject to the axioms: for all $x ; E A(i), X j E A U l, x k E A(k l, x ~ E A$,
Al: $\mathrm{x} \Phi \mathrm{x} . \mathrm{a}$,
A2: Xi E Xj.a implies Xj Ex;.a,
A3: $i>0$ implies $x$;.an $A(i-I)$ j:. 0 , and
A4: Xi E Xj. a implies i j. $j$.
An antimatroid closure operator is defined in the space. We then present connectivity concepts and a discrete Jordan surface characterization that is applicable to digital images and other discrete spaces.

Keywords: Discrete topology, closure, digital images, pixel spaces, Jordan surface

Some Anti-Ramsey numbers of large double stars Balazs Montagh, University of Memphis

Erdos, Simonovits and S6s initiated the investigation of anti-Ramsey problems for graphs. We say that a graph is totally multicolored (TMC, for short) if any two of its edges have different colors. Given a graph $G$, let $A R(n, G)$, the Anti-Ramsey numbers of $G$, be the largest integers $t$ for which it is possible to t-color the edges of $K n$ so that every color is used at least once, and there is no TMC copy of $G$.
The determination of these numbers turns out to be surprisingly hard. It has been done exactly only for cliques (Erdos, Simonovits, S6s, MontellanoBallesteros and Neumann-Lara), $C_{4}$ (Alon), $C 5$ and C5 (Jiang, West, Schiermeyer), stars (Jiang) and in some cases, for matchings (Schiermeyer). A modification of that problem proved to be considerably easier in some cases: here we look for Anti-Ramsey numbers of families of graphs. In other words, we look for the number of colors that guarantees a TMC copy of a member of a given family. In the previous two Southeastern Conferences we considered this easier version.
Now we turn to the original version, and present some of the Anti-Ramsey numbers of every double star, that is, of every tree of diameter 3 .

## 57 Independence on Triangular Hexagon Boards Heiko Harborth, Techn. Univ. Braunschweig

Triangular parts of the regular hexagon tessellation of the plane are considered as gameboards. The independence number of a chess-like piece $P$ is the maximum number of pieces $P$ on the gameboard not attacking one another. The independence numbers are determined and discussed for three types of knights, for a bishop, and for a queen. (Common work with Vincent Kultan, Katarina Nyaradyova, and Zuzana Spendelova.)

## 58 on Packing Subgraphs in a Graph

A. Kelmans, Rutgers University and University of Puerto Rico

Given a graph $H$, the $H$-packing problem is the problem of finding in a graph $G$ the maximum number of disjoint subgraphs isomorphic to $H$. This problem is known to be NP-hard for every connected graph H with at least three vertices. We discuss some recent results on the H-packing problem when $\boldsymbol{H}$ is a tree. Here are some of such results.
Theorem. Let $g$ : denote the set of graphs with every vertex of degree at least $r$ and
at most $s$ and let $r k(G)$ denote the maximum number of disjoint $k$-edge trees in $G$. Then (al) if $G \mathrm{E} 92$ ands 24 then $\mathrm{r} 2(\mathrm{G}) 2 v(G) /(s+1)$, (a2) if GE 9? and $G$ has no 5-vertex components, then $\mathrm{r} 2(\mathrm{G}) 2 v(G) / 4$, (a3) if $k 22, s 2.3, G \mathrm{E} 9 i$, and $G$ has no $k$-vertex components, then $r k(G) 2(v(G)-k) /(s k-k+1)$, (a4) the above bounds are attained for infinitely many connected graphs. There is a polynomial algorithm for finding the corresponding packings in a graph.

Keywords: graphs, packing subgraphs, tree packings, tree factors, NP-hard problems, polynomial approximation algorithms

## 59 The Fractional Flow Number of Rank 3 Orientable Matroids

Matt Edmonds* and Jennifer McNulty, The University of Montana
The circular chromatic number $\mathrm{x}_{\mathrm{c}}(\mathrm{G})$ of a graph $G$ is defined as $\mathrm{x}_{\mathrm{c}}(\mathrm{G})=$ $\inf \{\mathrm{k} / \mathrm{d}$ : there is a $(\mathrm{k}, \mathrm{d})$-coloriug of G$\}$. The definition of this parameter was extended to orientable matroids by Goddyn et. al. This extension is called the fractional flow number, denoted $\Phi^{*}(1 \backslash 1)$, and defined as $\Phi^{*}(M)=$ $\operatorname{minrl}<>*(H)$ for a matroid M with underlying hyperplane arrangement H , where

We show $\Phi^{*}(\mathrm{H}) \quad 4$ (and hence $\Phi^{*}(\mathrm{Af}) \quad$ 4) for all rank-3 orientable matroids $M$ with underlying hyperplane arrangement $H$.

Keywords: Circular chromatic number, orientable matroid, fractional flow number

## 60 Axiom of Choice and Chromatic Number of the Plane Saharon Shelah, The Hebrew University; and Alexander Soifer*, Rutgers University

Define a Unit Distance Plane as a graph $\mathrm{U}^{2}$ on the set of all points of the plane $\mathrm{R}^{2}$ as its vertex set, with two points adjacent iff they are distance 1 apart. The chromatic number x of $\mathrm{U}^{2}$ its is called chromatic number of the plane. In 1950 Edward Nelson posed the problem of finding X- Amazingly, the problem has withstood all assaults in p;cneral case, leaving us with disrespectfully wide range of $x$ being $4,5, G$ or 7 .
In their fundamental 1951 paper, Paul Erdos and N. G. de Bruijn have shown that chromatic number of the plane is attained in a plane's finite subgraph. This result has naturally channeled much of research in the direction of finite unit-distance graphs. One aspect of Erdos-de-Bruijn result, however, has remained a low key: they used quite essentially the axiom of choice. So, it is natural to ask, what if we have no choice?
We will present here an example of a distance graph on the line $R$, whose chromatic number depends a great deal upon our inclusion or exclusion of the axiom of choice. While the sdting of our example differs from that of chromatic number of the plane problem, the example illuminates how the value of chromatic number may he affected by presence or absence of the axiom of choice in the system of axioms for sets.

## 61 Teaching Introductory Combinatorics by Guided Discovery Kenneth Bogart, Dartmouth College

Over the past two years I have been developing materials for teaching introductory combinatorics in a style I call guided discovery. The course is taught from a set of notes that consist almost entirely of problems. Except for definitions, all the major intellectual content for the course is in the problems. Part of the students' activities is figuring out what the major theorems of the subject are and proving them. Students spend most of their time in class worl:ing on problems in small groups. Lectures are orient students and help students put what they have accomplished into a broader context. The notes are being tried at Dartmouth, Wesleyan, University of Minnesota, Marietta College, Howard University, and Illinois Institute of Technology. Reuslts are in from the first offerings at Dartmouth and Minnesota, and the preliminary evaluation is that student response has been quite favorable. Students interviewed at Dartmouth and Minnesota by a Faculty member from Harvey Mudd College showed greater confidence and similar knowledge after guided discovery teaching compared with traditional teaching. In this talk I will discuss some of the more interesting problems, some of the difficulties students had with the problems, and some of the positive and negative aspects of this style of teacing.

## 62 The orders of $G L(k, Z n)$ and $S L(k, Z n)$ <br> Michael Gilpin, Michigan Technological University

J. G. Sunday's formula gives the order of the special linear group of rank 2 over Zn as $\backslash \mathrm{S} £\left(2, \mathrm{Z}_{\mathrm{n}}\right) \backslash=\mathrm{n}^{3} 11(1-3 / 4 \mathrm{r})$ where the product is taken over all primes p that divide n . We state and verify similar formulas for the orders of GL(k,Zn) and SL(k,Zn) for arbitrary $k$.

Keywords: General Linear Group, Special Linear Group

## 63 On Perfect Independent Dominating Sets of Graphs

W. Gu, X. Jia* and J. Shen, Southwest Texas State University

Let $G=G(V, E)$ be a graph. An independent set $D$ of vertices is called a perfect independent dominating set (PIDS) of $G$ if every vertex of $G$ is in $D$ or adjacent to a vertex in $D$. In this paper, we discuss the existence of PIDs for some graphs. We prove a lower bound and a upper bound for the number of PIDSs that a graph cold possibly have. An algorithm is provided to count the number of PIDSs of trees. Among other results, it is also proved, for instance, that the $s$ by $t$ torus contains a PIDS if a:id only if both s and $t$ are multiples of 5 .

Keywords: Dominating set, Independent set, Perfect independent dominating set, Tree, Algorithm

## 64 Two theorems pertaining to the coloring of the edges of a graph <br> Matthieu Dufour, Universite du Quebec a Montreal; and Jean M. Turgeon*,

 Universite de MontrealThe marriage theorem of Phillip Hall (or its generalization, Tutte's theorem) belongs to a class of theorems that establish an equivalence between a property possessed by each subset of a set and a property of the whole set. Our first theorem is of the same type.
Suppose we color the edges of a graph with n vertices by means of the n - 1 colors of a set $C$. Suppose it was done in such a way that, if we remove the edges that belong to each subset $S \varsigma C$, then the number of connected components of the graph is increased by at most $\backslash S$. Then the graph with this coloring possesses a spanning tree with every edge of a different color. Moreover, we will give another property of such graphs and discuss applications.

Keywords: Graphs, Edge Colorings, Spanning Trees

65 Tiling Fringed Chessboards with Dominoes
Robert E. Jamison, Clemson University; and Natalie Lochner ${ }^{*}$, Rollins College
A fringe on an $n$ by $m$ chessboard is a set of additional squares added to the top row. These squares may be considered as colored ! lack and white consistently with the board. If the fringed board is to be tiled by dominoes, then the total number of black squares must equal the total number of white squares. In general, this parity condition is not sufficient. However, if the board is deep enough, then this parity condition does suffice to guarantee a tiling. In this talk, we present just,.., hat "deep enough" is and give examples showing our bounds are best possible.

Keywords: tiling, chessboard, grid graph, matching

## 66 Divisor Graphs

Gera Ralucca*, Ping Zhang, Western Michigan University; and Varaporn Saenpholphat, Srinakharinwirot University

For a finite nonempty set $S$ of positive integers, the divisor graph $G(S)$ of $S$ has vertex set $S$ and two vertices i and $j$ of $G(S)$ are adjacent if i divides $j$ or $j$ divides i , while the divisor digraph $D(S)$ of $S$ has vertex set Sand $(\mathrm{i}, \mathrm{j})$ is an arc of $D(S)$ if i divides $j$ A graph $G$ is a divisor graph if there exists a set $S$ of positive integers such that $G$ is isomorphic to $G(S)$. We present some results on divisor graphs.

Keywords: divisor labeling, divisor graph, divisor digraph

67 The Number of Minimum a-Dominating Sets in

## Tournaments

Larry Langley*, Sarah Merz, University of the Pacific
A set $S$ of vertices of a directed graph is a-dominating if every vertex not in $S$ has at least a of its in-neighbors in $S$. We count the number of minimum size a-dominating sets, 10 , of certain tournaments. If $a=1 / 2$, , o of a transitive tournament on $2 n$ vertices is then + I Catalan number, and for general $\mathrm{a}, 1_{\mathrm{o}}$ is a generalized Catalan number.

Keywords: Catalan Numbers, a-Domination, Transitive Tournament

68
Coloring Mixed Hypergraphs: theory, algorithms and applications
V. Voloshin, University of Delaware

The Theory of Graph Coloring has existed for more than 150 years. From its modest beginning of determining whether a geographic map can be colored with four colors, the theory has become central in Discrete Mathematics with many contemporary generalizations and applications. Historically, graph coloring involved finding the minimum number of colors to be assigned to the vertices so that adjacent vertices must have different colors. In this book we introduce the theory of coloring mixed hypergraphs where problems on both the minimum and maximum number of colors occur. Mixed hypergraphs contain two families of (hyper)edges: classic edges and their opposites which may be viewed as anti-edges. In every coloring, classic edges have at least two vertices of distinct colors, while the anti-edges have at least two vertices of the same color. The interaction between edges and anti-edges constitutes the main feature of coloring of mixed hypergraphs. This interaction brings many new properties to the theory of colorings, for example: colorability, unique colorability, lower and upper chromatic numbers, perfection with respect to the upper chromatic number, and broken chromatic spectra.
Perhaps the main conclusion is that in trying to establish a formal symmetry between the two types of constraints expressed by the edges and anti-edges we find a deep asymmetry between the problems on minimum and problems on maximum number of colors. This asymmetry pervades the theory, methods, algorithms and applications of mixed hypergraph coloring.

69 Some Narcissistic Half-and-Half Power-Sequence $Z_{p}$ Terraces with Segments of Different Lengths
Ian Anderson, University of Glasgow; and D. A. Preece*, Queen Mary, University of London

A power-sequence terrace for $\mathrm{Z}_{\mathrm{n}}$ is a $\mathrm{Z}_{\mathrm{n}}$ terrace that can be partitioned into segments one of which contains merely the zero element of Zn whilst each other segment is either (a) a sequence of successive powers of an element of $Z_{n}$, or (b) such a sequence multiplied throughout by a constant. If n is odd, $\mathrm{a} \mathrm{Z}_{\mathrm{n}}$ terrace ( $\mathrm{a} 1, \mathrm{a} 2, \ldots, \mathrm{a}_{\mathrm{n}}$ ) is a narcissistic half-and-half terrace if $a ;-a ;-1=a_{n}+2-i-a_{n+} I-i$ for $i=2,3, \ldots,(n+1) / 2$. Constructions are provided for some narcissistic half-and-half power-sequence terraces for $Z_{p}$ with p an odd prime; in each of these terraces, the 2 or 3 segments preceding the non-zero element have different lengths. For each construction, full details are given of all terraces obtainable with $p<200$. The constructions provide terraces for $Z_{p}$ for all prime $p$ with $5<p<200$.

Keywords: 2-sequences, Euler's function, Narcissistic half-and-half terraces, Power-sequence terraces, Primitive roots

## 70 cages and Voltage Graphs <br> Geoffrey Exoo, Indiana State University

A method for constructively attacking the cage problem is described. The method has produced the smallest known trivalent graphs of girths 14, 16, and 17. The new construction for girth 16 is a 96 -fold covering of the Petersen graph which admits a rather simple description. The method also has applications to the degree/diameter problem.

## 71 <br> Gallai-type Theorems and Domination Parameters in Digraphs

Sarah Merz* ${ }^{*}$, University of the Pacific; and Dustin Stewart, University of Colorado at Denver

Gayla, Dunbar, and Markus proved several Gallai-type theorems involving domination parameters in graphs. We apply these ideas to digraphs. For example, the lower domination number of a digraph $D$, denoted , ( $D$ ), is the least number of vertices in a set $S$, such that for every vertex $v$ in the digraph, either $v$ is in Sor a vertex in $S$ has an arc to $v$. The parameter i! $1+(\mathrm{D})$ denotes the maximum number of arcs leaving any vertex in D . A Gallai-type theorem has the form $\mathrm{x}(\mathrm{G})+\mathrm{y}(G)=\mathrm{n}$, where x and y are parameters defined on $G$ and n is the number of vertices in the graph. .For example, $x$ might be $I$ and y might be i.ll. For example, we show that ,(D)+.i!l+(D) ::; n. We consider other Gallai-type results for digraphs.

## 72 Upper bounds for Erdos-Rado numbers R.D.Morris, The University of Memphis

Let $E R(r, \mathrm{~m})$ be the minimum size of a set $S$ needed so that given any coloring of the r-sets of $S$, we can find a canonically colored m -set $M$ i.e. a set $M$ such that for each pair of r-sets in $M,\left(a 1, \ldots, a_{r}\right)$ and (b1, ... br), we have $c\left(a 1, \ldots, a_{r}\right)=c\left(b_{1}, \ldots, b_{r}\right)$ if and only if $a i=b$, for all $i$ in $I$, where $I$ is some subset of $\{1, \ldots, r\}$. We are interested in bounding $E R(r, m)$ as $m$ gets large. A lower bound can be found using Ramsey numbers which is a tower of height $r$; Lefmann and Rodi showed that $E R(r, m)$ has an upper bound which is a tower of height $\mathrm{r}+1$; Shelah claimed in a 1996 paper to have improved this to a tower of height r . Here we point out two possible errors in the proof of Shelah, and give counter-examples to illustrate the problems.

Keywords: Erdos-Rado number, Finite canonization

Generalized convexity notions and combinatorial geometry H Martini*, TU Chemnitz

In this lecture we discuss several generalized convexity notions which play a role in combinatorial geometry. More precisely, we will mainly speak about the notions of d-convexity, H -convexity and B-convexity, their relations to each other (e.g., in the spirit of Helly-type theorems) and various applications in the combinatorial geometry of convex bodies, namely for Euclidean as well as finite dimensional Banach spaces.

## 74 on the genus of finite categories <br> C. E. Ealy Jr., Western Michigan University

Let $C$ be a finite concrete category. In this paper, we will extend the definition of graph theoretic genus to a finite category C. After considering the general case, we focus our attention on a finite concrete category, M , with only one object. In this case, we bound the genus of M .

Keywords: Groups, finite categories, monoids, genus, graphs

75 Partial Domination Graphs of Extended Regular
Tournaments: Chords and Cycles James D. Factor*, Marquette University

The domination graph of a digraph $D, \operatorname{dom}(D)$, has $V(\operatorname{dom}(D))=V(D)$ and an edge between every pair of vertices in D that together be at all other vertices. A tie between two vertices $u, v \mathrm{E} V(D)$ is represented here by $\operatorname{arcs}(\mathbf{u}, \boldsymbol{v}),(\boldsymbol{v}, \mathbf{u}) \mathrm{E} A(D)$. For purposes of discussing results involving tournaments with ties, domination will be referred to as partial dcrrnination and denoted as $\operatorname{Pdom}(D)$. Only the regular tour!lament th has the odd cycle $C_{n}$ as its domination graph. In order to add an internal chord to $\mathrm{C}_{\mathrm{n}}$, one or more ties must be added to $\mathrm{U}_{\mathrm{n}}$. This creates an extended tournament $U ; ;$, where $C_{n}$ is a $\operatorname{sub}_{\mathrm{gr}}$ aph of $\operatorname{Pdom}(U ; ;)$. This talk explores the number and characteristics of the ties that must be added to $U_{n}$ to create an internal chord in $C_{n}$ without inducing additional chords. If an internal chord in $C_{n}$ requires m-ties to be added to $U_{n}$, then this chord is called an m-tie internal chord. Various induced cycles in $\operatorname{Pdom}\left(U ;!^{\prime \prime}\right)$ constructed from disjoint sets of $m$-tie internal chords will be explored.

Keywords: regular tournament, domination graph, tie, extended tournament, partial domination graph, m-tie internal chord, cycle

$$
76 \text { Local Colorings of Graphs }
$$

Gary Chartrand and Ping Zhang*, Western Michigan University, Ebrahim Salehi, University of Nevada, Las Vegas

A local coloring of a nontrivial graph $G$ is a function $\mathrm{c}: V(G) \rightarrow \mathrm{N}$ having the property that for each set $S \quad V(G)$ with 2 ISi 3 , there exist vertices $u$ vES such that $l c(u)-d(v) I \quad m s$, where $m s$ is the size of the induced subgraph (S). The maximum color assigned by a local coloring c to a vertex of $G$ is called the value of c and is denoted by $\boldsymbol{x e}(c)$. The local chromatic number of $G$ is $\operatorname{xe}(G)=\min \{\operatorname{xe}(c)\}$, where the minimum is taken over all local colorings c of $G$. Some results on local colorings are presented.

Keywords: local coloring, local chromatic number

## 78 Destroying automorphisms by fixing points

David Erwin*, Trinity College and Frank Harary, New Mexico State University
Let $G$ be a graph. The fixing number of $G$ is the minimum cardinality of a set $S \lessdot V(G)$ such that every nonidentity automorphism of $G$ moves at least one member of $S$. We give some results on the fixing number and mention some relationships with established invariants.

Keywords: automorphism, dimension, symmetry breaking

## 80 Broadcast Chromatic Numbers of Graphs

Wayne Goddard, Sandra Hedetniemi, Stephen Hedetniemi, Clemson University; John Harris and Douglas Rall*, Furman University

A function c : $V(G)$-> $\{1,2, \ldots, \mathrm{k}\}$ is called a broadcast coloring of order $k$ if $\mathrm{c}(u)=\mathrm{c}(v)$ implies $d(u, v)>\mathrm{c}(u)$. Equivalently, for each i, the vertices mapping to i form an i-packing in $G$. The broadcast chromatic number of $G$ is the minimum order, $X b(G)$, of a broadcast coloring of $G$. We introduce this new type of coloring, study some of it basic properties and investigate the broadcast chromatic number of some common classes of graphs.

Keywords: broadcast coloring, i-packing

82Some Characterizations of Unit Interval Bigraphs David E. Brown, J. Richard Lundgren*, University of Colorado at Denver

A unit interval bigraph (UIBG) is the intersection of graph of two distinct families of unit intervals with vertices adjacent if and only if their corresponding intervals overlap, and each belongs to a distinct family. We give several characterizations of UIBGs including bipartite graphs that are permutation graphs and asteroidal triple-free bipartite graphs. We also give characterizations involving valuation bigraphs, the structure of the adjacency matrix, and forbidden subgraphs. Some of these results were found independently by Hell and Hauang. We discuss possibilities of generalizing some of these results to unit intervals in k-graphs.

Keywords: Unit interval bigraph, asteroidal triple, bipartite permutation graph, interval k-graph, adjacency matrix

A set $S$ of vertices in a graph is a dominating set if every vertex either belongs to $S$ or is adjacent to a vertex in $S$ The smallest cardinality over all dominating sets is the domination number of a graph. If no two vertices in $S$ are adjacent, then $S$ is an independent dominating set. The smallest possible cardinality of an independent dominating set is the independent domination number of a graph.
We introduce a set of graphs that is critical with respect to the independent domination number. A graph is domination idot critical if collapsing any edge causes the independent domination number to decrease. We discuss how these graphs relate to the previously studied set of dot critical graphs whose domination number decreases with any edge contraction. In addition we present several classes of idot critical graphs and offer some open questions.

Keywords: domination number, independent domination number
84 How False is Kempe's Proof of the Four Color Theorem? Ellen Gethner ${ }^{\star}$ and William M Springer II, University of Colorado at Denver

In 1879 Sir Alfred Bray Kempe believed he had proved the now famous Four Color Theorem (FCT): the chromatic number of any planar graph G is at most four. Kempe's ill-fated proof, viewed through modern eyes, is a straightforward and implementable algorithm. The catch is that under certain conditions the algorithm may halt before the vertex coloring of $G$ is completed, thus rendering Kempe's proof invalid. This unfortunate fact was discovered independently by Percy John Heawood and Charles de la Vallee-Poissin in the 1890's.
But all is not lost: Kempe's algorithm sometimes succeeds in four-coloring planar graphs. In fact small examples of graphs upon which Kempe fails have surfaced only sporadically throughout the history of the FCT. We will talk about the smallest known graph on which Kempe fails and one measure of how bad this graph truly is. Related open questions abound.
(Of course the FCT was proved by Appel and Haken in 1977 and improved by Robertson, Sanders, Seymour, and Thomas in the 1990's.)

Keywords: graph coloring, Four Color Theorem, graph algorithms

# 86 Relationships among varieties of interval graphs, probe interval graphs, and ( 0,1 )-matrices 

David E. Brown*, J. Richard Lundgren, University of Colorado at Denver
We give correspondences between classes of bipartite graphs and combinatorial ( 0,1 )-matrices, some old and some new. Depending on one's perspective, the relationships may help provide structure theorems for the graph classes, or structure-theorems for the ( 0,1 )-matrices. Specifically, the adjacency matrices of several subclasses of interval bigraphs (including interval point bigraphs, X -, Y-consecutive bigraphs, convex bigraphs, unit interval bigraphs, bipartite permutation graphs, and asteroidal triple-free bigraphs) correspond to various classes of ( 0,1 )-matrices with combinatorial properties (including the zero-partitionable matrices, those with. consecutive l's in the rows, columns, or both, and those with a monotone consecutive arrangement). Other classes of graphs, e.g., bipartite probe interval graphs, have known containment relationships among the aforementioned graphs, and the matrix characterizations have provided insight into their structure. Consequently, we will give a few results regarding the bipartite probe interval graphs and the unit (or proper) probe interval graphs which follow from, or have been inspired by, the research presented.

Keywords: Interval bigraph, probe interval graph, (0,1)-matrix, consecutive l's property

## 87 Dot Critical vs. iDot Critical - The hazards of i: A Preliminary Report

Tamara Burton* and Melissa Matthews, Rochester Institute of Technology
A vertex is critical with respect to the domination number if removing it causes the domination number to decrease. A graph is critically dominated if the set of critical vertices forms a dominating set of the graph. A vertex is useable if it belongs to some minimum dominating set of a graph, and a graph is vertex useable if every vertex is useable. A graph is dot critical if collapsing any edge decreases the domination number.
Now add the letter i in front of these terms and redefine them in terms of the independent domination number. What theorems hold, what proofs can be adjusted, and where do things break down completely? With the introduction of a new class of graphs called idot critical graphs from smaller ones, a characterization of idot critical trees and other topics.

Keywords: domination number, independent domination number, critical vertex

## 88 A Combined Logarithmic Bound on the Chromatic Index of Multigraphs

Michael Plantholt, Illinois State University
For any multigraph $G$ the integer round-up $\$(G)$ of the fractional chromatic index $\times(G)$ yields a lower bound for the chromatic index $\times$ ' $(G)$. Kahn gave evidence of a sublinear upper bound by showing that for any real $c>0$ there exists a positive integer $N$ so that $\left.\left.x^{\prime}(G)<x\right](G)+c x\right](G)$ when $\mathrm{x} \|(G)>N$. We obtain a logarithmic upper bound, showing that for any multigraph $G$ with order $n$, x' $(G) 5 \Phi(G)+\log 312(\min \{n+1, \Phi(G)\})$.

## $90{ }_{\text {average Edge.Deleted Ecentricity }}$

Linda Eroh*, John Koker, Kevin McDougal, Hosien Moghadam and Steve Winters, University of Wisconsin Oshkosh

The edge-deleted eccentricity of a vertex v in a two-edge-connected graph $\boldsymbol{G}$ is the maximum, over every edge fin $\boldsymbol{G}$, of the eccentricity of v in $\boldsymbol{G} \boldsymbol{F}$ The average edge-deleted eccentricity of $v$ is the average, over every edge f in $G$, of the eccentricity of v in $G \boldsymbol{f}$. It is known that the edge-deleted eccentricity of two adjacent vertices can differ by an arbitrarily large value. We show that difference between the average edge-deleted eccentricities of two adjacent vertices is strictly less than $4 / 3$ and that this bound is best possible. We also address, by way of some examples and partial results, the relationship between the set of vertices of minimum eccentricity (the center of the graph) and the set of vertices of minimum average edge-deleted eccentricity (the average edge-deleted center).

Keywords: eccentricity, edge-deleted eccentricity, center

A patter of a matrix $M$ is a $(0,1)$ matrix which replaces all non-zero entries of $M$ with a 1 . Work in quantum algorithms is discrete situations has given rise to study of graphs and digraphs whose adjacency matrices are the pattern of a unitary matrix. A necessary condition for a matrix to be unitary is a property know as combinatorial orthogonality. If the adjacency matrix of a directed graph forms a patter of a combinatorically orthogonal matrix, we say the digraph is quadrangular. In particular, we look at eh quadrangular property in tournaments. We give some necessary conditions for tournaments to have the quadrangular property. We classify quadrangular tournaments which are not strongly connected in terms of lower bound on their domination number. We also give some constructions for quadrangular tournaments, and introduce a number of open problems.

Keywords: Combinatorial Orthogonality, Domination, Tournaments
92 Color Distribution in Minimal k-Rankings
Victor Kostyuk ${ }^{*}$, Darren A. Narayan, and Victoria A. Shults, Rochester Institute of Technology

A vertex coloring $f: V(G) \rightarrow\{1,2, \ldots, k\}$ is defined to beak-ranking of $G$ if every path connecting two vertices of the same color, contains a vertex of larger color. A ranking is minimal if the reduction of any label larger than 1 violates the ranking condition. We investigate properties involving the frequency and distribution of colors in a minimal k-ranking of a path. We use these properties to construct necessary conditions to determine if a given k -ranking is minimal.

Keywords: Vertex labeling, minimal ranking

# 94 Towards Minimal-Violations Rankings for Whist Tournaments 

David R. Berman, and Douglas D. Smith ${ }^{*}$, University of North Carolina at Wilmington

The English journal WHIST posed in 1891 what is now known as the whist problem: to devise a schedule of games wherein players enter as individuals, subject to the now-standard whist conditions. Much has since been accomplished in constructing tournament schedules, but little is known about the implicit second problem: to determine the rankings of individual players once the tournament has been completed. Suppose that a whist tournament is given along with an outcome (i.e., a winning and losing team) for each game. For a numerical ranking of players, we say that a given game is an upset if the sum of the rankings for the losing team exceeds the sum of the rankings for the winners, an indeterminate if the sums are equal, and a violation if the game is either an upset or an indeterminate. We describe outcomes for which there exist rankings with no violations and outcomes for which every ranking has violations.

Keywords: whist, ranking, upset, violation

Automated graph generation has been used extensively in graph theory research for several years. We describe some techniques and uses for matroid generation using the matroid software system Oid.
Keywords: graphs, matroids, software

## 96 Minimal k-Rankings and the A-rank Number of a Path Victor Kostyuk, Darren A. Narayan*, and Victoria A. Shults, Rochester Institute of Technology

Given a graph $G$ a function $\mathrm{J}: V(G)-+\{1,2, \ldots, \mathrm{k}\}$ is a k-ranking of $G$ if $f(u)=J(v)$ implies every $u-v$ path contains a vertex $w$ such that $\mathrm{f}(\mathrm{w})>J(u)$. A k-ranking is minimal if the reduction of any label greater than 1 violates the described ranking property. The a-rank number of $G$, denoted $\mathbb{P}_{\mathrm{r}}(\mathrm{G})$ equal the larges $k$ such that $G$ has a minimal k-ranking. We completely determine $\mathbb{P}_{\mathrm{r}}\left(\mathrm{P}_{\mathrm{n}}\right)$, a problem suggested by Laskar and Pillone in 2000. In particular we will show that $\mathbb{P}_{\mathrm{r}}\left(\mathrm{P}_{\mathrm{n}}\right)=\operatorname{llog} 2(\mathrm{n}+1) \mathrm{J}+\log 2\left(\mathrm{n}+1-\left(21_{\mathrm{g}}^{\mathrm{g} 2^{\mathrm{n}},}{ }^{1}\right)\right) \mathrm{J}$.

Keywords: Vertex labeling, minimal ranking, a-rank number

## 98 No Maximal Antichain of Tournaments With 3 Elements Brenda J. Latka, Lafayette College

A set of finite tournaments is an antichain if no element is a proper subtournament of another. Maximal (in the set theoretic sense) antichains of finite tournaments of every finite cardinality except 3 are constructed. A proof is given that no maximal antichain of finite tournaments with 3 elements is possible.

Keywords: Antichain, subtournament, tournament

99 On largest Circuits and Cocircuits in Matroids Nolan B. McMurray, Jr, University of Mississippi

Reid and Wu conjectured that if $C$ and $D$ are largest circuits in a $k$ connected matroid with IE (M)I $2(\mathrm{k}-1)$, then $r(C U D) \approx r(C)+r(D)-$ $k+1$. This conjecture generalizes the graph conjecture of Scott Smith that two longest cycles in a k -connected graph meet in at least k vertices. We establish the conjecture in certain special cases. In particular, progress on the conjecture is made for cographic matroids and matroids of small circuit size. Extensions of previous results on intsecting circui';s and cycles are given.

100 Forbidden Subgraph Edge Colorings
G Bullington*, L. Eroh, J. Koker, K. McDougal, H.Moghadam, S. Winters, University of Wisconsin-Oshkosh; and S. Stalder, University of Wisconsin-Waukesha

The edge-chromatic number of a graph $G$ can be thought of as the minimum number of colors necessary to color the edges of $G$ while forbidding a monochrome subgraph $K_{1,2}$. When we forbid both $K_{1,2}$ and $K 3$, notice that the minimum number of colors necessary is precisely the vertex covering number of $G$. These observations lead to the more general question: "Given a graph $G$ and class :F of graphs, what is the minimum number of colors necessary to color the edges of $G$ so that no monochrome subgraph of is contained in :F?" In this talk, we will present some general bounds for the minimum number of colors needed as well as some exact formulae for a variety of graphs $G$ and classes :F. Most results to be presented will address the cases when is complete or when :F contains Kl,n $\mathrm{mK} 1,2$, or odd-length cycles.
Keywords: edge colorings

## 102 Linear Arrangement of Trees

Robert Hochberg*, East Carolina University; and Matthias F. M. Stallmann, NC State University

Given an n-vertex tree $T$, one wishes to place the vertices of Ton the integers $\{1,2, \ldots, \mathrm{n}\}$ so that the sum of the lengths of the edges of $T$, called the cost, is as small as possible. Arrangements which are minimal with respect to costs, may have crossings. If we require our embedding to have no crossings, we refer to this as a one-page embedding. In this talk we will briefly outline a linear-time algorithm for optimal one-page embeddings (contrast with the besk-known $n$ algorithm of F . chung for embeddings which allow crossings) and prove that the ratio (cost of a one-page embedding) / cost of an optimal embedding) is never more than $a$, where $a=3 / 2$ today, but should be close to $7 / 6$ by the time of the talk.

## 105 An Upper Bound on the Basis Number of the Powers of

 the Complete GraphsSalar Y. Alsardary, University of the Sciences in Philadelphia

The basis number of a graph is defined by Schmiechel to be the least integer $h$ such that $G$ has an h-fold basis for its cycle space. Mac Lane showed that a graph is planar if and only if its basis number is 2. Schmiechel proved that the basis number of the complete graph $\mathrm{K}_{\mathrm{n}}$ is at most 3 . We generalize the result of Schmiechel by showing that the basis number of the $d$-th power of $K_{n}$ is at most $2 \mathrm{~d}+1$.

## 106 On the faces problem for perfect codes

 Olof Heden, KTH, Stockholm, SwedenPerfect I-error correcting codes $C$ in the hyper cube $Z!j$ are considered. The possibilities for the number $-\mathrm{y}(\mathrm{C})$ of code words in a k -face ' Y of the hyper cube are discussed. It is shown that the possibilities for the number $-(C)$ depend on the dimension of the face ' Y , the rank of C and the dimension of the kernel of $\boldsymbol{C}$. Especially we get an answer to a question of Sergey V. Avgustinovich whether there is a perfect code with no full ( $\mathrm{n}-1$ )/2-face or not.

107 Radiocolorings
Hemant Balakrishnan, University of Central Florida

Let $\boldsymbol{G}(\boldsymbol{V}, \boldsymbol{E})$ be an undirected graph on a vertex set $\boldsymbol{V}$ and a edge set $£$. The Radiocoloring on $G$ is defined as a function $\boldsymbol{f}: V+N$ such that $I f(x)-f(y) I \quad 1$ when $d(x, y)=2$ and $I f(x)-f(y) I \quad 2$ when $d(x, y)=1$, where $d(x, y)$ is the distance between the vertices $x$ and $y$ and $N$ is the set of nonnegative integers. The range of numbers used is called a span. The radiocoloring problem consists of determining the minimum span $>(G)$ for a given graph $\mathcal{G}$. In this paper we investigate the relationship between the minimum span $X(G)$ of a graph $G$ and that of its complement $>(G)$.

Keywords: Radiocoloring, $\mathrm{L}(2,1)$ labelling, $T$ - coloring, channel assignment, complement

108 Planar Minimally Rigid Graphs and Pseudo-Triangulations
Ruth Haas*, Smith College, David Orden, Francisco Santos, Univ.Cantabria;
Gunter Rote, Freie Univ. Berlin; Brigitte Servatius, Hermann Servatius,
Worcester Polytechnic Inst.; Diane Souvaine, Tufts Univ.; Ileana Streinu, Smith
College; and Walter Whiteley, York Univ.
A graph is rigid if when embedded on a generic set of points, with straight line edges, it is infinitesimally rigid. A pseudo-triangulation is an embedded planar graph, each of whose interior faces has exactly 3 convex vertices. We will describe these two classes of graphs and some of their uses. It was known that minimal (pointed) pseudo-triangulations (with respect to number of edges) are planar minimally rigid graphs. We have shown that the opposite statement is also true, i.e., every planar minimally rigid graph admits a pointed embedding. In fact this is true even under certain natural topological and combinatorial constraints.

Ranks of Graph Complements
George J. Davis, Gayla S. Domke ${ }^{\star}$ and Charles R Garner, Jr., Georgia State University

We will consider the rank of the adjacency matrix of the complement of a graph for various classes of graphs. A Nordhaus-Gaddum type result will also be given.

Keywords: rank, adjacency matrix, complement

110
New Optimal Self-Dual Codes of Length 106 Vassil Yorgov, Fayetteville State University

It is known that the minimal weight of any binary self-dual code of length 106 is at most 18. There is not known code with minimal weight 18. The best-known self-dual code has minimal weight 14 and was found by Gaborit and Otmani. Recently Gulliver, Harada, and Kim claimed that they found a pure double circulant code with minimal weight 16, but their code is not self-dual. In this talk we present three binary self-dual codes of length 106 with minimal weight 16 . The codes have 432,374 , and 400 words of weight 16, respectively, and are inequivalent. To obtain these codes we use the extended quadratic-residue $[104,52,20]$ code and the known decomposition techniques for codes with automorphisms. Since the group $\operatorname{PSL}(2,103)$ is an automorphism group for that code and has a permutation of order 17 with six cycles, we can find an equivalent code, $\mathrm{QR}(104)$, with automorphism $p=(1,2, \ldots, 17)(18,19, \ldots, 34)(35,36, \ldots, 51)(52,53, \ldots, 68)$ $(69,70, \ldots, 85)(86,87, \ldots, 102)$. Using Maple 7 we determine a subcode, E, of dimension 48 of $\mathrm{QR}(104)$ that is preserved by p. The code E contains only vectors that have even number of ones on the positions of each cycle of p and zeros on the entries 103 and 104. We extend E to a [ $106,48,20$ ] code, E ', by appending two zeros to each vector. Starting with a [10,5] binary self-dual code, $F$, we repeat 17 times each of the first six entries of its vectors to obtain a $[106,5]$ code, $\mathrm{F}^{\prime}$. Now $\mathrm{C}=\mathrm{E}+\mathrm{F}^{\prime}$ is a self-dual binary code of length 106 that has automorphism of order 17 with 4 fixed points. The following choices for F give the three new codes: $\{1000010000,0100001000,0010000100,0001000010,0000100001\}$, $\{0100100000,1000001000,0010000100,0001000010,0000010001\}$, and $\{0011000000,1000001000,0100000100,0000100010,0000010001\}$.

Keywords: self-dual codes, automorphisms, optimal codes

111Maximum Alliance-Free and Minimum Alliance-Cover

Sets
Khurram H Shafique* and Ronald D. Dutton, University of Central Florida
A defensive $k$-alliance in a graph $G=(V, E)$ is a set of vertices $A \quad V$ such that for every vertex $v E A$, the number of neighbors $v$ has in $A$ is at least k more than the number of neighbors it has in $\mathrm{V}-\mathrm{S}$ (where k is the strength of k -alliance). An offensive k -alliance is a set of vertices A V such that for every vertex $v E A$, the number of neighbors $v$ has in $A$ is at least k more than the number of neighbors it has in V - S . In this paper, we deal with two types of sets associated with these k -alliances: maximum k -alliance free and minimum k-alliance cover sets. Define a set $X \quad V$ to be maximum k -alliance free (for some type of k -alliance) if X does not contain any k-alliance (of that type) and is a largest such set. A set $Y \quad V$ is called minimum k-alliance cover (for some type of k-alliance) if $Y$ contains at least one vertex from each k -alliance (of that type) and is a set of minimum cardinality satisfying this property. We present bounds on the cardinalities of maximum k -alliance free and minimum alliance k -cover sets and explore their inter-relation. The existence of forbidden subgraphs for graphs induced by these sets is also explored.

112 The Hull Number of an Oriented Graph Gary Chartrand, Ping Zhang, Western Michigan University and John Frederick Fink ${ }^{\star}$, University of Michigan

For vertices u and v in an oriented graph $D$, the closed interval $J[\mathrm{u}, \mathrm{v}]$ consists of $u, v$, and all vertices lying in a $u-v$ geodesic or a $v-u$ geodesic in $D$. For $S \quad V(D), I[\mathrm{~S}]$ is the union of all closed intervals $I[u$, v] with $u, v$ E $S$. A set $S$ is convex if $\mathrm{J}[\mathrm{S}]=S$. The convex hull $[\mathrm{S}]$ of $S$ is the smallest convex set containing $S$. A set $S$ of vertices of $D$ is called a hull set of $D$ if $[\mathrm{SJ}=V(D)$. The minimum cardinality of a hull set in $D$ is called the hull number $h(D)$. If $\mathrm{I}[\mathrm{S}]=V(D)$ for $S \quad V(D)$, the set $S$ is called a geodetic set. The minimum cardinality of a geodetic set in $D$ is the geodetic number $g(D)$. For every nontrivial connected oriented graph $D, h(D)=g(D)$. It is shown that for every pair $a b$ of integers with $2 \$$ a $\$ b$ there exists a connected oriented graph $D$ such that $h(D)=a$ and $g(D)=b$ For every nontrivial connected graph $G$ $h-(G)=\min \{h(D): \mathrm{D}$ is an orientation of G$\}$ and $h^{+}(G)=\max \{h(D)$ : D is an orientation of G\}. We present characterizations of connected graphs $G$ of order $n 2: 2$ for which $h^{+}(G)=n$. It is shown that for every two integers n and m with $1 \$ \mathrm{n}-1 \$ \mathrm{~m} \$() \mathrm{n} 2$, that there exists a connected graph $G$ of order $n$ and size $m$ such that for each integer $k$ with $2 \approx k \$ n$ there exists an orientation of $G$ with hull number $k$.

113 Generalized Kloosterman sums over rings of order $2^{\mathrm{r}}$ and

Michelle R. DeDeo, University of North Florida

After presenting several applications of Kloosterman sums in analytic number theory, generalized Kloosterman sums attached to Gauss sums over the ring of $2^{\mathrm{r}}$ elements are explicitly evaluated and are shown to be sines and cosines. These sums are encountered in the evaluation of eigenvalues of the adjacency matrix associated to Euclidean graphs over rings modulo $2^{r}$. In addition, we recall several properties and the evahtation of the basic Kloosterman sum over Z2r due to Salie in 1929.

Keywords: Kloosterman, rings, Euclidean graphs, Salie, eigenvalues

## 114 Binary Strings without Odd Rums of Zeros

Silvia Heubach*, California State University Los Angeles; and Ralph Grimaldi, Rose-Hulman Institute of Technology

We look at binary strings of length $n$ which contain no odd run of zeros and express the total number of such strings, the number of zeros, the number of ones, the total number of runs, and the number of levels, rises and drops as functions of the Fibonacci and Lucas numbers and also give their generating functions. Furthermore, we look at the decimal value of the sum of all binary strings of length $n$ without odd runs of zeros considered as base 2 representations of decimal numbers, which interestingly enough are congruent (mod 3) to either 0 or a particular Fibonacci number. We investigate the same questions for palindromic binary strings with no odd runs of zeros and obtain similar results, which generally have different forms for odd and even values of $n$.

Keywords: Binary Strings, Fibonacci numbers, Lucas numbers, Runs

## One-factorization-based collective communication an a cluster of workstations

N. Deo, P. Micikevicius*, University of Central Florida.

Efficient collective communication is essential to a number of parallel algorithms. For example, algorithms for fast Fourier transform, matrix transpose, and sample sort utilize all-to-all personalized communication. We present a theoretical as well as empirical evaluation of collective communication on a Beowulf-type cluster. The proposed methods rely on graph one-factorization to achieve effi.ciericy, which is critical since cluster architecture has high communication overhead.
Keywords: Beowulf cluster of workstations, collective communication, one-factorization

## 116 Counting Small Latin Squares

Brendan McKay, Alison Meynert and Wendy Myrvold*, University of Victoria
A Latin square is an $n$ by $n$ array on $n$ symbols where each symbol appears exactly once in each row and column. Two Latin squares are isotopic if one can be transformed to the other by permuting rows, columns, and/or symbols. Being in the same main class means that in addition, the roles of rows, columns and/or symbols can be interchanged. A third form of equivalence is isomorphism which will be defined in the talk. By using the novel technique of generating only the Latin squares having non-trivial symmetry group, we are able to count the number of isotopy, main, and isomorphism classes up to $n=9$ and there is hope the results can be extended $t$ o $n=10$.
An outstanding open question is whether there exists an orthogonal triple of Latin squares of order 10 . The squares of order 10 which have been generated so far have been tested to see if they belong to some orthogonal triple. From these computations, we know that any square in such a triple must have isotopy group order $2^{\mathrm{k}}$ for some integer $k>=0$, as all other squares have been tested.

Keywords: Latin Squares, mutually orthogonal Latin squares, generation algorithms

117 Spacing Numbers of Graphs Michael Ferrara*, Emory University, Yoshiharu Kohayakawa, Universidade de Sao Paulo and Vojtech Rl>dl, Emory University

We consider a variation on a problem of embedding graphs into Cayley graphs considered by L. Babai and V. S6s. Let $G$ be a simple, undirected graph with $n$ vertices, and $\boldsymbol{N}$ and $\boldsymbol{S}$ be subsets of $\mathrm{t}=\{1,2, \ldots, \mathrm{t}\}$. We call the ordered pair ( $N, S$ ) a spacing of $G$ if we can associate each vertex $v i$ with a unique $n_{i} \mathrm{E} \boldsymbol{N}$ such that $V_{i V J}$ is in $\boldsymbol{E}(\boldsymbol{G})$ if and only if $\operatorname{lni}-n_{1} \mathrm{I}$ is in $\boldsymbol{S}$ The minimum $t$ such th:1.t S and $\boldsymbol{N}$ are contained in $[t$ and $(\mathbb{N}, \mathrm{S})$ is a spacing for $G$ is called the spacing number of $G$. A simple upper bound, based upon the theory of Sidon sets and the distribution of primes, is given for the maximum spacing number of a graph on $n$ vertices. Additionally, lower bounds are given for the maximum spacing number of all graphs on $n$ vertices and regular graphs on $n$ vertices. Several open problems will be given.

Keywords: Cayley graph, Sidon Set, embedding

## 118

 Compositions with no occurrence of a particular number Phyllis Chinn*, Humboldt State University; and Silvia Heubach, California State University Los AngelesA composition of $n$ is an ordered collection of one or more positive integers whose sum is $n$. The number of summands is called the number of parts of the composition. A palindromiccomposition or palindrome is a composition in which the summands are the same in the given or in reverse order. Compositions may be viewed as tilings of 1-by-n rectangles with 1-by-k rectangles, . We count the number of compositions and the number of palindromes of $n$ that do not contain any occurrence of $j$ for $j=3,4$ and give some formulas for the general case. We also explore patterns involving the number of parts and the total number of occurrences of each positive integer among all the compositions of $n$ without occurrences of $j=3,4$. These counting problems correspond to the number of rectangles used in tilings of length n and the number of rectangles of each size used among all the tilings of length $n$, respectively.
Keywords:Compositions, rectangular tilings, palindromes

Computing the diameter of random connected graph Ying Zhang* and Narsingh Deo, University of Central Florida

Large random graphs have recently been used to model and study the properties of the Internet, Intranets, the Web, and the like. A topic of continuing interest is to estimate the diameter of such graphs as well as the changes in the value of the diameter when nodes or edges are added at random to the graph (representing the changing nature of these networks). In this talk, we investigate the range and expected value of diameter of random, connected graph $G(n, p)$ of order $n$ and density p. More0ver, on-line algorithms for computing the diameter are also explored.
Keywords: Diameter, Random graph, on-line algorithm

120 Subsquare-rich Latin squares and their critical sets Richard Bean*, Institute for Studies in Theoretical Physics; and Ian Wanless, University of Queensland

In 1980 Heinrich and Wallis suggested the dihedral group table as a way to try to maximize the number of intercalates in a Latin square. In 1998 Donovan used the dihedral group table to construct a critical set of size $\left(5 m^{2}-3 m\right) / 2$ in such a Latin square. We examine the connection between constructions for subsquare-rich Latin squares and critical sets of large size in the literature, and present new constructions for such Latin squares and critical sets therein, in part by combining and extending the above results.

Keywords: latin squares, subsquares, intercalates, critical sets

121Domination Graphs of Symmetric Digraphs I: Stable Forms of Complete Biorientations of Disconnected, Complete, Bipartite, and Tripartite Graphs
Cara L. Cocking* and Kim A. S. Factor, Marquette University
Let $D$ be a digraph with arc set $A(D)$. If ( $x, y$ ) $\mathrm{E} A(D)$, we say that vertex $x$ beats vertex y . Iffor every vertex $z \mathrm{EV}(\mathrm{D})-\{x, \mathrm{y}\}$, either $(x, \mathrm{z}) \mathrm{E} \mathrm{A} \mathrm{(D)}$ or $(y, \mathrm{z}) \mathrm{E} A(D)$, then $x$ and $y$ are said to dominate in $D$. The domination graph of $D$, denoted dom( D ), is the graph on the vertices of $D, V(D)$, with an edge between two vertices $x$ and $y$ if $x$ and y dominate in $D$
The complete biorientation of an undirected graph $G$, denoted $G$ is a directed graph in which each edge $\{x, y\}$ of $G$ is replaced by $(x, y)$ and $(y, x)$. A symmetric digraph Dis one in which if $(x, y) \mathrm{E} A(D)$, then $(y, x) \mathrm{E} A(D)$. A symmetric digraph is a complete biorientation of some graph, and vice versa.
If the domination graph of a graph's complete biorientation is the graph itself (i.e., $\operatorname{dom}(\mathrm{G})=G$ ), we call the graph a stable form.
In this talk I present results regarding disconnected, complete, bipartite, and tripartite graphs and the stable forms they eventually reach when their biorientations are repeatedly subjected to the domination graph operator.

Keywords: domination graph, symmetric digraph, complete biorientation, stable form

## 122 The smallest k-regular h-edge-connected graphs without I-factors

John Ganci, Texas Instruments; and Douglas B. West*, University of Illinois
A well-known consequence of Tutte's I-Factor Theorem is that every k -regular graph that has even order and is $(\boldsymbol{k}-\mathrm{I})$-edge-connected has a Ifactor (a perfect matching). For $0 \mathrm{~h} \mathrm{k}-2$, we determine the minimum order of a k-regular h-edge-connected graph of even order with no I-factor. For $\mathrm{h}=\mathrm{k}-2$, the value is $\mathrm{k}^{2}+3 \mathrm{k}-2$ when k is odd and $\mathrm{k}^{2}+2 \mathrm{k}-2$ when k is even. The general formula is $(a+1)(t+2)-2$, where $a$ is the least odd integer greater thank, and tis $\mathrm{fk}_{\mathrm{f}}-\mathrm{l}$ when $\mathrm{k}-\mathrm{h}$ is even and is $\mathrm{fk}\{!-\mathrm{l}$ when $\boldsymbol{k}-\boldsymbol{h}$ is odd. We construct such graphs for all even orders at least this large.

Keywords: I-factor, matching, Tutte's Theorem, edge-connectivity, regular graph

Expected Value of the Diameter of a Random Graph and its Implications for the Web Graph Pankaj Gupta* and Narsingh Deo, University of Central Florida

The diameter of a graph (the maximum distance between any pair of nodes) is an important parameter. So far, no efficient algorithm running in linear or even quadratic time for computing the diameter is known. In this paper, we investigate the behavior of the diameter in large, random directed (as well as undirected) graphs and explore the algorithms for determining the diameter.
The World Wide Web can be modeled as directed graph in which a node represents a Web page and a directed edge represents a hyperlink. Albert et al. (Nature, 401 (1999): 130-131) reported that the diameter of the Web-digraph is logarithmic in the number of Web pages. We discuss the implications of our analysis of the diameter of random graph for the Web graph. This knowledge of the diameter can help us understand the \Veb topology, and lead to the design of efficient crawlers for search engines.

Keywords: Diameter, Random graph, Web graph

124The metamorphosis of 2 -fold 4 -cycle systems into 2 -fold 6-cycle systems
EMINE Sule Yazici, Auburn University

Let $\mathrm{c}^{*}=\mathrm{a} 6$-cycle with a double edged chord between opposite vertices. If we remove the double edge the result is a 6 -cycle. Let ( $X, \mathrm{C}$ ) be a 2 -fold 4 -cycle system without repeated 4 -cycles and ( $\boldsymbol{X}, \mathrm{C}^{*}$ ) a pairing of the 4 -cycles into copies of $\mathrm{c}^{*}$. If Ci is the collection of 6 -cycles obtained by removing the double edges form each copy of $c^{*}$ and $C_{2}$ is a reassembly of these double edges into 6 -cycles, then ( $X$, Ci U C:i) is a 2 -fold 6 -cycle system. We show that the spectrum for 2 -fold 4 -cycle system as described above is precisely the set of all $\mathrm{n}=0,1,9$, or $16(\bmod 24)$. This can be extended to maximum packings of $2 K_{n}$ with 6 -cycles.

125 Domination graphs of symmetric digraphs II: unipathic digraphs as biorientations of trees Kim A. S. Factor, Marquette University

For any tree $T$, the biorientation of T is the digraph $T^{\prime}$ where $V(T)=V(T 1)$ and for every edge $\{u, v\}$ E $V(T),(u, v) \mathrm{E} A\left(T^{\prime}\right)$ and $(v, u) \mathrm{E} A\left(T^{\prime}\right)$. This creates a unipathic digraph, where for every pair of vertices $u$ and $v$, there exists only one simple path from $u$ to $v$ and from $v$ to $u$. The outset of vertex u is denoted $\mathrm{O}(\mathrm{u})$ and is the set $\{v \mathrm{E} V(D) \mathrm{I}(u, v) \mathrm{E} \mathrm{A}(\mathrm{D})\}$. Two vertices $u$ and $v$ in a digraph Dare said to dominate if $V(D)-\{u, v\} \quad \mathrm{O}(\mathrm{u}) \mathrm{UO}(\mathrm{v})$. The domination graph of $D, \operatorname{dom}(D)$, is constructed by using $V(D)$ and placing edges between every pair of dominating vertices. This talk explores $\operatorname{dom}(T t)$.

Keywords: domination graph, unipathic, biorientation, tree

## 126 Isomorphic Factorizations of Some Linearly Recursive Trees <br> Gary E. Stevens*, Hartwick College; and Robert E. Jamison, Clemson University

It is known that some classes of graphs are 2 -splittable, i.e. their edge sets can be partitioned into two sets so that the induced subgraphs are isomorphic. Fibonacci and Lucas trees both have this property. In this paper we extend this idea to partitioning into k sets and construct a very general family of recursively generated trees that are all k -splittable.
Keywords: Isomorphic Factorizations, Splittable, Fibonacci Trees, Lucas Trees

Broadcasting is a problem of message dissemination in a communication network, where one of the nodes (called the originator) has an item of information and needs to transmit it to every other node. This communication pattern finds it main applications in the field of interconnection networks for parallel and distributed architecture. Numerous papers have investigated methods to construct sparse networks (graphs) in which broadcasting can be c0mpleted in theoretically minimum possible time from any originator. In this paper, we consider the the broadcast model called k-broadcasting. Under this model every informed vertex can call up to k of its neighbors simultanesily. Here we present a linear algorithm to determine the broadcast time of an arbitrary tree.

Keywords:communication networks, broadcast, algorithms

## 128 <br> Two-fold Maximum Packing C3 to $\mathrm{C}_{4}$ Metamorphoses

 D. G Hoffman*, C C Linder, Auburn UniversityFor all possible orders, we construct a two-fold maximum packing with three-cycles that admits a metamorphosis into a two-fold maximum packing with four-cycles.

The closed neighborhood packing number is dual to the domination number in simple graphs. It follows that their fractional analogues are equal. Let D be the set of all min. fractional domintaing functions and let P be the set of all maximum closed neighborhood packing functions. We look into the classes of graphs characterized by:
(i) D n $\mathrm{P}=0$
(ii) $D C P$
(iii) $P C D$
(iv) $D$ n $P=0$
(v) $D=P$

Keywords: fractional domination, linear programming

## 130

Large Hamiltonicity of digraphs for universal cycles of permutations
Garth Isaak, Lehigh University
The digraphs $\boldsymbol{P}(\boldsymbol{n}, \boldsymbol{k})$ have vertices corresponding to length $\boldsymbol{k}$ permutations of an $n$ set and arcs corresponding to ( $k+1$ ) permutations (with arcs ( $1 \mathrm{r} 1,1 \mathrm{r} 2,-. .1 \mathrm{rk})$-+ $(1 \mathrm{r} 2,1 \mathrm{r} 3, \ldots, 1 \mathrm{rk}+1)$ ). At this conference last year Klerlein, Carr and Starling showed that $\mathrm{P}\left(n_{2}\right)$ is Hamiltonian and asked about the general case. We show that these digraphs are Hamiltonian for $\boldsymbol{k}$ ::; $n-3$ using a lemma about Eulerian chains 'with restricted turns' and the fact that $\boldsymbol{P}(\boldsymbol{n}, \boldsymbol{k})$ is nearly the the line digraph of $\boldsymbol{P}(\boldsymbol{n}, \boldsymbol{k}-1)$. We also observe that the digraphs $\boldsymbol{P}(\boldsymbol{n}, \boldsymbol{n}-2)$ are not Hamiltonian for $\boldsymbol{n} 4$ using a result of Rankin on Cayley digraphs which originally arose from a problem in campanology (bell ringing).

Keywords: Hamiltonian cycle, Eulerian chain

131 Desirable Properties of Universal Formulas for Percolation Thresholds
John C Wierman and Dora Naor*, Johns Hopkins University
Percolation models are infinite random graph models that exhibit phase transition behavior at critical probability values called percolation thresholds. Several "universal formulas" that provide estimates of percolation thresholds have been proposed in the physics and engineering literature. The existing universal formulas make substantial errors for several infinite lclttice graphs. Our work proposes desirable properties of universal formulas, and begins to investigate which properties are satisfied by the existing formulas. The long-term goal of the investigation is to formulate improved universal formulas.

Keywords: percolation threshold (or critical probability), infinite graphs, average degree, dual graph, line graph, subgraph, contraction, subdivision

> 132 Title Coverings of type g-super -4 for $>.=3$ or 4 M.J. Grannell, T.S. Griggs, R.G. Stanton*, University of Manitoba

Coverings of type g-super-4 for $>=1$ were determined by Stanton and Stinson. Recently, the present authors discussed the case $>=2$. The present paper is a report on the behaviour of such coverings when $>=3$ and 4.

Limit Theory of the Domination Number for the Class Cover Catch Diagraphs
Pengfei Xiang*, and John Wierman, Johns Hopkins University

Dominating set is one of the NP complete core problems. In this paper, we will discuss the limiting behavior of the domination number of random class cover catch diagraphs (CCCDs). The CCCD problem is motivated by its applications in pattern classification. For the special case of uniformly distributed data in one dimension, Priebe, Marchette and Devinney found the exact distribution of the domination number of the random data-induced CCCD, and Devinney and Wierman proved the Strong Law of Large Numbers (SLLN). We will present progress toward the SLLN and the Central Limit Theorem (CLT) for general data distributions in one dimension. The ultimate goal is to establish SLLN and CLT results for higher dimensional CCCD.

Keywords: Class Cover Catch Diagraph, Domination Number, Strong Law of Large Number, Central Limit Theorem, Pattern Classification.

## 134 on super Edge-magic Graphs with Many Odd Cydes

 Sin-Min Lee*, San Jose State University; and Alexander Nien-Tsu Lee,Lynbrook High School
A $(\mathrm{p}, \mathrm{q})$ graph G is total edge-magic if there exits a bijection $f: V U E$-> $\{1,2, \ldots, p+q\}$ such that for each $e=(u, v)$ in $E$ we have $f(u)+f(e)+f(v)$ is a constant. A total edge-magic graph is called a super edge-magic if $f(V(G))=\{1,2, \ldots, p\}$. The super edge-magic properties of several classes of graphs with many odd cycles are studied.

Keywords: Total edge-magic, super edge-magic, sequential, harmonious, cordial, consecutive labeling, cycles

135 Improved methods for computing rigorous bounds on percolation thresholds
William D. May* and John C Wierman, Johns Hopkins University
The substitution method is a technique for calculating rigorous bounds on the percolation threshold for many bond and site percolation models. In this paper we describe two recent computational improvements to the substitution method. The theoretical foundation of these techniques also eliminates the need for individual proofs of correctness that were necessary in previous work. The first technique employs symmetry, in the form of a finite graph's automorphism group, to dramatically reduce the number of calculations needed for the substitution method calculation. The second improvement uses network flows to prove stochastic ordering between the probability measures on two graphs. Based on the special structure of the stochastic ordering problem, the network flow technique leads to much greater computational efficiency. Together, these two techniques allow us to apply the substitution method to larger regions of the infinite lattice, thus obtaining tighter bounds on the percolation threshold than were previously possible.

Keywords: percolation, network flows, stochastic ordering, automorphism group

136Metamorphosis of lambda-fold designs with block size four into 3-stars: the final case Elizabeth J. Billington, The University of Queensland

A ( $1,4,>$.) block design may be regarded as a decomposition of $>K_{v}$ into edge-disjoint copies of $\mathrm{K}_{4}$, that is, as a >.-fold $\mathrm{K}_{4}$-design. Let G be a (connected) subgraph of $K_{4}$. A metamorphosis of a $>$-fold $\mathrm{K}_{4}$-design into a >.-fold G-design is obtained by retaining all the copies of Gin the $K_{4}$ blocks, and then rearranging the edges in the remainder (the $K_{4}-G$ pieces) into further copies of $G$
Many authors have found metamorphoses of >.-fold $\mathrm{K}_{4}$-designs:

> Lindner and Rosa into $>$-fold triple systems;
> Lindner and Street into $>$-fold 4 -cycle systems;
> Lindner and Rosa into ( $K_{4}$ - e)-designs;
> $\mathrm{Kli}_{\varsigma} \mathrm{lik}_{\varsigma}$ if $_{4} \mathrm{i}$ and Lindner into >-fold ( $K_{4}-$ e)-designs, >. $>{ }_{1}$;
> Kil $_{S}$ lik $_{\text {dif }}^{4} \mathrm{i}$ and Lindner into $>$.fold ( $K_{3}+e$ ) (kite) designs.

The remaining nontrivial case is that of a metamorphosis into a $\mathrm{K}_{l} 3$ star design. We deal with that case here

Renu Laskar, Clemson University, Alice McRae, Appalachian State University and Charles Wallis*, Western Carolina University

The rook's graph, in which the vertices consist of the $n^{2}$ positions of an $n x n$ chessboard and adjacencies represent the possible moves of a rook, corresponds to the graph of the $\mathrm{L}_{2}$ association scheme. In a similar way, chessboard-like graphs can be defined based upon the relation known as the triangular association scheme; we designate these graphs triangulated $-:: h e s s b o a r d ~ g r a p h s . ~ I n ~ t h i s ~ p a p e r, ~ t h e ~ v a l u e s ~ o f ~ s e v e r a l ~ d o m i n a t i o n ~ p a r a m e-~$ ters of triangulated chessboard graphs are determined and compared to the known domination parameters of ordinary chessboard graphs.

138 Convex geometric graphs with no short self-intersecting paths
Debra Boutin, Hamilton College
A geometric graph is a straight-line graph drawing in the plane in which no three vertices lie on a single line and no three edges intersect at a single point. A convex geometric graph is a geometric graph all of whose vertices lie on the boundary of its convex hull. A principal question within geometric graph theory has been to find the maximum number of edges in a geometric graph that does not contain a given subgraph. Recently Pach, Pinchasi, Tardos and T6th proved that if a geometric graph on n vertices contains no self-intersecting path of length 3 then it contains at most $\mathrm{O}(n \log n)$ edges. Furthermore this bound is asymptotically tight. Geometric graphs with no short self-intersecting paths are naturally called locally planar. In this talk we will look at the class of convex geometric graphs without short self-intersecting paths. We call these graphs locally outerplanar. In contrast to the result for locally planar graphs, we show that locally outerplanar graphs have at most a linear number of edges.

Keywords: geometric graph, convex geometric graph, locally planar

140Constructing Resolvable ( $\mathrm{n}, 3,3,2$ ) Lotto Designs Using Resolvable Covering Designs and Kirkman Triple Systems Ben Pak Ching Li, University of Manitoba

An ( $n, k, p, t$ ) Lotto design is a set system $(X, B)$ where IXI $=n, \mathrm{~B}$ is a collection of k -subsets of $\boldsymbol{X}$, called blocks, such that, for any p-subset P of $\boldsymbol{X}$, there exists B E B such that $\mathrm{IB} \mathrm{n} \mathrm{Pl} t$. Let $L(n, k, p, t)$ denote the minimum number of blocks in any $(n, \boldsymbol{k} p, t)$ Lotto design. An ( $n, k, p, t$ ) Lotto design is called optimal if it has $L(n, k, p, t)$ blocks. In this presentation, I will discuss the resolvability of optimal $(n, 3,3,2)$ Lotto designs. Explicit constructions were obtained using Kirkman triple systems and resolvable covering designs

Keywords: Resolvable, Lotto Designs, Covering Designs, Steiner Triple Systems, Kirkman Triple Systems

Mark Anderson*, Jay Yellen, Rollins College; and Robert Brigham, University of Central Florida

The distance-k domination number of a graph $G, \mathrm{Y}<\mathrm{k}(\mathrm{G})$, is the size of a smallest set $D$ of vertices such that $\operatorname{dist}(\mathrm{v}, \mathrm{D}) \quad$ k-for every vertex v in G. Vertex set can be partitioned into sets, $\mathcal{V}-=\{\mathcal{V}$ E V: $\mathrm{Y}<\mathrm{k}(\mathrm{G}-\mathrm{v})<$ $\left.{ }^{\prime} \mathrm{Y}<\mathrm{k}(\mathrm{G})\right\}, \mathrm{v}^{0}=\left\{v \mathrm{E} \mathrm{V}:{ }^{\mathrm{C}} \mathrm{Y}<\mathrm{k}(\mathrm{G}-\mathrm{v})={ }^{\mathrm{\prime}} \mathrm{Y}<\mathrm{k}(\mathrm{G})\right\}$, and $\mathrm{v}^{+} \quad\{\mathrm{v} \mathrm{E} \mathrm{V}$ : $\left.\mathrm{Y}_{\mathrm{k}}(G-\mathrm{V})>\mathrm{Y}<\mathrm{k}(\mathrm{G})\right\}$. Carrington, Harary, and Haynes showed that ) whenk $=1$ and $\mathrm{v}^{0} \neq \mathrm{IV}^{0}, 2 \mathrm{j} V^{+} \mathrm{I}$. Anderson, Brigham, Carrington, Vitray, Williams, and Yellen generalized this inequality for arbitrary k to JV0 $\quad[2 /(2 \mathrm{k}-1)] \mathrm{jV}^{+} \mathrm{I}$ and also characterized those graphs for which $\mathrm{k}=1$ and $!\mathbf{0}=2 \mathrm{jV}+\mathrm{j}$. Here we discuss two classes of graphs for which $\mathrm{J}_{\mathrm{J}}=[2 /(2 \mathrm{k}-\mathrm{I})] \mathrm{jV}^{+} \mathrm{j}$ for any KZ 2
Keywords:distance-k domination number, domination, graph theory
142 Algorithmic advances in finding ( $\boldsymbol{a}$ mod 5 )-cycles in graphs Stephen E. Shauger* and Bin Zheng, Coastal Carolina University

Erdos first proposed problems of the following type. Given an infinite subset of the natural numbers $\boldsymbol{S}$, what is the smallest number $\boldsymbol{d}$, such that every graph with minimum degree at least $\boldsymbol{d}$ has a cycle whose length is in $\boldsymbol{S}$ We characterize the Hamiltonian graphs with minimum degree at least 3, which do not have a cycle of length a mod 5 , for each of the possible values of $a$.

Combinatorial Reconstruction using Polynomial

## Invariants

L. T. Pebody, Cambridge University and The Institute for Advanced Study, Princeton

A link is given between the Combinatorial Reconstruction problem looked at by Alon, Caro, Krasikov and Roclitty and the turn-of-the-century work on Polynomial Invariants by Hilbert, Noether and others.

A Constructive Approach for the Lower Bounds on the

## Ramsey Numbers $R(s, t)$

Xu Xiaodong, Xie Zheng, National University of Defense Technology; and Stanislaw P. Radziszowski*, Rochester Institute of Technology

Graph $G$ is a (k,p)-graph if $G$ does not contain a complete graph on $k$ vertices $\mathrm{K}_{\mathrm{k}}$, nor an independent set of order $p$. Given a (k,p)-graph $G$ and a (k, q)-graph $H$, such that $G$ and $H$ contain an induced subgraph isomorphic to some $\mathrm{K}_{\mathrm{k}_{-}} 1$-free graph $M$, we construct a $(k, p+q$ - 1)graph on, $\mathrm{i}(\mathrm{G})+n(H)+n(M)$ vertices. This implies that $R(k, p+q-\mathrm{l})$ $R(k, p)+R(k, q)+n(M)-1$, where $R(s, t)$ is the classical two-color Ramsey number. By applying this construction, and some its generalizations, we improve on 22 lower bounds for $R(s, t)$, for various specific values of $s$ and $t$ In particular, we obtain the following new lower bounds: $\boldsymbol{R}(4,15) 153$, $\mathrm{R}(6,7) \quad 111, \mathrm{R}(6,11) \quad 253, \boldsymbol{R}(7,12) \quad 416$, and $\mathrm{R}(8,13) \quad$ 635. Most of the results did not require any use of computer algorithms.
Keywords: Ramsey numbers, graph coloring

## 146 Large [r, t$]$-Colorings of Graphs Arnfried Kemnitz, Technische Universitat Braunschweig

Given non-negative integers $r, s$, and $t$, an $[r, s, t]$-coloring of a graph $G$ with vertex set $V(G)$ and edge set $E(G)$ is a mapping c from $V(G)$ UE (G) to the color set $\{0,1, \ldots, \mathrm{k}-\mathrm{l}\}$ such that $\mathrm{lc}(\mathrm{vi})-\mathrm{c}(\mathrm{v}) \mathrm{I} \quad r$ for every two adjacent vertices $v / V, \mathrm{lc}(\mathrm{ei})-\mathrm{c}(\mathrm{ej}) \mathrm{I} \quad s$ for every two adjacent edges e ;, ej, and lc(vi) - c(ej)I $t$ for all pairs of incident vertices and edges, respectively. The $[r, s, t]$-chromatic number $X_{r}, s, t(G)$ of $G$ is defined to be the minimum $k$ such that $G$ admits an $[\mathrm{r}, \mathrm{s}, \mathrm{t}]$-coloring.
This is an obvious generalization of all classical graph colorings since c is a vertex coloring if $r=1, s=t=0$, an edge coloring ifs $=1, r=t=0$, and a total coloring if $r=s=t=1$, respectively.
We present first results on $X_{r} s \mathrm{t}(\mathrm{G})$ such as some bounds and also exact values for specific classes of gr phs.

Keywords: Chromatic number, chromatic index; total chromatic number

147 Coloring paired graphs Alice McRae*, Dee Parks, Kelly Wise, Appalachian State University

The four-color theorem tells us that all maps can be colored with four colors. But what if countries in the map have colonies that need to be colored the same color as the mother country? Then, there may be many more than four colors required.
Let $G=(V, E)$ be a graph. We define a pairing of the graph as a set $\boldsymbol{S}=\left\{\mathrm{S}_{1}, \mathrm{~S}_{2}, \ldots, \boldsymbol{S}_{\boldsymbol{k}}\right\}$ of $\boldsymbol{k}$ pairwise disjoint subsets of the vertex set, where ISil $=2$ for $1 \quad \mathrm{i} \quad \mathrm{k}$. We refer to each of the s11bsets Si as a pair. The set $S$ is not required to be a partition of the vertex set; i.e., some of the vertices may be unpaired. The graph $P\left(G, S_{k}\right)$ is the graph obtained by identifying the k-pairs of the graph. In this talk we investigate coloring these paired graphs when the underlying graph $G$ is planar.

Keywords: graph coloring

148Bijections of Combinatorial Objects Wen-jin Woan, Howard University

Let $S_{n}$ be the $n$ by $n$ square sitting on the coordinate axes in the first quadrant, a polyomino $C \subset S_{n}$ is symmetric semi-convex, if $C$ is symmetric w.r.t. the diagonal $y=x$,every pair of points can be connected by two lines in $C$ Let $P_{n}$ be the set of all symmetric semi-convex polyominos with perimeter of length 4 n . Note that the upper perimeter is log concave of length 2 n . We use cut and paste technique to give a bijective proof that $I P_{n} I=(2::-i 2)$, the central binomial number.

Keywords: Polyominos, Catalan Paths, Binomial Paths, Noncrossing Trees, Rooted Plane Trees

A set of natural numbers is pseudo-arithmetic if $\{b-a l / a, b E A\}$ is a set of multiples of $\min \{/ b-a l / a, b E A\}$. When coloring a graph, we show one can always avoid monochromatic arithmetic sets. On the other hand, we prove a Ramsey Theorem asserting the eventual existence of monochromatic pseudo-arithmetic sets of any size. The associated Ramsey number for the 2 -color $\mathrm{K}_{3}$ case turn out to be 12 , while for three colors one needs at least 612 points! We use this Ramsy Theorem to prove a divisible Schur Theorem. We provide an upper bound for Pseudo-Arithmetic Ramsey Numbers.

150 Yet Another Algorithm for Generating the Gray Code Larry Dunning, Bowling Green State University

Another development of the Gray code generation algorithm of Bitner, Ehrlich and Reingold is given. It is shown that this algorithm can be obtained from a standard algorithm, which adds one to an integer in constant time. In addition to giving an alternate development of the original algorithm, an additional array-free version of the algorithm is obtained which can be useful when the sets generated are represented as words in a digital computer.

Keywords: Gray code, subset generation, subset enumeration

A graph G is Hall t-chromatic if there is a proper multicoloring of it from a constant $t$-set of colors whenever Hall's condition is satisfied. It has been shown previously that all graphs are Hall $t$-chromatic fort $=0$, l , and 2, that bipartite graphs, complete multi-partite graphs, and odd cycles are Hall t-chromatic for all $t$ and that there exist graphs which are Hall t-chromatic for only finitely many $t$. Here we show that each wheel (the join of a vertex and a cycle) of even order $>4$ is not Hall t-chromatic for all $t>2$ Some old questions are answered and some new ones are posed.

Keywords: list multicoloring, Hall's condition

152 Binary Strings with No Isolated l's in Even Positions Ralph P. Grimaldi, Rose-Hulman Institute of Technology

For a nonnegative integer $n$, we count those binary strings of length $n$ where the following condition is satisfied: If a 1 appears in an even position - say position 2 k , then there is a 1 in position $2 \mathrm{k}-1$ and a 1 in position $2 \mathrm{k}+$ 1 , unless $\mathrm{n}=2 \mathrm{k}$. The number of such n -bit strings with no isolated l's in even positions is Fn+2, where Fn denotes then-th Fibonacci number.
These strings arise in a natural way from the order ideals of the fence poset and this correspondence helps to determine results such as the total number of O's and l's in all of these Fn+2 binary strings. Further results consider the total number of runs as well as the total number of ascents (from Oto 1), levels (from Oto 0 , or 1 to 1 ), and descents (from 1 to 0 ) for these Fn+2 binary strings.

Keywords: Fibonacci numbers, Lucas numbers, binary strings, order ideals, fence poset

## 153 Size Ramsey numbers and linear programming

 Oleg Pikhurko, Cambridge UniversityThe size Ramsey number of a graph F is the smallest number of edges in an F-arrowing graph. I will present a linear programming approach which works for some bipartite $\boldsymbol{F}$. For example, the asymptotics of the size Ramsey number of $\mathrm{K}_{\mathrm{m}}, \mathrm{n}$, when $n-+$ oo, was computed for any fixed $m$ answering a question posed by Erdos, Faudree, Rousseau and Schelp (1978).

## 154 Combinatorial Algorithms for Computing Aggregate Functions in Probabilistic Relational Databases Ping-Tsai Chung, Long Island University

When multiple heterogeneous databases show different values for the same data item, its actual value is not know with certainty. To develop corporate data warehouse, which consolidate data from multiple heterogeneous data sources, has become an important issue for designing modern business information systems. Probabilistic relational databases have extended from the relational database model by incorporating probability measures to capture the uncertainty associated with data items. In this paper, we define a set of aggregate functions in probabilistic relational databases. Some properties of these aggregate functions are showed. We then develop combinatorial algorithms to compute these aggregate functions. Finally, we present some comparative results of computational complexity of these algorithms.

Keywords: combinatorial algorithms, computational complexity, probabilistic relational databases, aggregate functions, data uncertainty

155 k-fold coloring even cycles with Hall's condition
M.M. Cropper* Eastern Kentucky University; A.J.W. Hilton, Reading University; and P.D. Johnson Jr., Auburn University

For each positive integer $k$, the $k$-fold Hall number of a graph $\mathrm{G}, \mathrm{h}(\mathrm{k}>(\mathrm{G})$, is the minimum positive integer $\mathrm{m} \quad k$ such that if $L$ is any list assignment with IL(v)I $\quad m$ for every $v \mathrm{E} V(G)$ then there is a $k$-fold list coloring of $G$ whenever G and L satisfy Hall's condition (a natural necessary condition). This parameter has the unusual attribute of being more difficult for even cycles than for odd cycles. Indeed, it has been shown t.hat $h(k)\left(\mathrm{C}_{2 \mathrm{~m}}+\mathrm{i}\right)=$ $2 k$. It has also been shown that $h(k)\left(\mathrm{C}_{4}\right)=45 \mathrm{k} / 31$ and conjectured that $\mathrm{h}(\mathrm{k})\left(\mathrm{C}_{2 \mathrm{~m}}\right)=2 \mathrm{k}-\mathrm{lk} /(2 \mathrm{~m}+1) \mathrm{J}$. On this occasion we will demonstrate that $\mathrm{h}(\mathrm{k})\left(\mathrm{C}_{2 \mathrm{~m}}\right) \quad 2 \mathrm{k}-\mathrm{lk} /(2 \mathrm{~m}+1) \mathrm{J}$.

## 156 a Simple Generating Function for some Generalized Random Walk <br> Charles Moore, Howard University

Consider lattice paths in the plane using three types of weighted steps. (A) Up steps - $(1,1),(2,1),(3,1), \ldots$. (B) Level steps - $(1,0),(2,0),(3,0), \ldots$. (C) Down steps $-(1,-1),(2,-1),(3,-1), \ldots$. We derive a simple generating function counting the number of paths from $(0,0)$ to ( $n, 0$ ) using these three types of steps and then look at examples, implications, and related results.

## 157 From Ramsey-Graphs to Fast Matrix Multiplication

Vince Grolmusz, E/Itvlls University

We show astonishingly fast algorithms for computing some modulo 6 representations of elementary symmetric polynomials, the dot product and the matrix product.
We prove that the S! elementary symmetric polynomial modulo composite numbers $\mathrm{m}=\mathrm{p}_{1} \mathrm{p} 2$ can be computed with much fewer multiplications than over any field, if the coefficients of monomials $x i, X i, \cdots x i$, are allowed to be 1 either $\bmod p_{1}$ or $\bmod P 2$ but not necessarily both. More exactly, we prove that for any constant $k$ such a representation of $S$ ! can be computed modulo plp2 using only $\exp (\mathrm{O}(\mathrm{Jl} \overline{\mathrm{ogn}} \log \operatorname{logn})$ multiplications on the most restricted depth-3 arithmetic circuits,for $\min \{\mathrm{P} 1, \mathrm{p} 2\}>k l$.
Using these results, we show an algorithm for computing a similar representation of the dot-product of two length-n vectors with only $\mathrm{n}^{0}{ }^{1}$ ) multiplications, or, consequently, for the computing of the representation of the product of two $n \times n$ matrices with only $\mathrm{n}^{2+o(!)}$ multiplications. Our methods are closely related to those of ours what resulted new constructions for explicit Ramsey graphs and hypergraphs.

## 158

On incomparable and uncomplemented families of sets Yuejian Peng* and Cheng Zhao, Indiana State University

Let $A_{1}, \ldots, A_{k}$ be families of distinct subsets of $[n]$. These families are incomparable if no set in one family is contained in a set in another family. A family of subsets $A$ is uncomplemented if it contains no complementary pair of sets. Hilton conjectured that foi: k families of distinct subsets from [n] that are incomparable and uncomplemented, the sum of the sizes of the families is at most $2^{\mathrm{n}}-^{1}$. We prove that $(4-2 . / 2) 2^{\mathrm{n}}-1$ is an upper bound. We also prove the conjectured bound for some cases.

## 159 Ceneralized Chromatic Numbers and Additive

 Hereditary Properties of GraphsIzak Broere, Samantha Dorfling and Elizabeth Jonck*, Rand Afrikaans University

An additive hereditary property of graphs is a class of simple graphs which is closed under unions, subgraphs and isomorphisms. Let P and Qbe additive hereditary properties of graphs. The generalized chromatic number $\mathrm{X}_{\mathrm{Q}}(\mathrm{P})$ is defined as follows: $\mathrm{X}_{\mathrm{Q}}(\mathrm{P})=n$ iff $\mathrm{p} K_{-\mathrm{Q}}$ but $\mathrm{P} \% \mathrm{Q}-\mathrm{I}_{-}$We investigate the generalized chromatic numbi>rs of the well-known propertys of graphs Ik, Qk wk, sk and Dk
Keywords: property of graphs, additive, hereditary, generalized chromatic number

## 160 Counting Infinite Step Set Lattice Paths using Umbral Calculus <br> K. Humphreys ${ }^{\star}$ and H Niederhausen, Florida Atlantic University

A lattice path in the No x No plane with an infinite step set $S$ can go to infinitely many lattice points within the boundary but can only come from finitely many points. The path has its natural boundaries, the positive $x$ and y axes, or in addition the path is required to stay above the boundary line $y=a x-£\left(a, \mathrm{e} \mathrm{E} \mathrm{N}_{1}\right)$. We count the number of paths from the origin to point ( $n, m$ ) as well as the number of paths that contact the weak boundary, $y=a x-\mathrm{e}+1, \mathrm{c}$ times. A lattice path has privileged access to the boundary line if there is an additional set of (privileged) step vectors $P$ such that the lattice points ( $n$, an - £) can only be reached by steps from P. We find closed form solutions via Sheffer sequences and functionals using results of the Umbra! Calculus.
In this talk we give some general results and specific solutions with $S=\{(\mathrm{i}, \mathrm{j}) \mathrm{ENoxNo} \backslash(0,0)\}, \quad \mathrm{P}=\{(0, \mathrm{j}) \mathrm{ljENi}\}$ and boundary $y=2 x-3$.

Keywords: Lattice path counting, infinite step set, Umbral Calculus, boundary contacts, Sheffer polynomials, privileged access

Jens-P. Bode, Technische Universitat Braunschweig
Mosaic graphs are plane q -regular graphs partitioning the plane into p -gons only. For $(p-2)(q-2) \quad 4$ we define $B 1(p, q)$ to be one $p-g o n, B z(p, q)$ to consist of all p -gons surrounding one vertex, and $\mathrm{B}_{\ldots}(\mathrm{p}, \mathrm{q})$ to consist of $B_{\ldots, \ldots}(p, q)$ together with all neighboring $p$-gons. These sequences $B_{1 .,(p, q)}$ are used as host graphs for the mosaic graph Ramsey number $r_{p}, q(G, H)$ being the smallest number such that every 2 -coloring (green and red) of the edges of $B_{m}(p, q)$ contains a green Gora red $H$. First results on i;he existence of $\mathrm{r}_{\mathrm{p}}, \mathrm{q}(\mathrm{G}, \mathrm{H})$ and some exact values are presented. Common work with Heiko Harborth.

## 162 Forbidden subgraph conditions on the complements of a graph that insure a strong network design

L. Kazmierczak* F. Boesch, C. Suffel, Stevens Institute of Technology; and D. Gross, Seton Hall University

If a network is modeled by a simple graph in which the nodes represent the components of the network, then the connectivity of the graph is one measure of the "vulnerability" of the network. The authors have studied a somewhat related vulnerability parameter: the component order connectivity, i.e., the smallest number of nodes that must fail in order to insure that all remaining componenets have order less than some preassigned threshold value. In this talk we show that any almost regular graph have a $C_{4}$-free complement has the maxiumum possible connectivity and the maximum possible component order connectivity for all threshold values not less than three over all graphs having the same number of nodes and edges. It is also shown that every such graph has diameter at most two. Generalizations of this result are also discussed.

## 164 Lattice Paths on Parallel Planes

 Seyoum Getu, Howard UniversityLattice paths with weighted steps $\mathrm{E}, \mathrm{N}, \mathrm{U}$ or D starting at $(0,0)$ of the first quadrant of the first plane (first floor) and continuing to the first quadrants of succeeding parallel planes (floors), but not going to the ground floor, and ending in any one of the floors, are considered. Floor to floor steps are taken at any lattice point in any floor. The transfer matrix method is used to find the generating functions of these paths. The case of four floors with $E, U$, and D steps ending in any floor results in one form of Fibonacci numbers. Other conditions lead to Pell numbers and some 'new' sequences.
Keywords: Transfer matrix, Pell numbers

A graph is called $\mathrm{C}_{4}$-free if it contains no cycle of length four as an induced subgraph. The following question has been asked by Paul Erdos:is it true that $\mathrm{C}_{4}$-free graphs with n vertices and at least $\mathrm{c}_{1} \mathrm{n}^{2}$ edges must contain complete subgraphs of c 2 n vertices, where c 2 depends only on c 1 ? We give the affirmative answer with c2 $=0.4 \mathfrak{E}: \mathrm{I}$. We also give estimates on c 2 and show that a similar result does not hold for $H$-free graphs - unless $H$ is an induced subgraph of $\mathrm{C}_{4}$. The best value of $\mathcal{C}$ is determined for chordal graphs.

## 166

Reliability, T-Optimal Graphs, and the Multigraph Conjecture
Nathan Kahl, Stevens Institute of Technology
A central question in network reliability theory is the network augmentation problem: For $\boldsymbol{G} \operatorname{Er}(\mathrm{n}, \mathrm{m})$ fixed, what $\boldsymbol{H} \operatorname{Er}(\mathrm{n}, \mathrm{m}+\boldsymbol{k})$ such that $\boldsymbol{G} \subset \boldsymbol{H}$ is t-optimal, that is, maximizes the tree number $t(H)$ ? In the network synthesis problem, where $G$ is the empty graph, it is conjectured that all t-optimal graphs are simple. We domonstrate that, in the general network augmentation case, there exists an infinite class of non-empty $G$ for which the resulting t-optimal augmentations are multigraphs. This conclusion has ramifications for future attempts to prove or disprove the multigraph conjecture for the network synthesis problem.

168 Lattice Polynomials
Dave Hough and Louis Shapiro*, Howard University
Consider lattice paths from $(0,0)$ to ( $2 \mathrm{n}, \mathrm{n}$ ) using unit E and N steps, with the restriction that the paths never go above the line $y=x / 2$. We also want to give a weight of x to each step N with an odd abscissa. The number of weighted paths from $(0,0)$ to $(a, b)$ is then a polynomial in $x$ and these are the lattice polynomials. We will examine the connections of these polynomials to descent polynomials, to even trees, and to the Riordan group. We will also pose some questions about poset structure.

Generalised Irredundance in Graphs: Nordhaus-Gaddum Bounds
E.J. Cockayne and S. Finbow ${ }^{\star}$, University of Victoria

For each vertex $s$ of the vertex subset $\boldsymbol{S}$ of a simple graph $\boldsymbol{G}$ we define Boolean variables $p=p(\mathrm{~s}, S), q=q(\mathrm{~s}, S)$ and $r=r(\mathrm{~s}, S)$ which measure existence of three kinds of private neighbours of $s$ with respect to $S$ ( $S$ - pns). A 3-variable Boolean function $f=\boldsymbol{f}\left(\boldsymbol{p}, \boldsymbol{q}, r\right.$ ) $\mathrm{m}_{\mathrm{a}}$ be considered as a compound existence property of $\boldsymbol{S}$ - pns The subset $\boldsymbol{S}$ is called an $/$-set of $G$ if/ $=1$ foe all sES and the class of /-sets of $G$ is denoted by $n_{1}(G)$. Only 64 Boolean functions f can produce different classes $\mathrm{n},(G)$. Special cases of /-sets include the independent sets, irredundant sets, open irredundant sets and CO-irredundant sets of $G$
For each of the 64 functions $\boldsymbol{f}$ we establish sharp Nordhaus-Gaddum type bounds on the maximum cardinality of an /-set.
Keywords: graph, generalised irredundance, Nordhaus-Gaddum

## 170 Nonmagic and K -nonmagic Graphs

Gennady Bachman and Ebrahim Salehi*, University of Nevada Las Vegas
Given an abelian group $A$, a graph $G=(V, E)$ is said to be A-magic if there exists a labeling $l: E(G)$-> $A-\{\mathrm{O}\}$ such that the induced vertex labeling $z^{+}: V(G)->A$ defined by

$$
\left.z^{+}(v)=u E N(v) 2\right)(u v)
$$

is a constant map. A graph $G$ is said to be non-magic if for any abelian group A, it is not A-magic. Also, a Z-magic graph $G$ is said to be K nonmagic if $G$ is not $Z_{h}$-magic for all $h=1,2,3, \ldots \mathrm{~K}$. In this paper, we will introduce a few classes of non-magic graphs. Then we will focus on the groups $\mathrm{Z}_{\mathrm{h}}(h \mathrm{E} N)$ and will investigate certain K-nonmagic graphs.

171 Competition chromatic numbers of graphs Matt Walsh, Indiana-Purdue University Fort $\mathrm{W}_{\mathrm{a}}$ ne

The colouring game is played on a fixed graph $G$ The two players take turns selected a vertex to be coloured next; that vertex is given the lowestnumbered colour that preserves propriety. One player's goal is to maximise the number of colours used, the other's to minimise that number. Analysing rational $\mathrm{pl}_{\mathrm{ay}}$ in this game gives rise to the two competition chromatic numbers, one for when each player chooses first. In this talk I compute values for the parameter for several families of graphs.
Keywords:combinatorial games, graph colouring
172 Noncrossing Trees
Dave Hough* and Louis Shapiro, Howard University
Noncrossing trees are trees formed by arranging n points around a circle and using these points as the vertices of a tree where the edges do not cross. The number of such trees is the ternary number, $\mathrm{T}_{\mathrm{n}}=\overline{2 \mathbf{2 n}}-(3:)$. We refine this count by considering number of descents and degree at the root. We prove that the numbers involved give an element of the Riordan group with polynomial entries and we give some applications to other tree counting problems.

173 A New Look at Hamiltonian Walks I
Gary Chartrand ${ }^{*}$, Todd Thomas, Ping Zhang, Western Michigan University; and Varaporn Saenpholphat, Srinakharinwirot University

A Hamiltonian walk in a connected graph $G$ is a closed spanning walk of minimum length in $G$. This concept was introduced by S. E. Goodman and S. T. Hedetniemi. In this talk, we revisit this concept and take a new look at it.

Keywords: Hamiltonian cycle, Hamiltonian walk

## 174 Graceful Graphs and Graceful Labelings: Two

Mathematical Programming Formulations and Some Other New

## Results

Timothy A. Red!, Rice University
Given a graph $G$ consisting of vertices and edges, a vertex labeling of $G$ is an assignment $f$ of labels to the vertices of $G$ that produces for each edge $x y$ a label depending on the vertex labels $f(x)$ and $f(y)$. A vertex labeling $f$ is called a graceful labeling of a graph $G$ with $e$ edges if $f$ is an injection from the vertices of $G$ to the set $\{0,1, \ldots, e\}$ such that when each edge $x y$ is assigned the label $\operatorname{lf}(\mathrm{x})-\mathrm{f}(\mathrm{y}) \mathrm{I}$ the resulting edge labels are distinct. A graph $G$ is called graceful if there exists a graceful labeling of $G$. We present two mathematical programming formulations of the graceful labeling problem (first as an integer programming problem, second as a constraint programming problem), along with some new results on the gracefulness of three classes of graphs: generalized Petersen graphs $\mathrm{P}(\mathrm{n}, \mathrm{k})$, double cones $\mathrm{C}_{\mathrm{n}}+\overline{K 2}$, and product graphs of the form $K 4 \times \mathrm{P}_{\mathrm{n}}$.

Keywords: graceful labeling, graceful graph, generalized Petersen graph, double cone, $\mathrm{K}_{4} \times \mathrm{P}_{\mathrm{n}}$, integer programming, constraint programming

175 Ramsey Numbers for Circulant Colorings
Stefan Krause, Technische Universitat Braunschweig

A 2-coloring of the diagonals (and sides) of a convex n -gon is called a circulant 2-coloring of $\mathrm{K}_{\mathrm{n}}$ if all diagonals of the same length have the same color. For graphs $\boldsymbol{G}$ and $\boldsymbol{H}$, the circulant Ramsey number $\boldsymbol{R} \boldsymbol{C}_{l}(\boldsymbol{G}, \boldsymbol{H})$ is the largest number such that for every $n<\boldsymbol{R} \boldsymbol{C}_{l}$ a circulant 2-coloring without G in the first and H in the second color exists. The circulant Ramsey number $\boldsymbol{R} \boldsymbol{C}_{\mathbf{2}}(\boldsymbol{G}, \mathrm{H})$ is the smallest number such that for $n 2_{2} \boldsymbol{R} \boldsymbol{C}_{2}$ every circulant 2-coloring of $\mathrm{K}_{\mathrm{n}}$ contains G of the first or H of the second color. The numbers $\boldsymbol{R} \boldsymbol{C}_{1}$ and $\boldsymbol{R} \boldsymbol{C}_{2}$ are determined for paths and cycles, for small complete graphs and for graphs with up to five vertices. As a by-product the new lower bounds $R\left(K_{4}, \mathrm{~K} 10\right)$ 2: 92 and $R\left(K_{5}, \mathrm{~K}_{8}\right)$ 2: 101 for the classical Ramsey numbers are determined instead of 80 and 95 respectively.

176k-Trees, Catalan Identities and Applications - Part I Mahendra Jani* and Melkamu Zeleke, William Paterson University

We obtain analogs of the list of seventeen Catalan generating function identities of Deutsch and Shapiro for k-trees, and use these identities to enumerate k-trees with various statistics.

177 A New Look at Hamiltonian Walks II
Gary Chartrand, Todd Thomas॰, Ping Zhang, Western Michigan University; and Varaporn Saenpholphat, Srinakharinwirot University

This is a continuation of the preceding talk where additional results and problems are presented.

Keywords: Hamiltonian cycle, Hamiltonian walk

## 178 Edge Labeling and Deletion Games

B. Hartnell*, Saint Mary's University; and D. Rall, Furman University

For a given graph G, let Ebe the number of edges. Consider a labeling of the edges which is a one-to-one mapping of the set of integers $\{1,2, \ldots, \mathrm{E}\}$ onto the edges of the graph, with the property that for every vertex the sum of the labels assigned to all edges incident with it is some constant $k$. Although work to date has been on deciding which graphs admit such a labeling (called a vertex - magic edge labeling), we shall consider a game based on this concept. For certain families of graphs a winning strategy will be described. A related edge deletion game will also be introduced.

Keywords: labeling, game

For given graphs $G, H$ the rainbow number $r b(G, H)$ is the smallest number m of colours such that if we colour the edges of $G$ with at least m different colours, then there is always a totally multicoloured or rainbow copy of $H$. We will list the known rainbow numbers if $G$ is the complete graph and report about recent progress on the conjecture of Erdos, Simonovits and S6s on the rainbow numbers $r b\left(K_{n}, C_{k}\right)$ for cycles.
For $H=k K_{2}$ the values $\operatorname{ext}\left(n, k K_{2}\right)$ (the maximum number of edges in a graph containing no matching with k edges) have been determined by Erdos and Gallai.
Theorem 1 (Erdos and Gallai)
Let $G$ be a graph of order $n$ and with m edges. If $\mathrm{m}>$ $\max \left\{\left\{\backslash^{1}\right),(2+(\mathrm{k}-: \quad-\mathrm{k}+\mathrm{i})\}=\operatorname{ext}(n, k K 2)\right.$, then $k K_{2}>\mathrm{C} G$.
Theorem 2
$r b\left(n, k K_{2}\right) \quad \operatorname{ext}(n,(k-1) \mathrm{K} 2)+2$ for all $k \quad 2$ with equality if $2 \quad k \quad 4$ or $k \quad \gg 5$ and $n \quad 3 k+3$.
Finally, new results on the rainbow numbers $r b\left(Q_{n}\right.$, Q2) for the hypercube $Q_{n}$ will be presented.

180
80 k-Trees, Catalan Identities and Applications - Part II Mahendra Jani, and Melkamu Zeleke*, William Paterson University

We obtain analogs of the list of seventeen Catalan generating function identities of Deutsch and Shapiro for k-trees, and use these identities to enumerate k -trees with various statistics.

181 Generating Cycles in the Digraph $P(n, k)$ : An Algorithm
A. Gregory Starling*, Jacob Kier, University of Arkansas; and Joseph B. Klerlein, Western Carolina University

The digraph $\boldsymbol{P}(n, \boldsymbol{k})$ has vertex set that is the set of permutations of length $\boldsymbol{k}$ on $\{1,2,3, \ldots, \mathrm{n}\}$ and directed edge set determined by the following adjacency rules. There is an edge directed from the permutation nln2 ...nk to $\mathrm{n} 2 \ldots \mathrm{nkm}$ if and only if m is not one of the integers $\mathrm{n} 1, \mathrm{n} 2, \ldots, \mathrm{nk}$. At the CGTC conference in 2002, Carr, Starling, Sportsman and Klerlein presented a paper on the hamiltonicity of $P(n, 2)$ titled On the Digraph $P(n, k)$. In that paper an algorithm for producing Hamilton cycles was presented. This paper develops another algorithm for generating cycles of all possible lengths in $P(n, k)$.

Keywords: Digraph, Hamilton cycle, permutation

## 182 on The Edge-magic Cubic Graphs and Multigraphs

Sin-Min Lee and Ling Wang*, San Jose State University; and Yihui Wen, Suzhou Science and Technology College

Let $G$ be a ( $\mathrm{p}, \mathrm{q}$ ) graph in which the edges are labeled $1,2,3, \ldots \mathrm{q}$ so that the vertex sums are constant, $\bmod p$, then $G$ is called edge-magic. It is conjectured by Lee [7] that the cubic simple graph with $\mathrm{p}=2(\bmod 4)$ vertices is edge-magic. Lee and Shiu [16] showed that the conjecture is not true for disconnected cubic graphs and multigraphs. Some new edge-magic cubic multigraphs are exhibit in this paper.

## 183 on Extremal Rankings of Graphs Robert E. Jamison, Clemson University

A labeling of the vertices of a (finite) graph with positive integers is called a ranking provided any path joining two vertices of the same rank $r$ contains a vertex whose rank is strictly greater than $r$. In particular, rankings are proper colorings. The largest value taken on by a ranking is called its norm. The rankings on a graph $G$ form a partially ordered set $\operatorname{Rank}(G)$ under the pointwise order. Laskar and Pilone have investigated minimal rankings of graphs. Obviously, there are no maximal rankings in the usual sense since adding 1 to each value produces a larger ranking. We will say that a ranking is globally maximal iff it is maximal among all rankings with the same norm. A ranking $g$ is a lift of a ranking $f$ iff $f$ and $g$ agree on all vertices both one and there $g$ takes the larger value. A ranking is locally maximal iff it does not have a lift with the same norm. It will be shown that locally maximal rankings are in fact globally maximal. This gives insight into the structure of the ordered set $\operatorname{Rank}(G)$.

Keywords: graph, labeling, coloring, ranking, order set

$$
\begin{aligned}
& 184 \text { тwo more Fibonacci walks } \\
& \text { Asamoah Nkwanta, Morgan State University }
\end{aligned}
$$

We show that two classes of random walks are counted by the Fibonacci numbers. The calculations are performed by using a technique which involves Riordan matrices and the Riordan Group. The average heights of the walks are also computed.

185 on a Conjecture of Bermond David A. Pike, Memorial University of Newfoundland

A regular graph $G$ is said to be Hamilton decomposable if its edge set can be partitioned into Hamilton cycles (plus a !-factor if $G$ is regular of odd degree). Bermond has conjectured that for every Hamilton decomposable graph $G$ the line graph $L(G)$ of $G$ is also Hamilton decomposable.
For bipartite graphs, Bermond's conjecture remains unsettled when $G$ is regular of degree $8=2(\bmod 4)$. We will present some new results, one of which is that Bermond's conjecture holds in cases where $G$ is bipartite, has degree $8=2(\bmod 4)$, and has connectivity $k=2$

186 on Super Edge-magicness of Chain Graphs whose Blocks are Complete Graphs
Sin-Min Lee, San Jose State University; and Yung-Chin (Jack) Wang*, Stanford University

Barrientos [2] defines a chain graph as one with blocks B1, $B 2, i K, B m$ such that for every $\mathrm{i}, B$; and $\mathrm{B} ;+1$ have a common vertex in such a way that the block cut-point graph is a path.
A $(p, q)$ graph $G$ is total edge-magic if there exits a bijection $f: V U E+\{1.2, \mathrm{jK}, \mathrm{p}+\mathrm{q}\}$ such that for each $e=(u, v)$ in $E$, we have $\mathrm{f}(\mathrm{u})+f(e)+f(v)$ is a constant. A total edge-magic graph is called a super edge-magic if $f(V(G))=\{1,2, \mathrm{j} K, \mathrm{p}\}$. In this paper the super edge-magic properties of chain graphs whose blocks are complete graphs are studied.

Keywords: Total edge-magic, super edge-magic, sequential, harmonious, cordial, consecutive labeling, chain graph

187 on the chromatic number of intersection graphs of convex sets in the plane
Seog-Jin Kim*, Alexandr Kostochka, and Kittikorn Nakprasit, University of Illinois at Urbana-Champaign

Let $G$ be the intersection graph of a finite family of convex sets obtained by translations of a fixed convex set in the plane. We show that every such graph with clique number $k$ is $(3 k-3)$-degenerate. This bound is sharp. As a consequence, we derive that $G$ is $(3 k-2)$-colorable. We show also that the chromatic number of every intersection graph $H$ of a family of homothetic copies of a fixed convex set in the plane with clique number $\boldsymbol{k}$ is at most $6 k-6$.

Keywords: Intersection graph, convex sets, chromatic number

## 188 An Algebraic Approach to Counting Random Walks in Quadrants and Octants <br> Heinrich Niederhausen, Florida Atlantic University

We consider random walks restricted to shifted quadrants or octants, with step sets $\{--+, j,<-, l\}$ or $\{/ ', 1 /, " "-\}$ - They are enumerated by endpoint, number of steps, and number of contacts (adsorptions) with the boundanes. For example, there are $\frac{3\{\mathrm{n}+1)}{\left.\left.(\mathrm{k}+1-\mathrm{d}) \mathrm{k}_{\mathrm{k}}+1\right)^{(2 \mathrm{k}+1}\right)}(\mathrm{d}+\mathrm{l})\left(2_{k}^{\mathrm{k}-\mathrm{d}}\right)\left(\begin{array}{c}\left({ }_{k} \mathrm{k}-\boldsymbol{n}\right.\end{array}\right)_{+}$ $\frac{(2 n+d+2)(n d-k-H d)}{2(k+1)(2 k+1)}\left(\left(_{k-n}^{k+2}\right)\binom{2 k+1-d}{k}+\frac{2 n+d+2}{(k+n+2)\left({ }^{k}+1\right)}\binom{2 k}{k}\binom{2 k+1-d}{k-n-d}\right.$ walks from the origin to ( $n, n$ ) in the first octant taking 2 k steps from the step set $\{-+, j,<-, l\}$ and visiting the diagonal $d$ times. We use a linear operator approach to derive the contact generating functions.

Keywords: Random walks, adsorption

189 Hamilton Paths in Graphs Whose Vertices are Graphs Krystyna T. Balinska, The Poznan University of Technology; Michael L. Gargano and Louis V. Quintas*, Pace University

Let $U(n, \boldsymbol{J})$ denote the graph with vertex set the set of unlabeled graphs of order $n$ and having no vertex of degree greater than $\boldsymbol{f}$. Two vertices $H$ and $G$ of $U(n, \boldsymbol{f})$ are adjacent if and only if $H$ and $G$ differ (up to isomorphism) by exactly one edge. The problem of determining the values of $n$ and f for which $U(n, \boldsymbol{J})$ contains a Hamilton path is investigated. The only known non-trivial case for which a Hamilton path exists occurs when $n=5$ and $\mathrm{f}=3$. Results are given for f 3 , with $n \mathrm{f}+1$ and $(n, \boldsymbol{J}) 1$ $(5,3)$ that eliminate candidates for the existence of. a Hamilton path in a $U(n, f)$. The complete solution of this problem is unresolved.

Keywords: Hamilton path, bounded vertex degree

## 190

191 On Vertex Coloring Simple Genetic Digraphs
Geir Agnarsson•, George Mason University; and Agust Egilsson, U. of
California at Berkeley
Digraphs are frequently used in the life sciences to encode information. A new and an efficient querying of relations referencing an acyclic digraph $G$ employs a proper vertex coloring of $G$, where for each vertex $u$ of $G$, all the descendants $\mathrm{D}[\mathrm{u}]$ of $u$ in G , including $u$ itself, receive distinct colors. To determine the corresponding chromatic number $\mathrm{X}_{\mathrm{d}}(G)$, even in some very simple cases, reduces to known n.::mtrivial vertex coloring problems of simple undirected graphs.
By viewing $G$ as a partially ordered set (poset), we present some general upper and lower bounds for $\mathrm{X}_{\mathrm{d}}(\mathrm{G})$ for various classes of acyclic digraphs, and indicate when such vertex colorings can be done in a greedy fashion. Some open problems will be presented.

Keywords: Vertex coloring, chromatic number, digraph, poset
192
( Lynnell S. Matthews, University of Pennsylvania

Connections between the determinants of Hankel matrices and disjoint path systems have been analyzed by various authors such as Lindstrom, Viennot, Gessel, Mays and Wojeciechowski. This research is motivated by this relationship and presents preliminary results for the total number of Disjoint Path Systems formed using Motzkin paths. These initial results reveal an apparent connection with Fibonacci and Lucas numbers.

Keywords: Motzkin Paths, Hankel matrices, Hankel determinants, Disjoint Path Systems, Fibonacci Numbers, Lucas Numbers

193 Enumeration of Hamilton Paths in Cayley Digraphs Stephen Curran, University of Pittsburgh at Johnstown

We enumerate the Hamilton paths with initial vertex 1 in the Cayley digraph $\operatorname{Cay}(x, y: \boldsymbol{G})$ on a metacyclic group $\boldsymbol{G}$ in which $\left(y^{-1} \mathrm{x}\right)$ is a normal subgroup of $G$. Second, let $m$ and $n$ be positive integers such that $n$ is odd and $\operatorname{gcd}(\mathrm{m}, \mathrm{n}\}=1$. Let G be the semidirect product of cyclic groups given by $G=\left(x, y: x^{8 m}=I, y^{2} n=1\right.$, and $\left.y x y-{ }^{1}=x^{4 \mathrm{~m}+1}\right)$. Then the number of Hamilton paths in $\operatorname{Cay}(x, y: G$ ) (with initial vertex $1\}$ is two more than the number of visible lattice points in the interior of the triangle whose vertices are $(0,0),(2 n, 0)$, and $(n+2 m, 4 m\}$ plus the number of visible lattice points in the interior of the triangle whose vertices are $(0,0),(n, 4 m)$, and $(0, S m)$. Third, let $m$ and $n$ be positive integers such that $n$ is odd. Let $G$ be the semidirect product of cyclic groups given by $\mathrm{G}=\left(\mathrm{x}, \mathrm{y}: x^{4 \mathrm{~m}}=I, y^{2 \mathrm{n}}=1\right.$, and $y x y-{ }^{1}=\mathrm{x}^{2 \mathrm{~m}}-1$ ). Then the number of Hamilton paths in $\operatorname{Cay}(x, y: G$ ) (with initial vertex 1$\}$ is $(3 m-1\} n+m l(n+1\} / 3 j+1$.

Keywords: Cayley digraph, Hamilton path, finite group

## 194

## 195 Edge-Color Balance in $K_{n}$

Dean G. Hoffman, Sally A. Clark*; Auburn University
In this talk, we define a four-fold notion of (degree) balance with respect to a vertex partition in a graph, and then address the question,"When is an edge-colored graph H the homomorphic image of an edge-colored copy of $K_{n}$, in which each color class exhibits our four-fold balance?"

Keywords: graphs, edge-colorings, amalgamations

## 196 on the Topology of The Hamming Distance between Set Systems <br> Junichiro Fukuyama, Indiana State University

Let $n \mathrm{E} \mathrm{z}^{+}$and $m \mathrm{E}[1, \mathrm{n}]$. An $(n, m)$-set is a subset of $[\mathrm{n}]$ of size $\operatorname{lmJ}$. An $(n, m)$-set system $\boldsymbol{U}$ is a non-empty set of ( $n, \mathrm{~m}\}$-sets. Its sparsity $K(U)$ is defined by

$$
K(U)=-\ln \frac{\mathbb{U}}{(,,)}
$$

The Hamming Distance $8\left(t_{1}, t_{2}\right)$ between ( $n, m$ )-sets $t$; is the size of the symmetric difference between $t$;. It is naturally extended to the Hamming distance $8\left(U_{l}, \mathrm{U}_{2}\right)$ between two ( $\mathrm{n}, m$ )-set systems U ,, which is the minimum value of $8\left(t_{1}, t_{2}\right)$ over all t ; EU ;. It has been shown that

$$
8\left(U_{l}, U 2\right)=0\left(i \overline{i o} ; \overline{\mathrm{m}}+\log ^{3} \mathrm{n}\right)
$$

if $K(U ;)=\mathrm{O}\left(\mathrm{m}^{1}-\mathrm{E}\right\}$ for some fixed ${ }_{E} \mathrm{E}^{+}$. Here, mis regarded as a function $\mathrm{z}^{+}-->[1, \mathrm{n}]$ of $n$. For a given positive real number $\boldsymbol{d}$, the ( $\left.\mathrm{n}, \mathrm{m}\right\}$-set $\boldsymbol{t}_{1}$ is called ad-neighbor of $t_{2}$ if $8\left(t_{1}, t_{2}\right)=d$. The set $<l^{\prime \prime}\left(t_{1}, d\right)$ of the d-neighbor of $t_{l}$ is called the ball of radius $d$ about $\mathrm{t}_{l}$. It is significant to investigate a property of a ball to understand the topology of the Hamming distance. We will show that for any given reasonably dense $(n, m)$-set system $\boldsymbol{U}$, the balls of a small radius about many $\mathrm{t} \mathrm{E} U$ are filled by certain number of elements in $\boldsymbol{U}$. More precisely, we prove that there exists $\boldsymbol{U}^{\prime} \quad \boldsymbol{U}$ such that

$$
K\left(U^{\prime}\right\}=K(U)+O(l\} \text { and'v'tE } U^{\prime}, \frac{I G(t}{i \omega} \frac{d) n \mathrm{UI}}{i \cdot(i, u) /} 2:: \exp (-O(K(U))),
$$

whenever $\boldsymbol{d}, K(U)=o(m)$.

## 198

199 Grundy Coloring of Chessboard Graphs
Elizabeth Duea, Kim Overbay, Casey Parks*, Jill Rhyne, Appalachian State University

We look at the Grundy number of the king's graph, the bishop's graph, the knight's graph, the rook's graph, and the queen's graph on nom chessboards.

Keywords: Grundy coloring, chessboard graphs

## 200 The Determinant Sequence of Hankel Matrices Barbara Tankersley, Howard University

Let $\left\{a_{1}, \mathrm{a} 2, \ldots\right\}$ be a sequence. Define a Hankel Matrix of the sequence to be the infinite matrix, $\mathrm{H}=\left(\mathrm{h}_{\mathrm{i}_{\mathrm{j}}}\right) i, j>0$ such that $\mathrm{h}_{\mathrm{i}_{\mathrm{j}}}=\mathrm{ai}+\mathrm{i}$. We want to find the determinant sequence of the submatrix $\mathrm{H}_{\mathrm{m}} \mathrm{m}$. We calculated the determinant sequence of two classes of sequences.

214A Lexicographical Shelling for a New Lattice of Partitions Eric Gottlieb, Rhodes College

Let $\boldsymbol{f}:\{1, \ldots, \mathrm{n}\}+$ be a nondecreasing function. We define a sublattice II! of the lattice of partitions. II! generalizes some previously studied lattices, including the k-equal partition lattice, the $h$, k-equal signed partition lattice, and the $h$, k-equal Dowling lattice $\mathrm{Q} \cdot{ }^{\mathrm{k}}(\mathrm{e}\}$. We show that II! is lexicographically shellable and draw conclusions about its homology.

An array $T$ with two levels, m constraints, and $N$ runs (treatmentcombinations) is merely a matrix of size ( $\mathrm{m} \times N$ ) and with two symbols (say, 0 and 1). T is called an orthogonal array (O-array) of strength $t$ $(\$ \mathrm{~m}\}$ if in every $(\mathbf{t} \times \mathrm{N})$ submatrix T of $T$, every vector of weight i ( $0 \$ \mathrm{i} \$ \mathrm{t}$ the weight of a vector is the number of l 's in it) appears a constant number $\mu$ (say) times (it is independent of i\}. $T$ is called a Balanced array (B-array) if each vector of weight i appears $I$; times ( $\mathrm{i}=0,1,2, \ldots \mathrm{t}$ ). We will present some new results on the existence of some B-arrays, discuss their relationships with other combinatorial structures, and compare these results with the previous results.

Keywords: balanced array, orthogonal array, strength of an array, constraints of an array

> 216
> A Family of Well-Covered Graphs with Unimodal Independence Polynomials
> Vadim E. Levit* and Eugen Mandrescu, Holon Academic Institute of Tcchnoloy

If sk denotes the number of stable $s \mathrm{~d} s$ of cardinality $k$ in graph $G$, and a -(G) is the size of a maximum stable set, then $\gg \mathrm{J}(\mathrm{G} ; \mathrm{x}\}={ }_{k=0}^{\mathrm{a}(\mathrm{G})} \mathrm{Skx}^{\mathrm{k}}$ is the independence polynornial of $G$ (I. Gutman and F. Harary, 1983). J. I. Brown, K. Dilcher and R. J. Nowakowski (2000\} conjectured that the independence polynomial of a well - covered graph $G$ (i.e., a graph whose all maximal independent sets arc of the same size\} is unimodal, that is, there exists some k such that so $\$ s_{1} \$ \ldots \$ s_{k-1} \$ s_{k} 2^{\prime} . \gg$ sk+I 2 ... 2: Sa(G). T. S. Michael and N. Traves (2001\} provided examples of well-covered graphs whose independence polynomials are not unimodal.
Under certain conditions, any $w \ll 11$-covenxl graph equals $G^{+}$for some $G \gg$ , where $c \cdot$ is the graph ohtaincd $\mathrm{f}^{\text {rom }} G$ by appending a single pendant edge to each vertex of $G$ (A. Finbow, 13 Hartnell and R. J. Nowakowski, 1993). Y. Alavi, P. J. Malclc, A. J. Schwenk and P. Erdos (1987) asked whether for trees the independence polynomial is unimodal. V. E. Levit and E. Mandrescu (2002\} showed that the independence polynomials of some well-covered trees (e.g., P ;., $\mathrm{I}<\mathrm{j}$ '" where $\mathrm{P}_{l l}$ is a path on $n$ vertices and $\mathrm{K}_{1, \mathbf{n}}$ is then-star graph\} arc uni 10dal.
In this paper we show that for any graph $G$ with a $(G) \$ 4$, the independence polynomial of $G^{\bullet}$ is unimodal.

## 218

 Interesting Sequences in Star Graphs Ke Qiu, Acadia UniversityWe study the problem of finding the number of nodes whose distance from the identity node is $i$ in an $n$-star graph $S_{n}$, for $i=1,2, \ldots, D\left(S_{n}\right)$, where $D\left(S_{n}\right)$ is the diameter of the graph $S_{n}$. In doing so, we find several interesting sequences that can be found in the Sloant book (The Encyclopedia of Integer Sequences) and its on-line version.

Extended 5-Cycle Systems having a Prescribed Number of Idempotent Elements
Michael Raines, Western Michigan University
Let $\mathrm{I}<, \mathrm{t}$ denote the complete graph of order n in which exactly one loop appears at each vertex. An extended 5 -cycle system of order n is a partition of the edges and loops of $\mathrm{I}<, \mathrm{t}$ into loops (also called idempotent elements), 2-tadpoles (2-paths with a loop at exactly one of the end vertices), and 5 -cycles. In this talk, we give necessary and sufficient conditions for the existence of extended 5 -cyrle systems in which the number of idempotent elements is specified.

Keywords: cxt, ndcd cycle syst1)111, SJH ctrnrn, idempotent element

## 220 on line graphs with a unique set of cliques which covers all edges <br> Tao-Ming Wang, Tung Hai University

A finite simple graph $G$ is uniquely intersectable if up to isomorphisms there is only one set representation for $G$. We know that the study of edge clique covers is closely related to that of unique intersectability for graphs, therefore we consider the problem of characterizing the class of graphs which have a unique set of cliques covering all the edges. In 1980, M. Golumbic characterized trivially perfect graphs as P4-free and C4-free. We showed in 1997 that a graph is uniquely intersectable if and only if it is locally P4-free and C4-frec. In this talk, we characterized the line graphs with a unique set of maximal cliques which covers all edges. Some related results are also mentioned.

Keywords: edge clique cover, intersection graphs, uniquely intersectable graphs

## 222 Visibility Graphs on the Sphere

 Jay Bagga, John Emert* ${ }^{*}$ J. Michael McGrew, Ball State UniversityGiven a finite collection of line segments in the plane, their segment endpoint visibility graph (SEVG) has the endpoints of line segments as vertices, with two adjacent if the line segment joining them does not intersect any of the given segments in the collection. The segments in the collection are also considered as edges. Visibility graphs arise out of the study of such problems as the minimum number of security cameras necessary to guard an art gallery, or the optimum path for a robot to take through a set of obstacles. SEVGs and other types of visibility graphs in the plane have been studied extensively. The work described in this presentation extends this study to the case of visibility graphs on the sphere. In particular, we describe some of the insights we have gained using a software package called VGraph, which was developed at Ball State University specifically to enable us to visualize spherical visibility graphs. We give some properties of visibility graphs and present a number of open problems.

Keywords: Visibility Graph, Computational Geometry

223 A note on Hereditary Double Bound Graphs H Era, S. Iwai, K Ogawa, and M Tsuchiya*, Tokai University

In this talk, we consider hereditary double bound graphs. The double bound graph (DB-graph) of $\mathrm{P}=\left(\mathrm{X}, \mathrm{S}_{\mathrm{P}}\right)$ is the graph $\mathrm{DB}(\mathrm{P})=(\mathrm{X}, \mathrm{EDB}(\mathrm{P}))$, where $x y E_{\mathrm{on}_{(\mathrm{P}},}$ if and only if $\mathrm{x} \neq \mathrm{f}$ and there exist $\mathrm{m}, \mathrm{n} \mathrm{EX}$ such that $n-5 p u, v 5 p m$. A graph G is a hereditary double bound graphs if every induced subgraph is a double bound graph. We obtain a characterization of hereditary double bound graphs. Namely a double bound graph G is a hereditary double bound graph if and only if maximal posets of $\mathrm{PDa}(\mathrm{G})$ are ND-posets.

Keywords: double hound graph, edge clique cover

224 On a subclass of well-covered graphs
Rommel Barbosa*, Universidadc Federal de Mato Grosso and Domingos M Cardoso, Univcrsidadc de Avciro

A graph $G$ is well-covered if all maximal independent sets of vertices in $G$ have the same cardinality. A well-covered graph $G$ is $p$-regular-stable if for all maximal independent sets of vertices I in $V(G), \mathrm{INc}(\mathrm{v}) \mathrm{n} I I=p$, for $W_{\mathrm{v}} \mathrm{E} V(G) \backslash J$. Here we show some ways to build graphs with this property.

Keywords: Well-covered graphs, wgular-stable graphs, maximal independent sets of vertices

## 225

## 226 Unit Bar-visibility Graphs

Alice M Dean* and Natalia Veytsel, Skidmore College

A bar-visibility layout of a graph uses horizontal bars in the plane to represent vertices and unobstructed vertical visibility lines to represent edges. Graphs having such a layout have been completely characterized. $\mathbf{W}$ present results on unit bar-visibility graphs (UBVGs), which are those graphs having a bar-visibility layout in which all the bars have equal length. A variety of results are established, including necessary conditions for UBVGs and categorizations of some standard families of graphs. In particular, we give necessary and sufficient conditions for a tree to be a UBVG, and also for a circular ladder to be a UBVG. In addition, we determine whether certain outerplanar graphs are UBVGs by using properties of their dual graphs.

227 Independence, Radius and Path Coverings in Trees
Ermelinda DeLa Vina and Dill Waller*, University of Houston-Downtown

We discuss two conjectures of Graffiti generalizing the theorem that the independence number of a simple, connected graph is not less than its radius. The first of these conjectures states that the independence number . is not less than the radius plus the path covering number minus one. We demonstrate a family of counterexamples to this conjecture, which we also use to partially resolve one of Graffiti's follow-up conjectures, namely that the independence number is not less than the floor of half the radius plus the path covering number. In particular, we prove a stronger version of this inequality for trees.

Keywords: independence number, radius, path covering number, Graffiti
228 Characterizing a Subclass of Well-Covered Graphs Erika L.C. King, Hobart and William Smith Colleges

A graph $G$ is said to be well-covered if every maximal independent set of G is of the same size. It has been shown that characterizing well-covered graphs is a co-NP-complete problem. In an effort to characterize some of these graphs the author has focused on well-dominated graphs, a class of graphs Finbow, Hartnell and Nowakowski have proved to be a subclass of the well-covered graphs. A set of vertices $D$ is said to be a dominating set of a graph $G$ if every vertex of $G$ is either in $D$ or adjacent to a vertex in $D$. A graph is well-dominated if every minimal dominating set is minlmum. In an attempt to characterize the 3-connected, claw-free, planar, well-dominated graphs, the author was able to extend the result to a characterization of all well-covered graphs with these three properties. In this talk we will describe this characterization and outline the arguments of the proof.

Keywords: independence number, well-covered graphs, domination number, well-dominated graphs, planar graphs, claw-free graphs, 3-connected graphs

# 230 Classifying Trees with Edge Deleted Central Appendage Number 2 

Moghadam*, Eroh, Koker, Winters, University of Wisconsin, Oshkosh; and Stalder, University of Wisconsin, Waukesha

For a vertex v in a 2-edge-connected graph G, we define the edge-deleted eccentricity of v as the maximum eccentricity of v in G-e over all edges e of $G$. The edge-deleted center of $G$ is the subgraph induced by those vertices of $G$ having minimum edge-deleted eccentricity. The edge-deleted central appendage number of a graph $G$ is the minimum difference $I V(H) I$ - IV(G)I over all graphs H where the edge-deleted center of His isomorphic to G. In this talk, we determine the edge-deleted central appendage number of all trees and give necessary and sufficient conditions for a tree to have edge-deleted central appendage number 2.

The abdiff-tolerance edge clique cover number of a graph, $G=(V, E)$, is the size of the smallest collection of subsets, $S_{1}$, of $V$ such that (1) it is possible to assign to each vertex $v$ an integer $t$, and (2) and edge uv is in $G$ if and only if u and v appear together in at least $\mathrm{It}_{\text {, }}-\mathrm{t}_{\mathrm{u}}$ ! of the sets $S_{l}$. This paper places bounds on the asymptotic value of the abdiff-tolerance edge clique cover number of complete binary trees, $\mathrm{B}_{\mathrm{n}}$.

## 232 A generalized graph partitioning problem <br> P. Luo, Y. Peng, and C. Zhao, Indiana State University

This paper considers problems of the following type: given a graph $\mathrm{G}=$ $(V, E)$, sets $U p=\left\{u 1 p, u 2 p, \ldots, u k_{p}\right\}$ for $1 \$ p \$ r$, where each $U_{p}$ has $k$ specified vertices or terminals in $G$, a positive integer weight $w(e)$ for each $e \mathrm{E} E$ and $w(e)=0$ if $e \& E$, find a minimum weighted set of edges $E^{\prime} \quad E$ such that the removal of $E^{\prime}$ disconnects the graph $G$ into $k$ parts and the vertices in each $U_{p}$ arc in different parts for $1 \$ p \$$ r. In this paper, we show that, greedy local search when applied to the problems with the specified neighborhood and started from an arbitrary poor initial configuration, will reach the average cost in polynomial time. Some approximation algorithms for solving such problems are compared. Also, a simple approximation algorithm that is guaranteed to come within a factor of $2-2 / \mathrm{k}$ of the optimal cut weight is obtained.

