TWENTY-FIFTH SOUTHEASTERN INTERNATIONAL CONFERENCE ON COMBINATORICS GRAPH THEORY COMPUTING

PROGRAM SCHEDULE

Florida Atlantic University

March 7-11, 1994

MONDAY, MARCH 7, 1994

REGISTRATION begins at 8:00 A.M. in the downstairs lobby of the University Center, where COFFEE WILL BE SERVED. GCN (left or front) and GCS are the two Halves of the Gold Coast Room. FAU Rooms 202 A and C are reached through the second floor Lounge. Room 156 is downstairs.

	GCN	GCS	202 A	202 C	156
9:00 AM	OPENING and WELCOME PROVOST OSBURN; DEANS CARRAHER AND HOLLAND				
9:30	STANTON				
10:30	COFFEE				
10:50	1 K B REID	2 HURD	3 KLERLEIN	4 Y A LAI	5 LANDMAN
11:10	6 FLETCHER	7 EVANS	8 ASHLOCK	9 WILLIAMS	10 PIEPMEYER
11:30	11 MARSHALL	12 GODBOLE	13 LIU	14 OH	15 SHREVE
11:50	16 X LU	17 LICK	18 ALSPACH	19 STIVAROS	20 KLAMROTH
12:10 PM	21 JORGENSEN	22 FRANCEL	23 van den HEUVEL	24 PEDERSEN	25 BAJNOK
12:30	LUNCH (On your own -- Cafeteria open; there are many nearby restaurants)				
2:00	GRAHAM				
3:00	COFFEE				
3:20	26 MCDOUGAL	27 CARPENTER	28 BAGGA	29 SPRAGUE	30 CABANISS
3:40	31 J HUANG	32 WAHLAU	33 G CHEN	34 H LI	35 BEALER
4:00	36 LATKA	37 P GUAN	38 WALTERS	39 FAUDREE	40 SIMOSON
4:20	41 GIMBEL	42 McNULTY	43 KINGAN	44 BOYER	45 S-M LEE
4:40	46 HORAK	47 HARPER	48 JOHNS	49 OPATRNY	50 PIGG
5:00	51 DG HOFFMAN	52 SULLIVAN	53 ELLINGHAM	54 S CHEN	55 J GUAN
5:20	56 TELLE	57 CHEROWITZO	58 HEDETNIEMI	59 SCHWENK	60 SUN
5:40	61 BLAHA	62 EBERT	63 PRITIKIN	64 GHOSHAL	65 K-Y LAI
6:30	CONFERENCE RECEPTION in the BOARD of REGENTS ROOM on the THIRD floor of the ADMINISTRATION BUILDING.				

There will be Conference transportation back to the motels at 6:05 PM, returning to the reception about 6:35. There will be transportation from the reception back to the motels.

TUESDAY, MARCH 8, 1994

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. Room 156 is downstairs. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	202-A	202-C	156
8:30 AM	66 CORDERO	67 WEST	68 PADAYACHEE	69 ROELANTS	70 HARRIS
8:50	71 SHOBE	72 HIND	73 H ZHOU	74 ATKINSON	75 H WU
9:10	76 KEY	77 BURRIS	78 KOUNTANIS	79 PERUMALLA	80 SLATER
9:30	WILF				
10:30	COFFEE				
10:50	81 LEVAN	82 LEFMANN	83 BERRY	84 GRIMALDI	85 BEEZER
11:10	86 LEWIS	87 WANTLAND	88 KAYLL	89 CA ANDERSON	90 PIAZZA
11:30	91 MONROE	92 GROWNEY	93 ALDRED	94 O'REILLY	95 GARGANO
11:50	96 TAPIA	97 H-L FU	98 HEMMINGER	99 FOULIS	100 Z CHEN
12:10 PM	101 HOUGHTON	102 BARI	103 TIPNIS	104 SOBEL	105 BOZA
12:30	LUNCH BREAK (ON YOUR OWN)				
2:00	MILLS				
3:00	COFFEE				
3:15	PLESS				
4:20	106 JOB	107 LEHMANN	108 VARMA	109 GAVLAS	110 SCHULTZ
4:40	111 TMY WANG	112 STEINER	113 DANA	114 HOCHBERG	115 WINTERS
5:00	116 SAVAGE	117 PFALTZ	118 DIANEZ	119 VOGT	120 X-Y SU
5:20	121 SQUIRE	122 BROWN	123 THOMAS	124 CARRINGTON	125 DARRAH

6:00 CONFERENCE PARTY at the home of JACK FREEMAN : 741 AZALEA ST, 395 - 7921.

CONFERENCE TRANSPORTATION will leave for the motels at 5:40. There will be transportation from the UNIVERSITY CENTER to the party at about 5:45, and from the motels at about 6:20. There will be transportation from the party back to the motels. As always, we urge car-pooling, especially with parking spaces scarce near Freeman's. It is a pleasant walk to the Freeman home, should you be adventurous.

WEDNESDAY, MARCH 9, 1994

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. Room 156 is downstairs. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	202-A	202-C	156
8:30 AM	126 MARKUS	127 CACERES	128 PLANTHOLT	129 SOLTES	130 DUNBAR
8:50	131 BRIGHAM	132 FUREDI	133 FRAUGHNAUGH	134 PIKE	135 J KNISELY
9:10	136 EGGLETON	137 BERMUDEZ	138 KRISHNAMOORTHY	139 ALAVI	140 LASKAR
9:30	DILLON				
10:30	COFFEE				
10:50	141 HARBORTH	142 PULAPAKA	143 CHARTRAND	144 JACKSON	145 RALL
11:10	146 GRONAU	147 W WALLIS	148 MACULA	149 NAIR	150 C WALLIS
11:30	151 C-Q ZHANG	152 VALDES	153 COLLINS	154 W GU	155 KNILL
11:50	156 G FAN	157 HARTSFIELD	158 SANDERS	159 HAMBURGER	160 JIA

12:15 PM **CONFERENCE PHOTOGRAPH** at the OUTDOOR STAGE. We will lead you from the lobby, if you can't find it on your own, but PLEASE PARTICIPATE!

12:30 LUNCH BREAK (ON YOUR OWN)

2:00	CHUNG				
3:00	COFFEE				
3:15	THOMPSON				
4:20	161 K-C HUANG	162 LAWSON	163 KELMANS	164 EGECIOGLU	165 DOMKE
4:40	166 MOLINA	167 R MARTIN	168 NADON	169 JAMISON	170 OUYANG
5:00	171 EL-ZANATI	172 McRAE	173 GILBERT	174 KERR	175 FISCHER
5:20	176 L CLARK	177 B YU	178 ZHENG	179 BOZOVIC	180 HAVAS

The CONFERENCE BANQUET will be The Mystery Dinner Theater at the Embassy Suites--Yamato Road, just west of I-95 at the Tri-Rail Station. There will be a cash bar, an important part of the show, at 7:00PM--we'll buy you one drink. Conference transportation will be available to the motels at 5:45. There will be transportation from the University Center to the Embassy Suites at approximately 6:00 and from the motels to the Embassy Suites at approximately 6:35. There will be transportation back to the motels after the banquet.

THURSDAY, MARCH 10, 1994

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. Room 1 56 is downstairs. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	202-A	202-C	156
8:30 AM	181 SPICER	182 NOSTRAND	183 JOHNSON	184 S CHOI	185 HAXELL
8:50	186 WILL	187 WE CLARK	188 SNEVILY	189 S MEDIDI	190 PIOTROWSKI
9:10	191 RUSKEY	192 MEYEROWITZ	193 LUNDGREN	194 K HUANG	195 STUECKLE
9:30	ERDOS				
10:30	COFFEE				
10:50	196 WAXMAN	197 ULLMAN	198 SOIFER	199 SANTORO	200 GOLDSMITH
11:10	201 PERKEL	202 GRIGGS	203 O'DONNELL	204 PENDYALA	205 GINN
11:30	206 MARK	207 CHINN	208 BARR	209 QIAN	210 RASMUSSEN
11:50	211 PERVIN	212 E HARE	213 SCHELP	214 SUFFEL	215 MK GOLDBERG
12:10 PM	216 RAMIREZ	217 HULL	218 NP CHIANG	219 FARLEY	220 DD-F LIU
12:30	LUNCH BREAK (ON YOUR OWN)				
2:00	TARJAN				
3:00	COFFEE				
3:15	AJ HOFFMAN				
4:20	221 KREHER	222 LABELLE	223 DY GOLDBERG	224 GEORGES	225 STARLING
4:40	226 FINIZIO	227 NIEDERHAUSEN	228 MR DILLON	229 KUBICKA	230 RICE
5:00	231	232 RETI	233 KOOSHEESH	234 BEASLEY	235 M MEDIDI
5:20	236 RODNEY	237 HAGLUND	238 WN LI	239 ASMEROM	240 JONOSKA
5:40	241 ABRHAM	242 ROSENFELD	243 D KNISLEY	244	245 M KIM

There will be a reception honoring women in combinatorics, hosted by the FAU Mathematics Department, in Room 215 of the Science and Engineering Building, from 6:00 -7:00. [Tea will be served from 5:30 on.] This activity, organized by Carolyn Johnston and Kathryn Fraughnaugh, is open to individuals of both sexes. There will be an informal CONFERENCE PARTY 7:00-8:00 in the Cafeteria Patio area--to be moved indoors if weather dictates. There will be Conference transportation back to the motels at 6:00 PM and back to the party at 6:40. There will be transportation back to the motels after the party.

FRIDAY, MARCH 11, 1994

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:30 A.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. Room 156 is downstairs. There will be book exhibits in Room 232 from 9:00 to 11:30.

	GCN	GCS	202-A	202-C	156
8:30 AM	246 JIANG	247 SALZBERG	248 SCHLIEP	249 FISHER	250 OSSOWSKI
8:50	251 X ZHU	252 CRUZ	253 RR GOLDBERG	254 McKENNA	255 ANDRZEJAK
9:10	256 BENNETT	257 MORENO	258 STEPHENS	259 MERZ	260 McMAHON
9:30	MULLIN				
10:30	COFFEE				
10:50	261 LEONARD	262 WAGON	263 MARQUEZ	264 KUMAR	265 T REID
11:10	266 BURATTI	267 ROBINSON	268 AMIN	269 HUTCHINSON	270 RAJPAL
11:30	271 MYRVOLD	272 CRAIGEN	273 FRONCEK	274 A DEAN	275 WOJCIECHOWSKI
11:50	276 PHILLIPS	277 EATON	278 KINNERSLEY	279 BOWSER	280
12:10 PM	281 CHOPRA	282 ARKIN	283 ADHAR	284 LANGLEY	285 GORDON
12:30	LUNCH (ON YOUR OWN)				
2:00	CONWAY				
3:00	COFFEE				
3:20	286 CRAWFORD	287 N GRAHAM	288 LIVINGSTON	289 Y WU	290 HON
3:40	291 KLASA	292 SHAHRIARI	293 HARTNELL	294 MIKHALEV	295 BERMAN
4:00	296 WILLE	297 LONC	298 GUNTHER	299 OELLERMANN	300 ARSHAM
4:20	301 RAVIKUMAR	302 ISAAK	303 McKEE	304 STEVENS	305 G-H ZHANG
4:40	306 STOCKMEYER	307 CANFIELD	308 MICHAEL	309 ABELLO	310 ALI

There will be transportation back to the motels following the last talks.

THANKS FOR COMING!!

There will be an informal after-dinner SURVIVORS PARTY, at the home of Aaron Meyerowitz and Andrea Schuver, 454 NE Third Street, beginning about 8PM. Tell us if you need transportation.

WE'LL SEE YOU **HERE** FOR THE TWENTY-SIXTH SOUTHEASTERN INTERNATIONAL CONFERENCE ON COMBINATORICS, GRAPH THEORY AND COMPUTING,

***** ?? ??-??, 1995 *****

INVITED INSTRUCTIONAL LECTURERS

Monday, March 7, 1994

9:30 a.m. Ralph G. Stanton, University of Manitoba
Packings Since 1970

2:00 p.m. Ronald L. Graham, Bell Labs
Digraph Polynomials

Tuesday, March 8, 1994

9:30 a.m. Herbert S. Wilf, University of Pennsylvania
Computers Prove Identities: A 50-Year Study

2:00 p.m. William H. Mills, Institute for Defense Analyses
Balanced Incomplete Block Design

3:15 p.m. Vera S. Pless, University of Illinois-Chicago
Knowledge From Numbers in Coding

Wednesday, March 9, 1994

9:30 a.m. John F. Dillon, National Security Agency
A Quarter Century of Hadamard Difference Sets

2:00 p.m. Fan R.K. Chung, BellCore
Routing in Graphs

3:15 p.m. John G. Thompson, University of Florida
The nth Power Map in Profinite Completions

Thursday, March 10, 1994

9:30 a.m. Paul Erdös, Hungarian Academy of Sciences
Twenty-Five Years of Questions and Answers

2:00 p.m. Robert E. Tarjan, Princeton University
Minimum Spanning Trees: A Survey

3:15 p.m. Alan J. Hoffman, IBM Watson Research Center
A New Look at Some Old Results in Combinatorial Matrix Theory

Friday, March 11, 1994

9:30 a.m. Ronald C. Mullin, University of Waterloo
Room Squares and Related Designs: 1955-1994

2:00 p.m. John H. Conway, Princeton University
Problems in Combinatorics and Elementary Geometry from Quantum Theory

INDEX OF ABSTRACTS

This index contains a list of all authors of abstracts, not just presenters. The list is complete as of 2:00 p.m. on Wednesday, March 2, 1994.

INDEX (Continued)

1

TOURNAMENTS FOR TWO VOTING PROCEDURES

K. B. Reid, California State University, San Marcos

Tournaments are useful models of the outcome of majority rule when there is a majority choice for each pair of alternatives (e.g., when there is an odd number of voters each of whom linearly orders the alternatives). In fact, every tournament arises this way. Let T be a tournament which contains a spanning digraph consisting of a transitive subtournament W with transmitter w, a rooted spanning tree Q in T-W with root q, and arc $r \to q$, such that no vertex of Q dominates every vertex of W. Such a T has the property that the vertices can be ordered so that there is a single decision using two distinct majority voting processes known as sincere and sophisticated voting under amendment procedure. In this talk these two processes will be described, and tournaments such as T will be characterized as tournaments that do not contain the 3-cycle as their initial strong component. The consequences for voting will also be discussed.

2

Quantum Factorials
by

Spencer P. Hurd[1], Dept. of Math. and C.S., The Citadel, Charleston, SC, 29409
and Judson S. McCranie, 1503 East Park Ave., V-11, Valdosta, Ga., 31602

We define the double factorial symbol !! as follows:

$$0!! = 1!! = 1, \text{ and for } n = 2, 3, \ldots$$
$$n!! = n(n-2)!!.$$

Using this we define the integer-valued function G to be the least non-negative residue given by:

$$G(1) = G(2) = 1 \quad \text{and for } n > 2,$$
$$G(n) = (n-1)!! - (n-2)!! \text{ (reduced modulo n).}$$

The idea of the function G was suggested to us by a problem in Quantum Magazine (Sep/Oct 1992, Vol. 3, No. 1, p. 16), which asked if 1992!! - 1991!! was divisible by 1993, or, in other words, is G(1993) = 0? In this note we determine G(n) for all positive integers n. The circle of ideas we develop is then used to derive a curious parity relationship between the number of odd quadratic non-residues (QNR's) and the number of small QNR's for a prime of the form 4n + 3.

3

On Hamiltonian Cycles in CnXsCm

Joseph B. Klerlein,* Western Carolina University
Edward C. Carr, High Point University

In 1978 Trotter and Erdos gave necessary and sufficient conditions for the direct product, CnXCm, of two directed cycles to be hamiltonian. In this paper we give some sufficient conditions for hamiltonian cycles in CnXsCm, the directed graph obtained from CnXCm by joining vertices which are s units apart in a single m-cycle. These results generalize similar results obtained for CnX2Cm.

4

Edgesum of the sum of k sum-deterministic graphs

Y.L. Lai*, K.L. Williams, Western Michigan University

Determination of the edgesum, $s(G)$, for arbitrary graphs is known to be NP-complete. For $k \geq 2$ and $G = \sum_{i=1}^{k} G_i$, with each G_i sum-deterministic, we present a polynomial time algorithm to establish $s(G)$.

It is known that graph classes K_n, $\overline{K_n}$, C_n, P_n and $K_{1,n}$ are each sum-deterministic. We show that several additional classes of graphs also have this property.

A proper numbering of a graph that achieves $s(G)$ is said to be *optimal* (with respect to edgesum). We show by example that a proper numbering in the same order as an optimal non-proper numbering need not itself be optimal.

Keywords: edgesum, sum-deterministic, proper numbering, polynomial time algorithm.

5

Ramsey Functions for Sequences with Differences in a Specified Congruence Class

Bruce M. Landman* and Andrew F. Long, University of North Carolina–Greensboro

Given integers $m > 1$ and $0 \leq q < m$, define a k-term $q(\text{mod}\, m)$-sequence to be an increasing sequence of positive integers $\{x_1, \ldots, x_k\}$ such that $x_i - x_{i-1} \equiv q(\text{mod}\, m)$ for $i = 2, \ldots, k$. Numbers analogous to the van der Waerden numbers $w(k)$ are examined, where the collection of arithmetic progressions is replaced by the collection of $q(\text{mod}\, m)$-sequences. We show that, for most values of m and q, there exist partitions of the positive integers into two sets such that neither set contains such sequences, so that the associated Ramsey functions do not exist. However, when we add the set of those arithmetic progressions having constant difference m to the set of $q(\text{mod}\, m)$-sequences, the associated Ramsey function is always defined, and we obtain accurate lower and upper bounds for these Ramsey functions. We also show that the existence of arbitrarily long arithmetic progressions in a set neither implies, nor is implied by, the existence of arbitrarily long members of the collection of sequences just described.

MONDAY, MARCH 7, 1994
11:10 a.m.

6

Greatest Midpoint Groupoids and Tournaments

Raymond Fletcher, University of Texas of the Permian Basin

For any groupoid A we define the *graph of A*, $\Gamma(A)$, by $\Gamma(A) = (V,E)$ where the vertex set V consists of elements of A and the arc set E is given by $E = \{a \to ab : a,b \in A\}$. In case the correspondence $A \to \Gamma(A)$ is one-one over all groupoids in a given variety of groupoids we say the variety is *digraphical*. In this paper we introduce the study of two new digraphical varieties of groupoids: the variety of *trigramoids* given by the identities (1) $(xy)y = y$; (2) $x((xy)x) = xy$; (3) $(xy)(x((xy)z)) = x((xy)z)$, and the variety of *semitrigramoids* determined by the identities (1),(3) and (4) $x(xy) = xy$; (5) $(xy)[((xy)x)(xy)] = (xy)x$. The main feature of algebras in these varieties is that their associated graphs have the *unique greatest midpoint property*, i.e., if we take any pair (a,b) of vertices and if we let M denote the set of all midpoints of 2-paths from a to b, then there exists a unique point in M which dominates every other point in M.

After developping the basic properties of trigramoids we give a characterization of trigramoids whose graphs are tournaments. We then describe a larger class of tournaments which correspond to semitrigramoids. These tournaments are then used as components in more complicated constructions of semitrigramoids. [Key words: Groupoids, Tournaments, Digraphs, Varieties]

7

Cyclotomy and orthomorphisms: A survey.

A. B. Evans, Wright State University.

Orthomorphisms of finite groups have several combinatorial applications - for instance in the construction of designs and in the construction of mutually orthogonal sets of Latin squares. An especially useful class of orthomorphisms are those constructed using cyclotomy in finite fields. This talk will be a survey of what is known about cyclotomic orthomorphisms and their applications.

8

Equidimensional Grey Codes in Cayley Graphs

Daniel Ashlock, Iowa State Univesity

Key Words: Cayley Graphs, Hamilton Cycles, Grey Coding

A traditional Grey code is simply a Hamilton cycle in a hypercube. An interesting modification of the notion of Grey code is to find a Grey code that spreads its edge usage out as evenly as possible among the dimensions of the hypercube. Replacing the notion of "edge in dimension i" with "edge created by generator i" and this generalization can be examined for any Cayley graph. This talk will give a necessary condition and completely solve the existence problem in a few cases.

9

On Bandwidth and Edgesum of the Tensor Product of P_m with $K_{r,n}$

K.L. Williams, Western Michigan University

The *tensor product* of graphs G_1 and G_2, denoted $G_1(T_P)G_2$, is $G = (V(G_1) \times V(G_2), E)$ where $((x_1,y_1),(x_2,y_2)) \in E$ if $(x_1,x_2) \in E(G_1)$ and $(y_1,y_2) \in E(G_2)$. For graphs in general finding either bandwidth or edgesum is known to be NP-complete. Consider $G = P_m(T_P)K_{r,n}$ for $m > 2$ and $r \leq n$. This paper shows that the bandwidth of G is $n + r - 1$. Also, a linear time algorithm for finding an optimal numbering of G to achieve edgesum is provided.

Keywords: tensor product, bandwidth, edgesum, algorithm, NP-complete.

10

On the zero-sum Ramsey numbers r(K_n, Z_3)

Lothar Piepmeyer, Technische Universitaet Braunschweig

For natural numbers n and k, for which n(n-1)/2 is divisible by k, let r(K_n,Z_k) denote the minimum integer r such that for every labeling of the edges of the complete graph K_r with arbitrary integers there exists a subgraph K_n with edge lablels that add up to 0 mod 3.

The existence of the numbers r(K_n,Z_k) follows from the classical Ramsey numbers. The exact values of a large class of zero-sum Ramsey numbers is known. We determine the unsettled numbers r(K_4,Z_3) and r(K_6,Z_3).

MONDAY, MARCH 7, 1994
11:30 a.m.

11 **On the existence of k-tournaments with given automorphism group.**

Susan Marshall, Simon Fraser University

By a k-tournament on n vertices we mean a complete k-uniform hypergraph on n vertices in which each hyperedge has been linearly ordered. We refer to the ordered hyperedges of T as arcs. An automorphism of T is a permutation f of the vertices of T with the property that A is an arc of T if and only if f(A) is also an arc of T. The purpose of this talk is to present the following result. For a finite group G and an integer k, there exists a k-tournament whose automorphism group is isomorphic to G if and only if gcd(¦G¦,k)=1.

12 Probabilistic Methods in Design Theory

Anant P. Godbole, Michigan Technological University

A $t-(n,k,\lambda)$ covering design ($n \geq k > t \geq 2$) consists of a collection of k-element subsets (blocks) of an n-element set $\mathcal{X}$ such that each t-element subset of $\mathcal{X}$ occurs in at least λ blocks. We use a variety of probabilistic methods to obtain new results for covering designs: The method of alterations is employed, together with exponential probability inequalities, to derive general upper bounds on the number of blocks in a minimal $t-(n,k,\lambda)$ covering design, $\lambda \geq 2$, thereby extending a theorem of Erdős and Spencer. Next, the Janson correlation inequalities are used to exhibit the existence a threshold behaviour for random coverings of t-sets by k-sets: with $\lambda = 1$ and $k \leq 2t-1$, we consider a randomly selected collection $\mathcal{B}$ of blocks; $|\mathcal{B}| = \phi(n)$, showing that $\mathcal{B}$ exhibits a rather sharp threshold behaviour, in the sense that the probability that it constitutes a $t-(n,k,1)$ covering design is, asymptotically, zero or one - according as $\phi(n) \leq \binom{n}{t}/\binom{k}{t}\log\binom{n}{t}(1-o(1))$ or $\phi(n) \geq \binom{n}{t}/\binom{k}{t}\log\binom{n}{t}(1+o(1))$. Finally, the Stein-Chen method is used to extend the above result, making it valid for general values of k; various Poisson approximations are also obtained.

13 Pseudo-Cartesian Product and Hamiltonian Decompositions of Cayley Graphs on Abelian Groups

Cong Fan
Western Michigan University
Don R. Lick
Jiuqiang Liu*
Eastern Michigan University

Alspach has conjectured that any $2k$-regular connected Cayley graph $cay(A,S)$ on a finite abelian group A can be decomposed into k hamiltonian cycles. In this paper, we first generalize a Kotzig's result that the cartesian product of any two cycles can be decomposed into two hamiltonian cycles and show that any pseudo-cartesian product of two cycles can be decomposed into two hamiltonian cycles. Then, by applying that result we show that the conjecture is true for most 6-regular connected Cayley graphs on abelian groups of odd order and for some 6-regular connected Cayley graphs on abelian groups of even order.

14 **RESTRICTED CUTS IN HYPERCUBES**

A. Duksu Oh
Department of Mathematics and Computer Science
St. Mary's College of Maryland
St. Mary's City, MD 20686

For an n-dimensional hypercube Q_n and a given integer p ($1 < p \leq n$), the minimum cardinality of a p-restricted cut of Q_n, denoted $\kappa(n,p)$, provides a generalized measure of node fault tolerance in n-cube networks, where a *p-restricted* cut of Q_n is a vertex cut of Q_n which contains at most p neighbors of each vertex in Q_n. This paper is concerned with $\kappa(n,p)$ and the structure of the minimum cardinality p-restricted cuts of Q_n. It is shown that Q_n admits a p-restricted cut if and only if $n \leq 3p-2$, and that $\kappa(n,p) \geq p2^{n-p}$ when $n \leq 3p-2$. Further, it is shown that $\kappa(n,p) = p2^{n-p}$ if and only if $n \leq 3p-2$, and $n \leq 2p$ for $p > 3$. This paper also presents for each p and n ($1 < p \leq n \leq 3p-2$, and $n \leq 2p$ for $p > 3$), a characterization of all minimum cardinality p-restricted cuts of Q_n, and an algorithm for finding all such cuts of Q_n.

15 **Ramsey Numbers for irregular graphs.**

G. Chen, P. Erdos, W. Shreve*, North Dakota State Univ

Known graphs which furnish lower bounds for for Ramsey numbers are for the most part regular graphs or graphs which are close to being regular. A Ramsey number is defined for irregular graphs. It is shown be bear a linear relationship to ordinary Ramsey numbers.

MONDAY, MARCH 7, 1994
11:50 a.m.

16 **Unavoidable rooted spanning trees in tournaments**

Xiaoyun Lu

A digraph G is said to be *n-unavoidable* if every tournament of order n contains it as a subgraph. One such an example is the directed path of order n. Here we generalize this example and consider other unavoidable spanning rooted trees in tournaments. We also answer two questions proposed by Saks and Sós.

17 **STRONGLY DIAGONAL LATIN SQUARES**

Yousef Alavi, Western Michigan University

Don R. Lick*, Eastern Michigan University

Jiuqiang Liu, Eastern Michigan University

An $n \times n$ latin square $S = [s_{ij}]$ is *strongly left-diagonal* if for each k, $0 \le k \le n-1$,

$$s_{1,1+k}, s_{2,2+k}, \ldots, s_{n,n+k}$$

are distinct, and *strongly right-diagonal* if for each k, $0 \le k \le n-1$,

$$s_{1,n-k}, s_{2,n-1-k}, s_{3,n-2-k}, \ldots, s_{n,1-k}$$

are distinct, where the subscripts are taken modulo n. A latin square S is said to be *strongly diagonal* if it is both strongly left-diagonal and strongly right-diagonal.

We prove the following: (1) There are no strongly diagonal latin squares of even order. (2) If n is an odd integer not divisible 3, then there exists a strongly diagonal latin square of order n. (3) If n is a prime integer greater than 3, then there exists a set of $n-3$ mutually orthogonal strongly diagonal latin squares.

18 **Hamilton cycles in vertex-transitive graphs**

Brian Alspach, Simon Fraser University

A short survey on the search for Hamilton cycles in vertex-transitive graphs will be given.

19 **The all-terminal network reliability under optimal link assignment**

C. Stivaros, Computer Science Dept, F.D.U. at Madison, NJ
(e-mail: stivaros@fdumad.fdu.edu)

The Assignment problem is defined for the "all-terminal" network reliability model. This problem has been studied for the "residual" model (nodes fail) but was never considered in the all-terminal model (links fail) despite the model's popularity in the literature. Combinatorial tools are presented such as various graph reductions. Solutions for common network structures are discussed along with open questions.

20 **Ramsey Numbers for Sets of Graphs**

Kathrin Klamroth, TU Braunschweig, Germany,

Generalized Ramsey Numbers r(G,H) are investigated for sets of graphs G and H. An interesting relationship between these Ramsey Numbers and certain Turan-Graphs was worked out in the case that G only contains a complete graph with n vertices and that H=<p,q> is the set of all graphs with p vertices and q edges. This relationship can be transfered to other cases.

MONDAY, MARCH 7, 1994
12:10 p.m.

21

ISOMORPHIC SWITCHING IN TOURNAMENTS

Leif K. Jørgensen
Aalborg University, Fr. Bajers Vej 7, Aalborg, Denmark.

In a tournament T with a vertex x, let x^+ and x^- be the out-neighbours and the in-neighbours of x, respectively, and let T_x be the tournament obtained from T by reversing every edge joining two of the three sets $\{x\}, x^+, x^-$.

I investigate tournaments, T, with the property that T_x is isomorphic to T for every vertex $x \in T$, and the relation of such tournaments to (vertex-transitivity in) regular digraphs with the property that non-adjacent vertices have no common out-neighbours and adjacent vertices have λ common out-neighbours for some constant λ.

22

Ternary designs with replication numbers $\lambda(V-1)/(K-1)$ and $\lambda(V-1)/(K-2)$

Margaret Francel*, The Citadel and D. G. Sarvate, University of Charleston

A Balanced Ternary Design BTD(V,B;R;K,Λ) as we know today is an arrangement of V points in B blocks (i.e. multisets) each of size K, such that every point occurs 0,1 or 2 times in a block, every point occurs R times in the design, and every pair of distinct points occurs Λ times in the design. Tocher's original definition of ternary designs does not restrict the replication number R to be a constant. However he showed the replication number R will come between the two numbers Λ(V-1)/(K-1) and Λ(V-1)/(K-2). If we restrict the replication number to be these two constants we have a ternary design (TD) with two replication numbers, where the V points can be partitioned into two parts P_1 and P_2, $|P_1| + |P_2| = V$, where all points in P_1 occur singly in Λ(V-1)/(K-1) blocks and all points in P_2 occur doubly in Λ(V-1)/2(K-2) blocks.

We prove that the necessary conditions are sufficient for the existence of TDs with block size 3 and we obtain some partial results for the general case.

Study of these designs offer an interesting counterpart to the undergoing study of BTDs where the blocks are partitioned into blocks with repeat elements and blocks without repeat elements.

23

Hamiltonicity of Regular 2-Connected Graphs

Jan van den Heuvel
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In 1980, B. Jackson proved the following result.

Theorem 1 *Every 2-connected k-regular graph on at most $3k$ vertices is hamiltonian.*

Theorem 1 is essentially best possible. For 3-connected graphs the following strengthening was conjectured.

Conjecture *For $k \geq 4$, every 3-connected k-regular graph on at most $4k$ vertices is hamiltonian.*

A first result in the direction of this conjecture was proved by H. Li and Y. Zhu, who showed that for $k \geq 63$, every 3-connected k-regular graph on at most $\frac{22}{7}k$ vertices is hamiltonian. We improve both this result and Theorem 1 by establishing the following.

Theorem 2 *Let G be a 2-connected k-regular graph on at most $\frac{7}{2}k - 7$ vertices. Then G is hamiltonian or G belongs to a restricted class of nonhamiltonian graphs of connectivity 2.*

We immediately obtain the corollary that every 3-connected k-regular graph on at most $\frac{7}{2}k - 7$ vertices is hamiltonian.

Joint research with H. J. Broersma, B. Jackson and H. J. Veldman.

24

Graph Isomorphism and the Evolution of Cooperation

John Pedersen*, Stephen Suen and W. Richard Stark
Dept of Mathematics, University of South Florida, Tampa, FL

Nowak and May have proposed a simplified model for the evolution of cooperation (Nature, Oct. 1992), which showed interesting, sometimes chaotic, results. In their simulation, each vertex of an n x n grid graph is either a cooperator or a defector. Their simulation procedes in a way similar to Conway's game of Life, but with different rules based on simplified Prisoner's Dilemma playoffs. Huberman (Proc. Nat. Acad. Sci. USA, v. 90, 1993) has observed that if the simulation is performed serially instead of in the synchronous method of Life, different results are obtained, not including chaos. We have generalized the simulation to arbitrary graphs and schedules covering the entire spectrum from serial to synchronous. As well as examining the onset of chaos in this discrete setting, we suggest a novel approach to the isomorphism problem based on this simulation.

25

SOME RAMSEY-TYPE GRAPHS

Béla Bajnok, Gettysburg College

Let the Ramsey-type number $R_d(k,l)$ denote the smallest integer n such that every graph on n vertices with maximal degree d either contains a clique of size k or l independent points. Staton (1979) proved that $R_3(3,k)$=Floor[(14k-9)/5]. In this paper we will investigate graphs with maximal degree 3 on $v=R_3(3,k)$-1 vertices that are triangle free and have independence number less than k.

26 **COUNTING N-PLAYER SINGLE ELIMINATION TOURNAMENTS**

KEVIN MCDOUGAL
UNIVERSITY OF WISCONSIN-OSHKOSH

We present several methods for counting the number of n-player single elimination tournaments. These are rooted binary trees with n labelled leaves and n-1 unlabelled internal vertices. The history of this problem's solution is discussed. A newer perspective of one method of solution is presented and also a possibly new method of solution.

27 **Some Results on 2-ranks of Oval Designs**

Laurel L. Carpenter, Clemson University

Let Π be a desarguesian projective plane of even order, n, containing a hyperoval, $\mathcal{O}$. The oval design, $\mathcal{D}$, arising from this structure is the $2-(\frac{n^2-n}{2}, \frac{n}{2}, 1)$ design with points defined as the passants of $\mathcal{O}$, blocks as the points of $\Pi \setminus \mathcal{O}$, and incidence as that from the ambient plane. In the case where $\mathcal{O}$ is a regular hyperoval, we will show, using a result of Blokhuis and Moorhouse, that $rank_2 \mathcal{D} = 3^m - 2^m$. For irregular hyperovals, we will show some results and discuss a conjecture concerning the 2-rank of their oval designs.

28 **Intermediate Value Theorems for the Sizes of Some Classes of Visibility Graphs**

K. Jay Bagga*, John W. Emert, and J. Michael McGrew
Ball State University

William E. Toll
Taylor University

Given a set of n disjoint line segments (known as *obstacles*) in the plane, the *(segment endpoint) visibility graph* on these obstacles has the $2n$ endpoints of the obstacles as vertices, with two being adjacent if either they are the endpoints of the same obstacle or the line segment joining them does not intersect any obstacle. It is known that the number of edges in such a visibility graph is bounded by $5n - 4$ and $2n^2 - n$, and that these bounds are sharp. In this paper we show that for any q with $5n - 4 \leq q \leq 2n^2 - n$, there is a visibility graph on n obstacles which has q edges. Similar results on certain other classes of visibility graphs are also presented.

29 **An algorithm and implementation for distance computations in graphs**

Thomas B. Sprague, Alma College, Alma, MI.

We describe a simple algorithm for computing the distance between all pairs of vertices in a graph G of order p. It is based on the evaluation of a polynomial of degree p-1 or less in the adjacency matrix. The algorithm has been implemented in the MATLAB language, from which it inherits the capabilities of interactive computations, graphical displays and extensibility. We discuss the extensibility feature by showing how we were able to interactively compute and display the antipodal and radial graphs of all trees on 10 vertices. We conclude by describing how the code may be obtained via the internet from the Alma College archives.

30 **2-Regular Edge-Graceful Graphs**

*Dr. Sharon Cabaniss
David Bealer

A graph G(p,q) is said to be edge-graceful is there exists an injective map ϕ:E(G) → Z_q such that the map ψ:V(G) → Z_p defined by $\psi(v) = [\phi(e_1) + \phi(e_2) + ... + \phi(e_k)]$ modulo p, where $e_1, e_2, ..., e_k$ are all the edges incident with v, is injective.

It has been conjectured that almost all 2-regular graphs are edge-graceful, (there is a known exception, namely C_3UC_7). We have been successful in discovering a number of 2-regular edge-graceful graphs. In fact an infinitely large family of 2-regular graphs which have this property.

We will discuss a method of determining that 2-regular graphs are indeed edge-graceful using arrays of numbers, in particular the addition table for the group Z_p.

By the use of this method it is an easy matter to show that for any odd positive integer k, $C_kUC_{2t(k+1)}$ and C_kUC_{4tk} are edge-graceful, along with many others, for arbitrary positive integers t.

31 **Optimal recognition of local tournaments.**

Joergen Bang-Jensen (Odense University, Denmark), Pavol Hell, and Jing Huang* (Simon Fraser University, Canada).

A local tournament is an oriented graph in which the inset as well as the outset of each vertex is a tournament. Local tournaments extend the class of tournaments, keep many nice properties of tournaments, and are related to the class of proper circular arc graphs. Applying a recent structural characterization of local tournaments (due to the speaker), we give a linear time algorithm for recognizing local tournaments.

32 **The Largest Linefree projective Subset of PG(n,2)**

Dr. A. Bruen, Dr. L. Haddad, Dr. D. Wehlau*
University of Western Ontario and Royal Military College of Canada

Let PG(n,2) denote projective space of dimension n over the field, GF(2), of order 2 and let AG(n,2) denote n dimensional affine space over GF(2). It is easy to show that the largest line free subset of PG(n,2) is AG(n,2). In this talk we describe the largest line free subset of PG(n,2) which is not contained in any copy of AG(n,2). This set is unique (up to isomorphism) and possesses a number of interesting properties, some of which we will discuss.

33 **NOTE ON WHITNEY'S THEOREM FOR K-CONNECTED GRAPHS**

Guantao Chen*, North Dakota State University, Fargo, ND 58105
Ralph J. Faudree, Memphis State University, Memphis, TN 38152
Warren E. Shreve, North Dakota State University, Fargo, ND 58105

In this paper we refine Whitney's Theorem on k-connected graphs for k >2. In particular we show the following: If G be a graph with k>2, then for any two distinct vertices u and v of G there are k internally vertex paths
P1[u,v], P2[u,v], ..., Pk[u,v]
such that G -V(Pi(u,v)) is connected for each i=1,2,...,k, where Pi(u,v) denotes the internal vertices of the path Pi[u,v].
Further one of the following properties holds:

A: G-V(Pi[u,v]) is connected for i=1,2,3.

B: G-V(Pi[u,v]) is connected for i=1,2 and G-V(Pi[u,v])has exactly two connected components for i=3,4,...,k.

Some other properties will be discussed too.

34 **Hamiltonicity in 2-connected graphs with claws**

Hao LI * *
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M. Matthews and D. Sumner have proved in [?] that if G is a 2-connected claw-free graph of order n such that $\delta \geq \frac{n-2}{3}$, then G is hamiltonian. Li has shown that the bound for the minimum degree δ can be reduced to $\frac{n}{4}$ under the additional condition that G is not in Π, where Π is a class of graphs well defined in [?]. On the other hand, we say that a graph G is almost claw-free if the centres of induced claws are independent and their neighbourhoods are 2-dominated. Broersma, Ryjáček and Schiermeyer have proved that if G is 2-connected almost claw-free graph of order n such that $\delta \geq \frac{n-2}{3}$, then G is hamiltonian. We generalize these results by considering the graphs whose claw centres are independent. If G is a 2-connected graph of order n and minimum degree δ such that $n \leq 4\delta - 3$ and if the set of claw centres of G is independent, then we show that either G is hamiltonian or $G \in F$, where F is a class of graphs defined in the paper. The bound $n \leq 4\delta - 3$ is sharp.

35 **Edge-Graceful r-Regular Graphs**

Dr. Sharon Cabaniss
*David Bealer

A graph G(p,q) is said to be edge-graceful is there exists an injective map ϕ:E(G) → $\mathbb{Z}_q$ such that the map ψ:V(G) → $\mathbb{Z}_p$ defined by $\psi(v) = [\phi(e_1) + \phi(e_2) + ... + \phi(e_k)]$ modulo p, where $e_1, e_2, ..., e_k$ are all the edges incident with v, is injective.

It is from this definition and some elementary number theory a routine matter to prove that give a number of edge-graceful r-regular graphs with some particular properties in common that there union is also edge-graceful. An example is, given three 2-regular graphs ($C_3 U C_8$), C_{11}, and C_{11}, it can be shown that $C_3 U C_8 U C_{11} U C_{11}$ is edge-graceful.

Though the proof involving number theory is indeed routine, we will be discussing a more intuitive and informal approach involving the use of factor groups. This method, although less rigorous, is in many ways more aesthetically pleasing.

36 **The structure of tournaments omitting IS4 and S4I**

Brenda J. Latka, Lafayette College

A tournament is a complete directed graph. A tournament T is an obstruction to a class of tournaments if T is not a subtournament of any tournament in the class. Some obstructions completely determine the structure of the tournaments in their corresponding classes. We show that the tournaments IS4 and S4I are such obstructions.

37 **A Secret Sharing Scheme**

Puhua Guan, Dept of Math Univ. of Puerto Rico, Rio Piedras PR 00931

A (t,w)-threshold secret sharing scheme is a method of sharing a secret key among a finite set of w paticipants, in such way that any t participants can compute the value of K, but no group of t-1 participants can do so. In this talk we present a (t,w)-threshold scheme based on the chinese remainder theorem. The rate of information of this scheme is good and it alows a fast algorithm to recover the secret by t participants.

38 **Constructing Cospectral Expander Graphs with Unequal Expander Coefficients**

Ian C. Walters Jr., Western Michigan University

An (n,k,c) expander graph is a k-regular graph with n vertices so that any subset S of V(G) satisfies the inequality

$$|N(S) - S| \geq c\left(1 - \frac{|S|}{n}\right)|S|$$

where $N(S) = \{v \in V(G) \mid uv \in E(G) \text{ and } u \in S\}$ and c is a real number. The largest c which satisfies the inequality for all subsets S is called the expander coefficient, denoted expan(G). Two compositions that result in cospectral graphs are used to construct several examples of cospectral pairs of graphs with unequal expander coefficients.

39 **FORBIDDEN PAIRS OF GRAPHS AND HAMILTONIAN PROPERTIES**

Ralph J. Faudree* — Memphis State University
Ronald J. Gould — Emory Univesity

A characterization of pairs of forbidden subgraphs that are sufficient to imply various hamiltonian type properties in graphs will be discussed. Similar to the results of Bedrossian for hamiltonian graphs and pancyclic graphs, all forbidden pairs sufficient, along with the appropriate connectivity condition, to imply a graph is traceable, panconnected, or cycle extendable will be determined.

40 **On Edge-Graceful Spiders**

Andrew Simoson, King College, Bristol, TN 37620

Graceful and edge-graceful graph labelings are dual notions of each other in the sense that a graceful labeling of the vertices of a graph G induces a labeling of its edges, whereas an edge-graceful labeling of the edges of G induces a labeling of its vertices. In this talk we demonstrate an algorithm for generating balanced, graceful labelings of paths which in turn provides a means of edge-gracefully labeling certain trees, so advancing *Lee's Conjecture* that all trees of odd order are edge-graceful. To complement the well known result that all spiders of odd order with legs of equal length are edge-graceful, we demonstrate in particular that all spiders of odd order with three or four legs are edge-graceful, and give partial results for n-legged spiders with $n \geq 5$.

41 Source-sink pairs in comparability graphs

By John Gimbel, University of Alaska, Fairbanks, Alaska 99775

Given G, a comparability graph and A,B, disjoint sets of vertices in G, we say (A,B) is a source-sink pair if there is a transitive orientation of G in which each vertex of A has indegree zero and each vertex of B has outdegree zero. We present a characterisation of source-sink pairs which answers a question of Szwarcfiter, et al. Further, we show that given A and B, we can determine in polynomial time if (A,B) is a source-sink pair.

42 An Axiomization of the Flats of an Affine Hyperplane Arrangement

Jennifer McNulty, The University of Montana

The flats of an arrangement of hyperplanes in projective space form a geometric lattice under reverse inclusion in which the atoms correspond to the hyperplanes of the arrangement. In a similar manner, the flats of an affine arrangement yield an inf-semilattice. An affine hyperplane arrangement can be embedded into projective space producing a projective arrangement containing one additional hyperplane. Likewise, the inf-semilattice can be embedded into a geometric lattice which has one additional atom. An axiomization of inf-semilattices with this property will be given. In graph theoretic terms, the flats of a projective arrangement correspond to the cutsets of a graph, while the flats of an affine arrangement correspond to the (s,t) cutsets. The presentation will concentrate on this graphic aspect of the axiomization.

43 A GENERALIZATION OF D. W. HALL'S GRAPH RESULT

Sandra R. Kingan, Louisiana State University, Baton Rouge LA 70803

D. W. Hall proved that if a 3-connected graph has a minor isomorphic to the complete graph on five vertices, K5, then it must also have a minor isomorphic to the complete bipartite graph with 3 vertices in each class, K(3,3), the only exception being K5 itself. In this paper, we prove that if a 3-connected binary matroid has a minor isomorphic to K5 or its dual, then it must also have a minor isomorphic to K(3,3) or its dual, the only exceptions being K5, its dual, and a particular highly symmetric, 12-element, self-dual matroid.

44 **Biclique Decompositions of the Complement of Trees**

Elizabeth D. Boyer* **Bryan L. Shader**
University of Wyoming

A *biclique* is a complete bipartite graph. A *biclique decomposition* of a graph G is a set of edge subgraphs of G, each of which is a biclique, which partition the edges of G. The *biclique decomposition number* of G is the minimum number of bicliques in any biclique decomposition. In a survey paper, S. Monson, N. Pullman and R. Rees posed the question of determining the biclique decomposition number of the complement of a Hamilton path in $K_{n,n}$. We answer this question and generalize the result to determine the biclique decomposition number of the complement of any tree with a perfect matching in $K_{n,n}$.

45 **On edge-magic regular complete k-partite graphs**

Sin-Min Lee
Department of Mathematics and Computer Sciences,
San Jose State University,
San Jose , CA 95192, U.S.A.

A graph G=(V,E) is said to be edge-magic if there exists a bijection f : E -> {1,2,...,|E|} such that the induced mapping f^+ : V -> N defined by $f^+(u) = \Sigma\{ f(u,v) : (u,v) \in E \}$ (mod |V|) is a constant map.

We complete characterize regular complete k-partite graphs which are edge-magic. Algorithms for edge-magic labelings for these graphs are given.

Keywords: complete k-partite graphs, edge-magic labeling.

46 **BISECTABLE TREES**

K.Heinrich, Simon Fraser University, Vancouver, BC
P.Horak(*), Slovak Technical University, Bratislava, Slovakia and Southern Illinois University, Carbondale, IL

A graph is called bisectable(=even) if its edge set can be partitioned into two isomorphic subgraphs. R.Graham and R.Robinson asked if the problem: "Given a tree T, decide whether T is bisectable" is NP- complete. In contrast to this, F.Harary and R.Robinson conjectured that there are only two trees of maximum degree 3 which are not bisectable. We present results concerned with the conjecture and a related problem of P.Erdos. An infinite version of the problem will be discussed as well.

47 The Isoperimetric Problem in Finite Projective Planes

L. H. Harper, University of California at Riverside

The following problem is a natural analogue of the isoperimetric problem of plane geometry. Over all sets of k points in the projective plane over GF(q), k between 0 and q^2 + q + 1, minimize the number of lines which contain points of the set. Despite the obvious appeal of this problem very little progress has been made. The methods which solve closely related problems such as that of Kruskal-Katona do not work. The author will show that B. Segre's theory of ovals can be used to solve the following special cases:

i) For k between 0 and q + 1, the points of a conic are solutions. If q = 2^h, a power of 2, then the nucleus of the conic may also be thrown in to give a solution for k = q + 2.

ii) By duality, the solutions in i) give solutions for k =or> (q^2 - q)/2

iii) If q = 2^h then for n = 2^i, i between 0 and h, Segre's maximal (k,n)-arcs give solutions for k = 1 + (n - 1)(q + 1). These sets are unions of conics with their common nucleus.

48 **THE ANNULUS OF A GRAPH**

Gary Chartrand, Western Michigan University
Garry L. Johns*, Saginaw Valley State University
Songlin Tian, Central Missouri State University
Steven J. Winters, University of Wisconsin Oshkosh

KEY WORDS: distance, eccentricity, annulus, interior

The eccentricity $e(v)$ of a vertex v in a connected graph G is the distance between v and a vertex furthest from v. The annulus Ann(G) of a connected graph G whose vertices have at least three distinct eccentricities is the subgraph induced by the vertices whose eccentricities lie strictly between the radius and the diameter of G. In this paper, several results related to the annulus are given, including a characterization of those graphs which are isomorphic to the annulus of some connected graph.

49 **Broadcasting in Linear Congruential Graphs of Degree 4**

K.T. Lin and J. Opatrny
Dept. of Computer Science, Concordia University, Montreal, Canada

A *Linear Congruential Graph*, or *LCG graph* for short, of size n and degree $2k$ is a graph in which the set of vertices is the set $\{0, 1, \ldots, n-1\}$, and there is an edge from u to $f_i(u)$ mod n for any u and any function in a set $\{f_1, f_2, \ldots, f_k\}$ of linear functions.

Broadcasting in a graph is a process of sending a message from a vertex to all other vertices in which any vertex that already received the message can send a message, in one time unit, to at most one of its neighbours. Broadcasting is an important part of many distributed algorithms, and it is thus important to find what graphs are suitable for fast broadcasting of messages.

We investigate the problem of broadcasting in LCG graphs of degree 4, in which f_1 generates a Hamiltonian cycle, and f_2 generates a small number of disjoint cycles. First we exhibit functions for which the broadcasting is done in time $O(\log_2 n)$, which is asymptotically optimal. We than consider the broadcasting problem for two-dimensional LCG graphs and discuss strategies that improve the broadcasting time.

50 On edge-magic cubic graphs conjecture

Thomas Cox, Sin-Min Lee, and William Pigg*
Department of Mathematics and Computer Sciences,
San Jose State University,
San Jose , CA 95192, U.S.A.

A graph G=(V,E) is said to be edge-magic if there exists a bijection f : E -> {1,2,...,|E|} such that the induced mapping f^+ : V -> N defined by $f^+(u) = \Sigma \{ f(u,v) : (u,v) \in E \}$ (mod |V|) is a constant map. Lee conjectured a cubic (p,q)-graph is edge-magic if and only if p ≡ 2(mod 4) . We report some positive results of this conjecture and also illustrate the difficult nature of this problem.
Keywords: edge-magic, labeling, cubic graphs.

MONDAY, MARCH 7, 1994
5:00 p.m.

51 **Bipartite Designs - Partitioning the Edges of K(c,d) into Copies of K(a,b)**

D.G. Hoffman*, Mark Lloyd Liatti - Auburn University

We determine necessary and sufficient conditions on integers a, b, c, d for the existence of a partition of the edges of the complete bipartite graph K(c,d) into copies of the complete bipartite graph K(a,b).

52 **Some Comments on Weight-1 Vectors In Binary Codes**

F. E. Sullivan, Clemson University

The design of points and lines of the projective geometry of dimension d over the field GF(2) is a Steiner triple system (STS) whose binary code C is the binary Hamming code H(d+1) of minimum weight 3. Let E be the vector space spanned by C and a weight-1 vector e. We conjecture that E always contains a set of weight-3 vectors whose supports form the blocks of an STS. The binary code of this STS is E. We also make preliminary comments on the binary code of E', the vector space spanned by C, e and a distinct weight-1 vector f.

53 **Girth, minimum degree and circumference**

M. N. Ellingham*, Vanderbilt University and
D. Kirk Menser, Austin Peay State University

Key words: girth, minimum degree, circumference

We present a new lower bound on the circumference (length of the longest cycle) of a graph with given girth (length of the shortest cycle) g and minimum degree d. Previous lower bounds due to O. Ore, C. Peyrat and C. Q. Zhang are all roughly proportional to g times d. For fixed g at least 3, our bound is asymptotic to c (d-2)^k as d goes to infinity, where c is 1, 2, 3 or 4 depending on the value of g, and k is the integer part of (g+1)/4. Our bound is as good as the best of the Ore, Peyrat and Zhang bounds for d at least 3 and g at least 3, and is strictly better for d at least 3 and g at least 7.

54 **UNDIRECTED LOOP NETWORKS**

Sheng Chen* and Xing-De Jia, Southwest Texas State University

Let M, d and k be positive integers. We say that M is *feasible* with respect to d and k if there exists a set $A = \{0, \pm 1, \pm a_2 \cdots, \pm a_k\}$ such that the Cayley graph associated with $Z/(M)$ and A has diameter less than or equal to d. Such a Cayley graph is a model for undirected loop communication networks. Denote $M(d,k)$ the maximal feasible number M with respect to d and k. In the paper, an explicit formula for $M(d,2)$ is obtained. And when $k \geq 3$, a lower bound for $M(d,k)$ is established.

55 **AN EFFICIENT LABELING ALGORITHM FOR MAGIC GRAPH**

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A graph $G=(V, E)$ is magic if we can find an edge labeling assignment $L:E\rightarrow\{1, 2, ...\}$ such that the sum of all edge labels incident to each vertex has the same value, and this value is called the magic index w.

In this paper we present an efficient labeling algorithm for magic graph. Given a graph represented as an adjacency matrix, the algorithm uses a modified depth-first search strategy to label the graph. If the graph is magic, there exists at least one value of $w \leq |V|$. Heuristics based on properties of magic graph are employed to terminate the search for a magic labeling if the graph is not magic and to reduce the search space if the graph is magic.

56

Vertex Partitioning Problems on Partial k-Trees

Jan Arne Telle(*) and Andrzej Proskurowski, University of Oregon

Let D_q be a q by q matrix with entries being subsets of $\mathcal{N} = \{0, 1, 2, \ldots\}$. A D_q-partition in a graph G is a partition $V_1, V_2, \ldots, V_q$ of its vertices such that for $1 \leq i, j \leq q$ we have $\forall v \in V_i : |N(v) \cap V_j| \in D_q[i, j]$, where $N(v)$ is the (open) neighborhood of v in G. Many well-known graph problems, such as domination, coloring, H-covering, can be defined as optimizations over, or existence of, D_q-partitions. We give algorithms for solving any problem in this class on partial k-trees (equivalently, graphs of treewidth bounded by k), accounting for dependency on the treewidth k in the complexity of these algorithms. For example, the chromatic number problem, taking an n-vertex graph G and a width k tree decomposition of G as input, is solved in $O(n2^{2k \log k})$ steps, whereas the domatic number problem is solved in $O(n2^{2k^2})$ steps.

57

Hyperovals and Flocks of Quadratic Cones

William Cherowitzo*, Univ. of Colorado at Denver
Ivano Pinneri, Tim Penttila, Gordon Royle, Univ. of Western Australia

A flock of a quadratic cone in PG(3,q) is a partition of the points of the cone (without its vertex) into conics (plane intersections with the cone). Flocks have been used to produce generalized quadrangles and translation planes. We use a connection between flocks and hyperovals in a Desarguesian plane of even order to construct a new family of flocks of even order. This family of "Subiaco" flocks corresponds to a new family of hyperovals which includes the previously sporadic Lunelli-Sce hyperoval in PG(2,16).

58

Maximal Paths and Maximal Induced Paths in Graphs

S.M. Hedetniemi, S.T. Hedetniemi*, Clemson University
M.S. Jacobson, University of Louisville

The detour number DT(G) of a graph G is the maximum length of a path in G and the induced detour number ID(G) is the maximum length of an induced path in G. We initiate the study of the minimum lengths of maximal paths and maximal induced paths in graphs. Let dt(G) denote the minimum length of a maximal path in G and ID(G) denote the minimum length of a maximal induced path in G. We study the values of these four numbers in a variety of chessboard graphs and discuss their algorithmic complexities.

59

Cycle Shortness Exponents for Maximal Planar Graphs

Peter Christopher, Worcester Polytechnic Institute and
Allen J. Schwenk*, Western Michigan University

If a graph of order n has maximum cycle length c, we define the cycle exponent to be the power e such that c = n^e. We study various families of maximal planar graphs. The first family contains all nonhamiltonian maximal planar graphs. The second consists of eulerian nonhamiltonian maximal planar graphs. The third family has minimum degree 5 maximal planar graphs. We seek to determine the minimum cycle exponent in each family. Within each family we construct an infinite sequence of graphs whose limiting cycle exponent is log 7 / log 8 = 0.935 784 974 019.

60

CONSTRUCTION OF MAGIC GRAPHS

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A graph G=(V,E) is magic if we can find an edge labeling assignment L:E->{1,2,...} such that the sum of all edge labels incident to each vertex has the same value, and this value is called the magic index w. We denote the set of all magic assignments of G by M(G). For each L in M(G), we define the strength of L as S(L)=max{L(e): e in E}. The magic strength of the graph is min{S(L):L in M(G)} and is denoted by m(G).

In this paper, we investigate the magic property of the resulting graph after applying a constructive operation of sum or product on two magic graphs. Five classical products (Castesian, Conjunctive, Normal, Lexicographic, and Disjunctive) and one sum of two graphs are studied. We conclude that the products and the sum of two magic graphs are magic. The magic index and magic strength of the constructed graph are formulated.

MONDAY, MARCH 7, 1994
5:40 p.m.

61 **P-Complete Permutation Group Problems**

Kenneth Blaha*, Pacific Lutheran Unv., Tacoma, WA
Eugene Luks, University of Oregon, Eugene, OR

It was shown by Furst, Hopcroft and Luks that a variant of Sims's elegant algorithm for membership-testing in permutation groups could be implemented in polynomial time. Because this well-known method employs a ''sifting'' process which seems inherently sequential, McKenzie and Cook conjectured that the membership problem was P-complete. Later, Babai, Luks, and Seress, relying, in part, on the classification of finite simple goups, developed methods that bypassed the sifting obstruction. However, the parallelizability of Sims's method remained open. We now justify the earlier intuition by showing that sifting is P-complete.
We also demonstrate the P-completeness of some other permutation group problems. Among these is the problem of computing the proposed canonical forms for the class of vertex-colored graphs with bounded color multiplicities. This opens a gap, in parallel computation, between isomorphism-testing and finding canonical forms; the former problem is in NC for this graph class.

PROJECTIVE BUNDLES

62 R.D. Baker(Delaware), G.L. Ebert*(Delaware), J.C. Fisher(Regina)

A projective bundle in PG(2,q) is a collection of q^2 + q + 1 conics that mutually intersect in a single point and hence form another projective plane of order q. The purpose of this paper is to investigate the possibility of partitioning the q^5 - q^2 nondegenerate conics of PG(2,q) into (q-1)*q^2 disjoint projective bundles. We are able to show that for odd q half the conics of PG(2,q) can be partitioned into projective bundles. For q even the best general result we have is a method for constructing q-1 disjoint bundles. The construction for odd q is probably close to optimal, at least if one uses only the known types of projective bundles. For q even one can undoubtedly do much better. For instance, using the software package Magma we were able to find 30 mutually disjoint projective bundles in PG(4,q).
An interesting connection with perfect difference sets is discussed along the way.

63 **INDUCED PATHS AND CYCLES IN QUEENS GRAPHS**

Dan Pritikin, Miami University, Oxford, OH 45056

The Queens graph Q(n) is the graph whose vertex set forms an n by n grid, with two vertices adjacent when on the same row, column or diagonal. The Superqueens graph SQ(n) is defined similarly, where diagonals extend with wraparound. Given a graph G, let MIP(G) [resp. MIC(G)] be the maximum number of vertices in an INDUCED subgraph H of G such that H is isomorphic to a path [resp. cycle]. This report concerns bounds for lim sup MIP(Q(n))/n and for lim sup MIC(Q(n))/n and the corresponding parameters for SQ(n). Of particular use are the results that lim sup MIP(Q(n))/n >= MIP(Q(k))/k and lim sup MIC(Q(n))/n >= MIC(Q(k))/k for all k.

64 **Further Results on Mod Sum Graphs**

J. Ghoshal*, R. Laskar, D. Pillone Clemson University
G. Fricke Wright State University

A graph G = (V, E) with $V = \{v_1, v_2, ..., v_p\}$ is a mod sum graph (MSG) if there exists a positive integer n, called the modulus, and a one to one function $\sigma : V(G) \to \{1, 2, ..., n-1\}$ such that $(v_i, v_j) \in E(G)$, where $v_i \neq v_j$, if and only if $\sigma(v_i) + \sigma(v_j) = \sigma(v_k)$ (mod n) for some $v_k \in V(G)$. In this paper, we study some classes of graphs which are not MSGs and extend some results by Boland, Domke, Laskar and Turner in an earlier paper.

65 **On graceful permutation graphs conjecture**

Sin-Min Lee and Kuen-Yu Lai *
Department of Mathematics and Computer Sciences,
San Jose State University,
San Jose, CA 95192, U.S.A.

Yon-Sian Wang and Min-Kuan Kiang
Department of Mathematics
National University of Education at Changhua,
Changhua, Taiwan, R.O.C.

In 1983 the first author conjectured that for any n >1 and any permutation f in S(n) the permutation graph P(Pn,f) is graceful. Some progress of this conjecture is given. A more general conjecture is also proposed.
Keywords: graceful, permutation graphs.

66 **Semifield Planes With a Transitive Autotopism Group**

Minerva Cordero-Vourtsanis(*) Department of Mathematics,Texas Tech University, Lubbock, Texas 79409
Raúl F. Figueroa Department of Mathematics, University of Puerto Rico, Rio Piedras, Puerto Rico 00931

Let π be a non-Desarguesian semifield plane of order p^n, where p is a prime number and $n \geq 3$. Let G be the autotopism group of π relative to an autotopism triangle Δ. We prove that if the group $\bar{G}$ induced by G on a side of Δ is transitive on the non-vertex points of that side, then π is a generalized twisted field plane.

67 **The Superregular Graphs**

Douglas B. West, University of Illinois-Urbana

A graph is *superregular* if it has no vertices, or if it is regular and the subgraphs induced by the neighbors and by the nonneighbors of each vertex are superregular. We prove that a graph is superregular if and only if it is mK_p (disjoint union of isomorphic cliques), $K_m \square K_m$ (cartesian product of two isomorphic cliques), C_5 (the five-cycle), or the complement of one of these graphs.

68 **The Dimension of the Linear Space Generated by Incidence Vectors of Minimu m T-joins**

Krishna Padayachee, University of Waterloo

We consider the linear space of incidence vectors of minimum cardinality T-joins. A formula for the dimension of the convex - hull of incidence vectors of minimum cardinality T-joins is given which generalizes a result due to Edmonds, Lovasz and Pulleyblank, on the dimension of the perfect matching polyhedron. The main technique used is a graph decomposition that generalizes a decomposition procedure due to Lovasz and again Lovasz and Plummer based on results due to Kotzig and Lovasz. Furthermore this decomposition is shown to be unique in a sense to be made precise.

69 **Loopless Generation of Subsets with a Given Sum**

Dominique Roelants van Baronaigien, Computer Science
Eugene Neufeld, Mathematics
University of Victoria, Victoria, B.C., V8W 3P6, Canada

A loopless generation algorithm is an algorithm that generates combinatorial objects such that the computation necessary to determine each successive object is O(1) worst case. In this talk, we present a loopless algorithm for listing all k element subsets of the set {1,2,...,n} that have sum p.

70 **A Stochastic Optimization Algorithm for Steiner Minimal Trees**

Frederick C. Harris, Jr., fredh@cs.clemson.edu
Department of Computer Science, Clemson University

The Optimization problem is simply stated as follows: Given a set of N cities, construct a network which has minimum length. The problem is simple enough, but the catch is that you are allowed to add junctions in your network. Therefore the problem becomes how many extra junctions should be added, and where should they be placed so as to minimize the overall network length. This intriguing optimization problem is also known as the Steiner Minimal Tree Problem, where the junctions that are added to the network are called Steiner Points. A Simulated Annealing approach is proposed for this NP-Hard problem, and some very exciting results from it are presented.

71

Bagchi and Bagchi designs with their automorphisms and codes.

Franklin D. Shobe, Clemson University

The designs of Bagchi and Bagchi include the only two known unitals with an automoprphism group that contains a single cycle acting regularly on the points. One of these unitals, which was discovered slightly earlier by R. Mathon, is the first known unital whose parameter, 6, is not a prime power.

Some of the Bagchi and Bagchi designs have a structure of subdesigns similar to the flats in an affine geometry. This occurs when the parameter q is a power of the parameter p. When q = p, the Bagchi and Bagchi design BB(p, p, 1) is isomorphic to the Desarguesian affine plane, which follows from a result of Dembowski and Ostrom. When $q = p^d$, for d > 1, $BB(p, p^d, 1)$ is a $2\text{-}(p^{d+1}, p, 1)$ design but is different from the design of points and lines of $AG_{d+1}(F_p)$.

The automorphism groups of many of the Bagchi and Bagchi designs act imprimitively on points with one particular set of blocks forming a complete system of imprimitivity.

72

A sufficient condition for graphs to be class one

Hugh Hind, University of Waterloo

An early conjecture, possibly due to Dirac, claims that a d-regular graph of even order p is class one if $d \geq \frac{1}{2}p$. It has been shown that the conjecture is correct if the $\frac{1}{2}$ is replaced by $\frac{5}{6}$. In this talk, it will be shown that there are corresponding degree sequence based sufficient conditions for irregular graphs. In particular, it will be shown that a graph having even order p, maximum degree Δ, minimum degree δ and deficiency def(G) is class one if $\Delta - \delta \leq \frac{1}{2}\text{def}(G)$ and $7\delta - 6\Delta \geq \frac{5}{6}p$.

73

Assignment Problem and Some of Its Generalizations

Huakang Yang, Mathematics, Yunnan University, China
(Now Visiting Mathematics and Statistics, Simon Fraser University, Canada)
Huishan Zhou*, Mathematics and Computer Science, Georgia State University

In this paper we survey algorithms for the solution of the general assignment problem and propose an improvement for the Kuhn-Munkres' algorithm. The personnel-assignment problem is the problem of choosing an optimal assignment of n man to n jobs, assuming that numerical ratings are given for each man's performance on each job. An optimal assignment is one which makes the sum of the men's ratings for their assigned jobs a maximum. This problem is equivalent to finding a maximum-weight matching in a weighted bipartite graph. In practice more constraint conditions are often added to the assignment problem or some concepts are generalized. If some ratings equal zero, it is possible that $m(m < n)$ workers can maximize the total effectiveness. We can determine such a set of minimal workers. We can find an optimal matching in which the difference between the maximal weight and the minimal weight is minimum. We can also introduce the fuzzy concept into the assignment problem to reflect more closely the real world problem. At the end we will mention our improvement for the original Kuhn-Munkres' algorithm.

Key Words: assignment, weighted bipartite graph, maximum matching, fuzzy, Kuhn-Munkres.

74

Polynomial representations of binary functions.

Bruce W. Atkinson, Palm Beach Atlantic College

Fix a positive integer n and let S be the set of n-tuples of elements of the set {0,1}. A function with domain S is called a binary function. It is shown that any binary function is a polynomial in the coordinates on S. This results as a corollary of the invertibility of the {0,1}-valued matrix on subsets of {1,2,...,n}, B(A,C), where B(A,C) = 1 iff C is a subset of A. The polynomial representation of binary functions is then used to derive an explicit inverse of B. This, in turn, has the classical Mobius inversion formula as a corollary.

75

On contractible and vertically contractible elements in 3-connected matroids and graphs

Haidong Wu, Louisiana State University

An edge e in a 3-connected graph G is contractible if the contraction G/e is still 3-connected. The problem of bounding the number of contractible edges in a 3-connected graph has been studied by numerous authors. In this paper, the corresponding problem for matroids is considered and new graph results are obtained. Two matroid notions of 3-connectedness will be used: vertical 3-connectedness, the analogue of 3-connectedness for graphs; and (Tutte) 3-connectedness, which is vertical 3-connectedness with the additional requirement that the matroid is simple. An element e in a 3-connected matroid M is contractible or vertically contractible if its contraction M/e is, respectively, 3-connected or vertically 3-connected. Cunningham and Seymour independently proved that every 3-connected matroid has a vertically contractible element. In this paper, we study the contractible and vertically contractible elements in 3-connected matroids and get best-possible lower bounds for the number of vertically contractible elements in 3-connected and minimally 3-connected matroids. We also prove generalizations of Tutte's Wheels and Whirls Theorem for matroids and Wheels Theorem for graphs.

76 Bases for codes of designs from finite geometries

Jennifer D. Key, Clemson University

The codes associated with the designs of points and fixed-dimensional subspaces (or flats) of a finite-dimensional projective (or affine) geometry over the finite field $\mathbf{F}_q$ are Reed-Muller or generalized Reed-Muller codes. The minimum weight in each case is the cardinality of the subspace (or flat) and the minimum-weight vectors are the characteristic functions of these subspaces (or flats). The dimension of the code is also known in each case.

When $q = p$ is a prime the associated designs and codes have consequent properties that have led to some conclusions about *bases* for the codes that consist of minimum-weight vectors. We will illustrate some recent constructions, in particular for the case of designs of points and lines: for planes (Moorhouse, Dougherty and Blokhuis and Moorhouse); for $p = 2$ and 3, all dimensions (Key and Sullivan). We pose some natural questions that arise in the general case.

77 On Graphs with Irregular Coloring Number Two

A.C. Burris, Mathematics Department, Youngstown State University, Youngstown, Ohio 44555

An edge-coloring is called *irregular* if every two distinct vertices are incident to different multisets of colored edges. The minimum number of colors required for an irregular edge-coloring of a simple graph G is denoted by $c(G)$. It is easy to show that $c(K_n) = 3$. In this paper we prove that if H is any non-empty graph with $|E(H)| < n/2$ then $c(K_n - H) = 2$. We also exhibit edge-minimal graphs with irregular coloring number 2.

78 **The Target Optimization Problem Has a Polynomial Solution**

D. Kountanis*, K.L. Williams, Western Michigan University

The Target Optimization (TO) problem deals with a set of sites, a number of projectiles available at each site, and a set of targets with possibly differing hitting requirements. The objective is to efficiently find an optimal solution such that each target is targeted by its hitting requirement, each projectile for each target is sent from a different site, and the sum of the distances travelled by all projectiles to all targets is minimal.

We first develop a graph model of TO. Then we show that the corresponding decision problem may be polynomially mapped to a special case of the Minimum Product-Cost Flow problem which has been shown to have a polynomial time solution, therefore the decision problem is also polynomial. The cost of an optimal solution is computed and this mimimal cost is then used to derive a polynomial solution of TO. Finally, a fast polynomial solution of a special case of the problem is provided.

79 **Parallel Algorithms for Maximum Subsequence and Subarray**

Kalyan Perumalla, Georgia Tech *and* Narsingh Deo, UCF

Given a sequence Q of n numbers (positive and negative), the *maximum subsequence* of Q is the contiguous subsequence that has the maximum sum among all contiguous subsequences of Q. Given a two-dimensional array A of $n \times n$ numbers (positive and negative), the *maximum subarray* of A is the contiguous subarray that has the maximum sum among all contiguous subarrays of A. We present two $O(\log n)$-time parallel algorithms—one for finding the *maximum subsequence sum* of a given sequence, and the other for finding the *maximum subarray sum* of a given array. The former is optimal on EREW PRAM. The latter is optimal on a CREW PRAM; however, the time-complexity of $O(\log n)$ is still achieved on an EREW PRAM, with an increase in the cost by a factor of $O(\log n)$. Our solutions to the problems, using parallel prefix sums and its variations, are substantially different from the ones that have appeared so far in the literature.

80 **MAD Partitioning for Grid Graphs**

D. L. Meiers and P. J. Slater* - University of Alabama in Huntsville

Multifacility location problems for networks involve selecting a certain number p of points at which to locate facilities, each customer vertex is serviced by the nearest facility, and one is interested in minimizing such things as the average or maximum customer-facility travel time. Here the modified problem in which the p facilities are all different is considered.

The general problem is to partition the vertex set V(G) of a graph of order t into a set F of p "facility vertices" and a set C of $t-p$ customer vertices so as to Minimize the Average Distance between customer and a facility. We focus on the grid graphs $G_{m,n} = P_m \times P_n$.

81

Computing Kernels of non-linear codes.

Kevin T. Phelps, Mike LeVan*, Auburn University

Given a binary code C, the set, K, of all vectors which leave C invariant under translation is called the kernel of C. The main concern of this paper is the development of an efficient algorithm for computing the kernel of C. Using the most obvious brute force method, one can find the kernel of C in $|C|^2 \log |C|$ steps. However, this shall be shown to be an inefficient algorithm, and a more efficient algorithm, which finds the kernel in a magnitude of $|C| \log |C|$ steps shall be presented.

82

Totally Multicolored Subgraphs

Hanno Lefmann,
Universitaet Dortmund, FB Informatik, LS 2, Dortmund, Germany

Several problems in Graph Theory or Combinatorics can be described in terms of edge-colorings of graphs. A typical question is the following: Given an edge-colored complete graph on n vertices, what is the maximum size f(n) of the largest totally multicolored complete subgraph? In this talk we will give new bounds for f(n) for some particular colorings, focussing on colorings induced by the distances between points in geometric configurations like grids and others. For grids for example, these improve earlier results of Erdoes and Guy. It turns out that the sum of the squares of the multiplicities of the occuring colors is of importance here. In particular, for convex sets of n points in the plane we can bound this sum by cn**3, which shows a conjecture of Erdoes and Fishburn. (joint work with Torsten Thiele)

83

Statistical Properties of the MAX-CUT Problem

Jon Berry*, Mark Goldberg, Rensselaer

The MAX-CUT problem is one of several NP-hard graph partitioning problems that have important practical applications. There are many heuristic approaches to the problem, most of which iteratively improve upon a complete initial partitioning. The alternatives are constructive heuristics which build a partitioning step-by-step, typically in a greedy manner. These are seldom used because of their poor relative performance. However, experimental evidence indicates that a very small percentage of nongreedy steps are actually necessary to construct an optimal or near-optimal solution. Moreover, the distributions of nongreedy steps have remarkably stable characteristics that are tightly correlated with some simple parameters. In this talk, we present experimental results describing the number and distribution of nongreedy steps necessary to build a near-optimal partitioning. We discuss the correlation between these steps and some statistics present with every vertex placement. We then discuss some observations drawn from the experimental data, including an anomaly in the famous Kernighan-Lin heuristic which led to the discovery of a new and competitive partitioning procedure.

84

The Catalan Numbers via a Partition

Ralph P. Grimaldi
Rose-Hulman Institute of Technology

For $n \geq 0$, $c_n = \frac{1}{n+1}\binom{2n}{n}$ denotes the n-th Catalan number, where we find that

$$c_0 = c_1 = 1$$
$$c_2 = 2$$
$$c_3 = 5 = 2+3$$
$$c_4 = 14 = (2+3) + (2+3+4)$$
$$c_5 = 42 = [(2+3)+(2+3+4)] + [(2+3)+(2+3+4)+(2+3+4+5)].$$

Here it *appears* that for $2 \leq n \leq 4$, we obtain c_{n+1} from c_n by replacing each 2 by $2+3$, each 3 by $2+3+4$, and each 4 by $2+3+4+5$. Does this scheme hold true in general? In this paper we set about showing how these partitions continue to generate the Catalan numbers,

85

Counting Subgraphs of a Regular Graph

Robert Beezer University of Puget Sound

We are interested here in the cardinalities of the isomorphism classes of edge-induced subgraphs of a regular graph. Specifically, we will demonstrate a method for determining linear equations relating these cardinalities. These equations are remarkable in that the coefficients depend on the specific regular graph only through its order and degree. By solving systems of these linear equations, we can express all of the cardinalities in terms of just the order and degree of the graph, together with the cardinalities of the isomorphism classes represented by subgraphs containing no vertices of degree one.
These equations turn out to be a powerful, general tool for analyzing the structure of a regular graph. We will demonstrate this by showing how they can be used to prove that certain cages are characterized by the numbers of matchings they contain.

86

Enumeration of Maximal Codes

Norman J. Finizio and James T. Lewis*
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The maximum number of words in a code whose words have length m, use q symbols, and differ in at least two positions is known to be q^{m-1}. We study the number of such maximal codes. There is a connection with Latin squares and with permutation cubes (a three dimensional generalization of Latin squares).

87

Embedding edge-colorings into 2-edge-connected k-factorizations of complete multipartite graphs

Evan B. Wantland Auburn University

We find necessary and sufficient conditions for the embedding of an edge-colored complete multipartite graph into an edge-colored $K_t^{(s)}$ in which the edges of each color induce a k-factor. Necessary and sufficient conditions are also obtained when we require the extra condition that each k-factor be 2-edge-connected.

This presentation will focus mainly on the method used involving *amalgamations of graphs* and *outline graph decompositions*. We will also present a complete characterization of the embedding of an edge-colored $K_{a,b}$ into a k-factorization of $K_{t,t}$ including the case where each k-factor is required to be connected.

88

Asymptotically Good Hyperedge Covers from their Fractional Counterparts

Jeff Kahn and Mark Kayll*, Department of Mathematics, Rutgers University

In the early 1980's, V. Rödl proved the Erdős-Hanani Conjecture, sparking a remarkable sequence of developments in the theory of packing and covering in hypergraphs with bounded edge sizes. Generalizations were given by P. Frankl and Rödl, by N. Pippenger, and by others. In each case, an appropriate *semirandom* method is used to "construct" the desired optimal object (covering, matching, coloring) in several random stages, followed by a greedy stage. The current work, which further generalizes some of the above results, is again probabilistic, and uses both earlier ideas and connections with so-called "normal" distributions on the set of matchings of a graph. For fixed $k \geq 2$, $\mathcal{H}$ a k-bounded hypergraph, and $t : \mathcal{H} \to \mathbb{R}^+$ a fractional cover, a sufficient condition is given to ensure that the edge cover number $\rho(\mathcal{H})$, *i.e.*, the size of a smallest set of edges of $\mathcal{H}$ with union $V(\mathcal{H})$, is asymptotically at most $t(\mathcal{H}) = \sum_{A \in \mathcal{H}} t(A)$. This settles a conjecture first publicized in Visegrád, June 1991.
Keywords: hypergraph, covering, fractional, asymptotic, semirandom, normal distributions on graph matchings.

89

Lists the Size of Catalan Numbers

Charles A. Anderson* and Stanley E. Payne
University of Colorado at Denver

One of the authors encountered the Catalan numbers unexpectedly while trying to enumerate a class of graphs. Categorizing these graphs by their clique structure led to a sequence of lists with some unusual properties; neither author has seen these lists in the previous literature. Each list can be generated recursively from the preceding list by a simple rule. We discuss some of the background leading up to these lists, and we show that the sum of the entries in the nth list is the nth Catalan number, and the number of entries is the (n-1)st Catalan number.

90

On k-hallian graphs and their binding number

Michelle Kittrell, Barry Piazza*, U. of Southern Mississippi

A graph G is *hallian* if G has a (1,2)-factor and it is *k-hallian* if the graph G-A is hallian for every subset A of vertices of order at most k. The largest k such that G is k-hallian is called the *hallian index* of G and is denoted by h(G).

The *binding number* of G is given by $b(G) = \min_{S \subseteq V} \left\{ \frac{|N(S)|}{|S|} \mid S \neq \emptyset, N(S) \neq V \right\}$, where the N(S) denotes the neighborhood of S. It is easy to see that $\frac{|V|-1}{|V|-\delta}$ is an upper bound for b(G).

Boroweicki and Michalak showed that if $\kappa(G) \geq h(G) + \delta(G) - 1$, then $b(G) = \frac{|V|-1}{|V|-\delta}$.

We exploit this idea to determine the binding number of generalized Petersen graphs to provide alternative proofs to theorems of Goddard dealing with Harary graphs and inflation graphs.
KEYWORDS: binding number, k-hallian graphs, generalized Petersen graphs, Harary graphs, inflation graphs.

TUESDAY, MARCH 8, 1994
11:30 a.m.

91

Greedy Codes

Laura Monroe, University of Illinois at Chicago

A greedy code is an error correcting (n, k, d) code generated the application of a greedy algorithm to the vectors of length n over GF(2**(2**a)), arranged in some ordering. These codes have been studied for several years, by Levenstein (1960), Conway and Sloane (1986), Pless and Brualdi (1993), and others.

The greedy code generated over GF(2**(2**a)) using a natural B-ordering is linear. Codes produced via this greedy algorithm using a triangular basis for the B-ordering have had dimension at most one less than the best known codes having the same n and d. Parity check matrices may also be generated over any base field by a greedy method applied to column vectors. These also produce very good codes. A new, simple proof of the linearity of greedy codes over the binary field will be presented and the structure of such codes will be discussed. Results obtained for specific d will be presented.

92

A NEAR CHARACTERIZATION OF UNIQUELY COLORABLE MAPS

Wallace J. Growney, Susquehanna University

A 2-edge-connected plane graph without loops, multiple edges, or vertices of degree less than 3 is called a map. For a map G any face n-coloring partitions the face set F(G) into n color classes. If all face n-colorings induce the same partition, then G is said to be uniquely face n-colorable and in the case n = 4 we simply say G is uniquely face colorable. Twenty-five years ago Chartrand and Geller proved every uniquely face colorable map is cubic. A cubic map G is uniquely edge colorable iff it is edge 3-colorable and every such coloring induces the same partition of the edge set E(G). Since G is uniquely edge colorable iff it is uniquely face colorable, we simply call it uniquely colorable. Can we characterize uniquely colorable maps as those cubic maps with exactly three Hamiltonian circuits? If a cubic map G is uniquely colorable, then G possesses exactly three Hamiltonian circuits. Although the converse is not proven I have shown (a) the set of all "known" cubic maps with exactly three Hamiltonian circuits is arbitrarily large and each element is uniquely colorable and (b) for any cubic map with exactly three Hamiltonian circuits, the "natural" edge 3-coloring based on each circuit induces the same partition of E(G).

93

Matching extensions in 4-connected 4-regular planar graphs

R.E.L. Aldred*, S. Bau and D.A. Holton
University of Otago

Let G be a 4-connected 4-regular planar graph. We determine conditions which guarantee that a given pair of independent edges in G belong to a perfect matching in G.

94

Computation of the Probability Density Function for the Collector's Problem

Thomas J. O'Reilly, Saint Joseph's University, Philadelphia, PA 19131

If there are a number of equally likely outcomes for each independent trial, and an experiment consists of repeating trials until each of the possible outcomes has occurred at least once, then computing the expected value for the length of an experiment is the collector's problem. A difference equation with boundary and initial conditions is developed for this problem. From the solution of these equations, the probability distribution function, the mode, the median, and the mean are determined.

The collector's problem is generalized to where more than one occurrence of each of the possible results must occur before the collection is complete. An expression for the probability density function is derived for this general case, and the probability distribution function, the mode, the median, and the mean are presented.

95

GAUSSIAN GRAPHS

MICHAEL L. GARGANO*, JOHN W. KENNEDY
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A CONNECTED (2, 2)-REGULAR DIGRAPH IS GAUSSIAN IF THERE EXISTS A PLANAR REPRESENTATION THAT ADMITS A GAUSSIAN CIRCUIT (I.E., A DIRECTED EULER CIRCUIT WHOSE REPRESENTATION IN THE PLANE USES ONLY CROSSING PATHS). A GRAPH THEORETIC PROOF OF GAUSS' THEOREM ON THE SELF-INTERSECTION OF A JORDAN CURVE IN THE PLANE IS PRESENTED.

96 An extension of some results of J. Bruck and M. Blaum on Neural Networks, Error-Correcting Codes, and Polynomials over the n-Cube

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C. Rentería (I. Politécnico Nacional, México)

(Keywords: Neural networks, Goppa codes, Energy Functions)

In " Neural Network, Error-Correcting Codes, and Polynomials over the Binary n-cube", (IIIE Trans. on Inf. Theory, vol. 35, No.5, 1989, 976-987), J. Bruck and M. Blaum introduced a function on a linear block code over the field Z_p which plays a similar role to the energy function on a neural network. In this talk we show that the definition of this "energy function" can be extended to error-correcting linear codes over any finite field with q elements. Similar results as those obtained by the above authors are given, and they are illustrated with examples of codes defined on algebraic curves (Goppa codes).

97 Some Results on Equalized Total Coloring

Hung–Lin Fu

A total k–coloring of a graph G is a map $\pi : V(G) \cup E(G) \to C$ such that $|C| = k$ and no incident or adjacent pair of elements of $V(G) \cup E(G)$ receive the same color. A total coloring π is equalized if for each pair of distinct colors c_1 and c_2 in C, $||\pi^{-1}(c_1)| - |\pi^{-1}(c_2)|| \leq 1$. In this paper, we first pose a conjecture about equalized total coloring : for each graph G, G has an equalized total k–coloring for every $k \geq \Delta(G) + 2$. Then we show that the conjecture is true for several classes of graphs.

98 THREE CONNECTED GRAPHS, PERFECT MATCHINGS, AND CONTRACTIBLE EDGES

R.E.L. ALDRED, Univ. of Otago and R.L. HEMMINGER*, Vanderbilt Univ.

If G is a 3-connected graph with at least five vertices and if M is a maximum matching in G, then it is known that M contains at least one contractible edge of G; moreover, those containing only one such edge have been characterized. In this paper we characterize the 3-connected graphs having a perfect matching M that contains at most two contractible edges of G.

99 Statistics on Graphs

David J. Foulis, University of Massachusetts

Events affiliated with an experimental situation may be represented by the nodes of a graph, two events being connected by an edge if and only if they are simultaneously testable and disjoint. Such a representation is useful for dealing with censored data or with incompatible data sources. Frequency data yield a nonnegative-integer valued function F on the nodes of the graph. We consider the problem of making probabilistic inferences from the data encoded by F, either using maximum likelihood, or the mechanisms of Bayesian statistics. *Key words and phrases*: Graph, frequency data, censored data, probability model, maximum likelihood, Bayesian inference.

100 **Spanning Eulerian Subgraphs Involving Distances**

Zhi-Hong Chen, Butler University, Indianapolis, IN 46208

Let G be a 2-edge-connected simple graph of order n. A graph G is said to satisfy the Fan-type condition if max{d(u),d(v)}>= n/p-s for any u, v in V(G) with dist(u,v)=2, where p> 1 is a fixed integer, and s in {0,1,2}. In this paper, we study the existence of spanning eulerian subgraphs of a graph satisying the Fan-type condition.

TUESDAY, MARCH 8, 1994
12:10 p.m.

101 **Construction of (48,24,12) doubly-even self-dual codes**

Sheridan Houghten, Concordia University

According to Mallows and Sloane (1973), the largest minimum weight of a self-dual, doubly-even code of length n and dimension n/2 is d = 4[n/24]+4.
Self-dual codes having the largest possible minimum weight are called extremal codes. Of such codes, the Golay code is the only (24, 12, 8) code and the Extended Quadratic Residue code is the only known (48, 24, 12) code. There is no known (72, 36, 16) extremal code. Such codes are of particular interest because for any non-zero weight w, the codewords of weight w form a 5-design. The search for other extremal codes of length 48 may be divided into three cases, based on the structure of the generator matrix. Using estimates of the search size based on work done on a sequential computer, we designed an algorithm to complete the search on a parallel computer assuming one of the cases. For this case, we found only codes that are isomorphic to the Extended Quadratic Residue Code.

102 TOTAL CHROMATIC POLYNOMIALS OF GRAPHS

Ruth A. Bari, The George Washington University

Let G be a simple graph, with E edges and V vertices.

A total coloring of G is an assignment of colors to the elements of G (vertices and edges), such that no two adjacent or incident elements are colored alike. The total chromatic number $\chi_T(G)$ is the smallest integer n such that G has a total n-coloring.

The total chromatic polynomial of G, T(G,x), is a function that counts, for each integer n>0, the number of total n-colorings of G. To compute T(G,x), we associate with the graph G a supplemental graph G*, in which each edge ij of G is represented by a vertex e_{ij}, adjacent to vertices i and j in G*. The chromatic polynomial of G* is the total chromatic polynomial T(G,x), and $\chi_T(G)$ the smallest integer n such that $T(G,n) \neq 0$.

103 **Extensions of factorisation results on simple graphs to multigraphs**

Shailesh Tipnis, Illinois State University.

It has been shown that an rs-regular multigraph G with maximum multiplicity less than or equal to r can be factored into r regular, simple graphs if we allow the addition (deletion) of a small number of hamilton cycles to (from) G. We will use this theorem to obtain extensions of some factorization results on simple graphs to new results on multigraphs.

104 **Hypergeometric Analogues of Multinomial Type-1 Dirichlet Problems.**

Milton Sobel*, UCSB and K. Frankowski, UMinn.

This paper deals with basic probability questions in a hypergeometric setting. It develops a uniform notation and a framework analogous to the one developed by the authors for the multinomial setting and published in volume 4 of Selected Tables of Mathematical Statistics. Some remarkable new analogies have been found and proved between multinomial and hypergeometric problems and these lead to similar efficient algorithms to obtain highly accurate hypergeometric results. The paper gives a unified development for solving a wide class of hypergeometric problems and presents a vast world of applications to combinatorial and finite sampling situations. The analogous results for negative hypergeometric sampling will be put in a separate paper.

KeyWords: Multivariate hypergeometric problems, Finite population sampling without replacement, Dualities among joint probabilities.

105 **Composition of the operators line, middle and total given an Eulerian Graph**

L. Boza* and A. Marquez. Departamento de Matematica Aplicada I. Universidad de Sevilla (Spain)

In this Note, we characterize when a given composition of the operators line, middle and total applied to a graph gives an eulerian graph.
Defining the appropiates families of graphs, and by using the above characterization, we give a linear time algorithm solving the mentioned problem. Moreover, we repeat the construction in the case of infinite graphs.

106

A Minimum Change Algorithm for Generating All Subsets of a Fixed N-Set Which Contain K or Fewer Elements

Beverly Jamison and Vanessa Job*
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Marymount University, Arlington, VA 22207

We present a minimum change algorithm for generating all subsets of an n-set which contain k or fewer elements. It is more efficient than (a) using a full gray code and discarding the unwanted subsets or (b) generating all 1-sets, 2-sets, ..., k-sets because it (a) is linear in the number of sets produced and (b) gives a change of 1 or 2 elements between successive subsets. For combinatorial searches where the complexity of creating the next object of the search is proportional to the change between the subsets from which the objects are built, this algorithm is more efficient than existing algorithms.

107

Combining Concept Hierarchies "Modulo Symmetries" of a Relational Structure

Fritz Lehmann, *GRANDAI* Software, 4282 Sandburg, Irvine, CA 92715, fritz@rodin.wustl.edu

A description may use concepts from several different hierarchies ordered by generality (for time, place, substance, purpose, formal scales, lexical inclusion, etc.); these may be structured as trees, lattices, chains or posets. If a description is a vector of values from such structured domains, the induced hierarchy of all possible descriptions ordered by generality is the direct product of the posets of the domains. These feature-vectors fail to account for descriptions which have relational structure. So do (asymmetric) functional "feature structures," in which you can have a mother but not two sisters.

An order-sorted relational structure (semantic network) is a graph or hypergraph labeled with type-names from one or more concept hierarchies. What is the induced order structure on semantic networks ordered by generality (subsumption)? There are two sources of order: the structures of the various concept hierarchies which are combined, and the "universal" poset of graph inclusion since every descriptive graph is subsumed by its subgraphs. The latter poset is a congruence. Its quotients are striking: when graph inclusion is factored out, it turns out that order-sorted networks with particular "skeleton" graphs form distinctive posets which depend on the symmetries in the graphs. The skeleton product of hierarchies is the poset of descriptions sharing the same graph, and the fret product of hierarchies is the poset of descriptions with all graphs. They are by no means lattices. Skeleton product is direct product only when the controlling graph is "rigid" (has no symmetries). This theory now covers undirected graphs ordered by (injective) subgraph isomorphism; it must be extended to directed hypergraphs ordered by noninjective graph morphism, which corresponds to implication in typed predicate calculus without negation. Embedding the resulting poset into a Boolean lattice yields corresponding bit-strings for near-instantaneous inference using bit-wise logic in conventional computers.

Keywords: type-labeled graphs, order theory, subgraph isomorphism, graph automorphism, fret product, skeleton product, relational structures, semantic networks, concept hierarchies, concept lattices, conceptual graphs, posets, type theory, order-sorted logic, facets, artificial intelligence, knowledge representation, taxonomy.

108

Cycles in Bipartite Graphs

K. Jay Bagga
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Muncie, Indiana

Badri Varma*
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Menasha, Wisconsin

Several sufficient conditions in terms of degrees or number of edges for a graph to have hamiltonian like properties have been improved in the case of bipartite graphs by us and others. This paper presents several new interesting results in this area.

109

***H*-DISTANCE IN GRAPHS**

Gary Chartrand, Western Michigan University
Heather Gavlas*, Western Michigan University
Michelle Schultz, Western Michigan University

For a connected graph H of order at least 3, the H-distance $d_H(G_1, G_2)$ between two edge-induced subgraphs G_1 and G_2 of the same size in a connected graph G is defined. Some properties of this concept are discussed. For a graph G of size q, and an integer n with $1 \le n \le q$, the H-rotation graph $H_n(G)$ is that graph whose vertices are the edge-induced subgraphs of size n in G such that two vertices G_1 and G_2 of $H_n(G)$ are adjacent if and only if $d_H(G_1, G_2) = 1$. The graph $H_1(G)$ is referred to as the H-line graph of G and is also denoted by $HL(G)$. We show that if H is a connected graph of order at least 3 such that every two adjacent edges belong to a common 3-, 4-, or 5-cycle, then $HL(G)$ is hamiltonian.

110

FORCED ORIENTATION NUMBERS OF GRAPHS

Gary Chartrand, Western Michigan University
Frank Harary, New Mexico State University
Michelle Schultz*, Western Michigan University
Curtiss E. Wall, Norfolk State University

If, for a given assignment of directions to a subset S of the edges of a 2-edge-connected graph G, there exist orientations of $E(G) - S$ so that the resulting digraph is strong, then we say that the given assignment can be extended to a strong orientation of G. The forced (strong) orientation number $f(G)$ of G is the minimum cardinality among the subsets of $E(G)$ to which some assignment of orientations can be uniquely extended to a strong orientation of G. It is shown that if G is a 2-edge-connected graph of order n and size m, then $f(G) = m - n + 1$. Similar considerations are discussed for unilateral orientations of G.

TUESDAY, MARCH 8, 1994
4:40 p.m.

111 Generating Neckties: Algorithms, Gray codes, and Parity Differences

Frank Ruskey
University of Victoria, Victoria, B. C., Canada

*Terry MinYih Wang**
North Carolina State University, Raleigh, North Carolina

Following Polya, we define a necktie of n bands in k colors to be an equivalence class of k-ary n-tuples under reversal. In this talk, we present an algorithm to generate all of the n-band k-color neckties in constant amortized time. Moreover, we give a necessary and sufficient condition for generating neckties in Gray code order, so that successive neckties differ only in one band. For neckties of two colors, 0 and 1, by calculating parity differences, we further give a necessary condition for a Gray code listing of all of the n-band neckties containing exactly d 1's, so that only one adjacent interchange of a 0 and a 1 is allowed between successive neckties.

112 FINDING THE LARGEST SUBORDER OF FIXED WIDTH IN POSETS

George Steiner McMaster University, Hamilton, Canada

Finding the maximum weight suborder of fixed width in a partially ordered set is an interesting combinatorial problem with applications in combinatorial optimization and scheduling. We present a polynomial time solution for this problem by transforming it into a minimum cost network flow problem in an auxiliary network.

113 Spanning trees in a periodic infinite graph

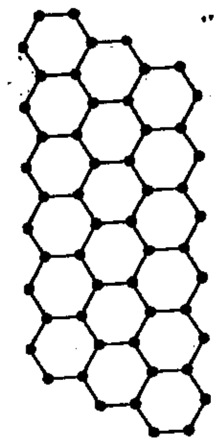
J.C. Dana* and A. Marquez. Departamento de Matematica Aplicada I. Universidad de Sevilla (Spain)

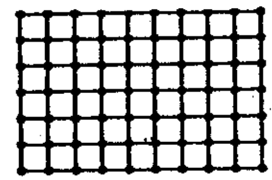
In this paper we define a family of infinite graphs generated by finite ones (periodic graphs), where it is possible to study many problems from an algorithmic point of view. Those graphs appear in very different contexts as in tilings or patterns, in infinite graphs defined by systems of equations or in some Cayleys diagrams. We develop an algorithm which gives as output the spanning tree of a graph in that family.

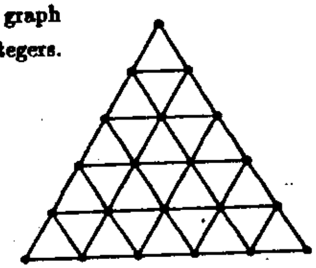
114 OPTIMAL LABELINGS of GRAPHS: BANDWIDTH

Robert Hochberg, Rutgers University

A mathematician with some free time might enjoy assigning distinct integers to the vertices of a given graph G, so that adjacent vertices are assigned nearby integers.







Indeed, the *Bandwidth* of a graph G is the least natural number B such that a clever labeler can avoid exceeding B as the difference between the labels on adjacent vertices. This paper describes a method for lower bounding the bandwidth of some planar graphs, occasionally showing that the "natural" labeling is also the best.

115 Multivertex Digraphs With Prescribed Center and Periphery

Steven J. Winters, University of Wisconsin Oshkosh

The directed distance $d(u, v)$ from u to v in a strong digraph D is the length of a shortest u–v path in D. The eccentricity $e(v)$ of a vertex v in D is the directed distance from v to a vertex furthest from v in D. The center $C(D)$ of D is the subdigraph induced by those vertices of D having minimum eccentricity; while the periphery $P(D)$ is the subdigraph induced by those vertices of D having maximum eccentricity. We show that every two asymmetric digraphs can be the center and periphery, respectively, not only of some strong asymmetric digraph, but also of some multivertex digraph.

116 **A Gray Code for Compositions of a Multiset**

Frank Ruskey, Computer Science, University of Victoria
Carla Savage(*), Computer Science, North Carolina State University

Let C(k; n_0,n_1,...,n_t) denote the set of all integer solutions to the equation x_0 + x_1 + ... + x_t = k, subject to the constraints 0 <= x_i <= n_i for 0 <= i <= t. We show that this set can be listed so that between successive solutions, one coordinate increases by 1 and one coordinate decreases by 1 and the other coordinates are unchanged. This improves an earlier solution of Ehrlich in which only two coordinates change, but possibly by more than 1.

The elements of C(k; n_0,n_1,...,n_t) can be regarded as compositions of a multiset and in the corresponding listing, successive combinations differ by one element. These objects generalize combinations and compositions for which minimal change listings are known.

117 **Closure Spaces**

John L. Pfaltz
University of Virginia

A closure space (S, φ) is a set S of elements and a closure operator φ defined on the power set of S. Besides the three standard closure axioms, we require a fourth, that, $X.\varphi = Y.\varphi$ implies $(X \cap Y).\varphi = X.\varphi = Y.\varphi$. This extra axiom appears to characterize discrete closure spaces.

Given φ one can partially order S by $X \leq_\varphi Y \stackrel{def}{\equiv} Y \cap X.\varphi \subseteq X \subseteq Y.\varphi$. It is easy to show that $\leq_\varphi$ is a partial order and, more importantly, that $(S, \leq_\varphi)$ is a lattice, $\mathcal{L}_{(S,\leq_\varphi)}$ This provides an important framework for deriving results about the underlying closure space (S, φ) itself.

Partial orders, whether arising naturally in a variety of computer science applications (which is our primary interest), or simply as abstract, formal systems provide a rich source of closure spaces.

One of the interesting applications is the concept of continuous transformation of closure spaces, several examples of which will constitute the bulk of this presentation.

118 **The center of an infinite graph (*)**

L. Boza, A. Dianez*, A. Marquez (Departamento de Matematica Aplicada I. Universid ad de Sevilla)

In this Note we extend the notion of center of a graph to infinite graphs. Thus, a vertex is center of the infinite graph G if it is center of an increasing family of finite graphs covering G. We give different characterizations of when a vertex is center of an infinite graph and we prove that any infinite graph, if it has center, has infinite centers and that any infinite graph with at least two ends has center.

119 **On the Additive Bandwidth of Simple Trees**

Michael P. Vogt* and Robert C. Brigham, University of Central Florida

A numbering of graph G = (V, E) with |V| = p is a bijection f: V → {1, 2, ..., p}. The additive bandwidth of numbering f for G is $B^+(G, f)$ = max {|f(u) + f(v) - (p+1)|: uv ∈ E} and the additive bandwidth of G is $B^+(G)$ = min {$B^+(G, f)$: f a numbering of G}. Preliminary results are given on the additive bandwidth of some simple trees.

120 **Paths and cycles in arc-connected digraphs**

Xiang-Ying Su, Department of Mathematics,
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Paths and cycles in undirected edge-connected graphs have been investigated by many people. Here we consider mainly arc-disjoint paths and cycles in arc-connected digraphs. A path or a cycle may pass a vertex more than once but can pass an arc at most once. Let D be a k-arc-connected digraph (multiple arcs allowed). We proved, among others, the following theorems: (1) Let x be a vertex of D and let l be an integer with $0 \leq l \leq k$. Then, for any l disjoint arc pairs $\{f_1, g_1\}, ..., \{f_l, g_l\}$, where $f_1, ..., f_l$ are arcs with head at x and $g_1, ..., g_l$ are arcs with tail at x, there exist in D l arc-disjoint cycles $C_1, ..., C_l$ such that $\{f_i, g_i\} \subseteq E(C_i)$ for each i ($E(C_i)$ denotes the arc set of C_i) and such that $D - \cup_{i=1}^{l} E(C_i)$ is $(k-l)$-arc-connected. (2) Let l be an integer with $0 \leq l \leq k$. If $f_1, ..., f_l$ are l distinct arcs of D, then there exist in D l arc-disjoint cycles C_i with $f_i \in E(C_i)$, $i = 1, ..., l$, such that $D - \cup_{i=1}^{l} E(C_i)$ is $(k-l)$-arc-connected. And (3) For any k triples $(x_1, f_1, y_1), ..., (x_k, f_k, y_k)$, where $x_1, ..., x_k, y_1, ..., y_k \in V(D)$ (not necessarily distinct) and $f_i \in E^+(x_i)$, $i = 1, ..., k$, (resp. $f_i \in E^-(y_i)$, $i = 1, ..., k$,) which are distinct, there exist in D k arc-disjoint $x_i - y_i$ paths P_i with $f_i \in E(P_i)$, $i = 1, ..., k$. Our results generalize the early works of Mader and Shiloach.

121 **GRAY CODES FOR ACYCLIC ORIENTATIONS**

Matthew B. Squire, Computer Science, North Carolina State Univ

Let G be a simple undirected graph, and let A(G be the set of acyclic orientations of G. A Gray code for A(G) lists its elements such that successive orientations differ by the reversal of a single edge. It has been shown that a Gray code for A(G) is not always possible. A relaxed Gray code for A(G) lists its elements such that successive orientations differ by at most two edge reversals. In this paper, we show that a relaxed Gray code for A(G) is always possible. We give an algorithm which produces this Gray code in O(m) time per orientation, where m is the number of edges of G.

122 **Complementation of Finite Topologies and Posets**

Jason I. Brown* and Stephen Watson, York University

The lattice of all topologies on a given set form a complemented lattice. There is a strong connection between finite T0 topologies and and posets, and this leads to a natural definition of complemetation for two posets P and Q on the same set X, namely P and Q are complementary if and only if (as directed graphs) they share no arcs except loops but the transitive closure of their union is X x X. We survey some new combinatorial results on complementation for finite topologies and posets. In particular, asymptotic upper and lower bounds for the maximum number of pairwise complementary posets on a set of size n will be discussed, as well as the problem of determining when a finite topology has a homeomorphic complement.

123 **Spanning paths in infinite planar graphs**

Robin Thomas, Georgia Tech

We prove a conjecture of Nash-Williams that if G is a 4-connected infinite planar graph such that the deletion of any finite set of vertices leaves exactly one infinite component, then G has a one-way infinite spanning path.
This is joint work with Nate Dean and Xingxing Yu.

124 **The Additive Bandwidth of a Union of Stars**

Frederick W. Hackett, Richard M. Caron,
Dept. of Mathematics, University of Central Florida, Orlando, FL 32816
Julie R. Carrington, Mathematical Sciences,
Rollins College, Winter Park, FL 32789

The primary purpose for studying the graphical invariants bandwidth and additive bandwidth is to condense the data of a graph's adjacency matrix into as small a region as possible. Bascuñan, Ruiz and Slater showed that the additive bandwidth of a $K_{m,n}$ graph is $B^{+}(G) = n - \max\left\{\left\lfloor \frac{n-2m+2}{2} \right\rfloor, 1\right\}$. This paper deals with determining the additive bandwidth of unions of a sub-class of the $K_{m,n}$ family, namely the star graphs, $K_{1,n}$.

125 **Cycles of All lengths in Semi-complete Digraphs**

Marjorie Darrah*, West Virginia University
Yi-Ping Liu, Nanjing Normal University, China
Cun-Quan Zhang, West Virginia University

A digraph is called semi-complete if for each pair of vertices $u, v \in V(D)$, either uv, vu, or both are in $A(D)$. Obviously, this generalizes the idea of a tournament graph; therefore, many theorems true for tournaments are also true for semi-complete digraphs. For tournaments, it has been shown that except for a family of counterexamples, all arcs of an arc-3-cyclic tournament are contained in cycles of all possible lengths k, $3 \le k \le |V(D)|$ (this property is referred to as arc-pancyclic). However, this result does not hold for semi-complete digraphs since Bang-Jensen found a semi-complete digraph that is not arc-pancyclic and does not fit into the family of counterexamples. We generalize this result to semi-complete digraphs and show that in certain cases arc-3-cyclic semi-complete digraphs are arc-pancyclic.

126 Disjoint Cycles in $K_{1,r}$-free Graphs
Lisa R. Markus, Furman University

Let p denote the number of vertices of a graph and let q denote the number of edges. A family of cycles in a graph is *disjoint* if no two of them have a common vertex. Posá proved that any graph with at least $3p - 5$ edges contains two disjoint cycles. In this talk, we present results for the number of edges necessary to ensure the existence of two disjoint cycles in $K_{1,r}$-free graphs and of k-disjoint cycles in claw-free graphs. This talk includes joint work with Guantao Chen, Richard H. Schelp and Hunter S. Snevily.

127 **W-p-outerplanar graphs**
Jose Caceres*, Alberto Marquez

In this Note, we extend Oubina and Zuchello's W-outerplanar graphs (those graphs with a plane embedding so that all the vertices of a non-empty subset W of vertices lie on the exterior face -Discrete Mathematics 51(1984) 243-249) to locally finite countable graphs with embeddings such that the vertex set has no accumulation point in the plane. We consider two cases: when all the vertices of W must lie on a bounded face or on a unbounded face.

128 **An Improved Order-based Bound On The Chromatic Index Of Multigraphs**
Mike Plantholt, Illionis State University

A standard lower bound for the chromatic index of a multigraph is given by the combined maximum of the vertex degrees and the odd set quotients. This lower bound is sometimes called the fractional chromatic index. Longstanding conjectures by Goldberg and Seymour would imply that no multigraph can have a chromatic index that exceeds the fractional chromatic index by more than 1 . We verify this result for multigraphs of order at most 16, and show, more generally, that the chromatic index of any multigraph of order n cannot exceed its fractional chromatic index by more than the integer round-up of n/8 - 1.

129 **CYCLE-EXTENDABLE POWERS OF GRAPHS**
Lubomir Soltes, Memphis State University

We generalize a result of Sekanina who proved that the cube of any connected graph with at least 3 vertices is hamiltonian. A cycle C in a connected graph G is extendable, if there is a cycle in G that contains all the vertices on C plus one extra vertex. Our main result states that if G and H are connected graphs on at least 3 vertices such that the k-th power of G is the subgraph of H and H is a subgraph of the t-th power of G, t<(3k)/2-1 and k>2, then each nonhamiltonian cycle in H is extendable. The result does not hold for t=2k+1, but to find the largest value of t, for which the conclusion is valid, is an open problem.

130 **Minus k-Subdomination in Graphs**
Izak Broere, Rand Afrikaans University
Jean Dunbar*, Converse College
Johannes Hattingh, Rand Afrikaans University

Let G = (V,E) be a graph and let let k <= |V| be a positive integer. A minus k-subdominating function {kSF} is a function f: V --> {-1,0,1} with the property that the closed neighborhood sum f(N[v]) >= 1 for at least k vertices of G. The weight of a minus kSF is the sum of all function values, f(V). The minus k-subdomination number of a graph G equals the minimum weight of a minus kSF of G. We present preliminary results characterizing this parameter of a graph.
Keywords: domination, minus-, signed- and majority domination

131 **Nordhaus-Gaddum Results for the p-Vertex Clique Cover Number**

Robert C. Brigham* and Ronald D. Dutton, University of Central Florida

The p-vertex clique cover number of graph G is the smallest positive integer r for which there are subsets $S_1, S_2, \ldots, S_r$ of the vertex set of G such that the intersection of any p of the subsets is empty or induces a complete subgraph of G, and all such p-intersections form a vertex clique cover of G. Nordhaus-Gaddum results are presented for this graphical invariant.

132 **Random Graphs of Given Diameter**

Zoltán Füredi
Dept. of Mathematics, University of Illinois, Urbana, IL 61801-2917

Let $G(n,d)$ be the number of graphs of diameter d on n labeled vertices. It is well-known (and easy) that almost all graphs have diameter 2, $G(n,2) \sim 2^{\binom{n}{2}}$. Here a conjecture of Tomescu is proved that $\lim_{n\to\infty} G(n,d+1)/G(n,d) = 0$ for every fixed $d \geq 2$. Actually,

$$G(n,d) = (1+o(1))\,\frac{d-2}{2}\,n^{d-1}\,3^{n-d+1}\,2^{\binom{n-d+1}{2}}$$

holds for every $d \geq 3$, and a typical graph of diameter d is a combination of a path of length $d-1$ and a random graph on $n-d+1$ vertices.

133 **Conditional Chromatic Numbers with Forbidden Cycles**

Karen Casey and Kathryn Fraughnaugh *
University of Colorado at Denver, Denver, Colorado 80217-3364

The *conditional chromatic number* $\chi(G,P)$ of a graph G with respect to a graphical property P is the minimum number of colors needed to color the nodes of G such that each color class induces a subgraph of G with property P. The conditional chromatic number with respect to various properties has been studied by numerous authors. We investigate $\chi(G,P)$ where P is the property of having no cycle of length j for fixed $j \geq 3$. We find the conditional chromatic number with forbidden cycles for graphs of large size and for all graphs with order at most six.

134 **HAMILTON DECOMPOSITIONS OF SOME LINE GRAPHS**

David A. Pike
Department of Discrete and Statistical Sciences
Auburn University, Auburn, Alabama, USA. 36849–5307

The proof of the following theorem is the main result of this paper: If G is a bipartite $(2k+1)$-regular graph that is Hamilton decomposable, then the line graph, $L(G)$, of G is also Hamilton decomposable.

135 **Edge Minimal Cyclic Gossiping Graphs**

James Knisely*, Bob Jones University
Renu Laskar, Clemson University

Gossiping is the total exchange of information between vertices in a graph. A gossiping scheme can be described by the sequence of matchings that are used. The color classes of a proper coloring of the edges define matchings that can be used to gossip in a graph. Cyclic gossiping is gossiping that occurs by using those matchings in a cyclic manner.

Edge minimal cyclic gossiping graphs of order n are graphs that permit gossiping in an optimal amount of time (log n) such that any graph of order n with fewer edges cannot gossip in an optimal amount of time. Edge minimal graphs will be presented for even orders between 14 and 24 plus constructions will be given for several other orders.

136 Recent Results on Primitive Minimal Triangle-Saturated Graphs

Roger B. Eggleton*, Math. Dept., Illinois State University,
& James A. MacDougall, Math. Dept., University of Newcastle, Australia.

A graph is *triangle-saturated* if insertion of any additional edge creates a triangle. Such a graph is *minimal* if deletion of any one edge permits some edge to be added without creating a triangle; and is *primitive* if no two of its vertices have the same set of neighbors. We explain the role of primitivity in this context, survey a number of recent results concerning primitive minimal triangle-saturated graphs, and demonstrate the new result that the number of primitive minimal triangle-saturated graphs of order n is at least $(n/8)^n$, for all sufficiently large n.

Key words: triangle, triangle-saturated graphs.

137 Debugging Functional Programs Using Sub-tree Transformations

Manuel E. Bermudez, University of Florida, Gainesville, FL 32611

We consider the problem of graphically displaying the current status of a functional program, for purposes of debugging. We begin with the premise that to debug functional programs, source-level information is indispensable, but cannot realistically be displayed in textual form. Instead, we propose to display it in tree form, and to display the progress of the execution of the program via transformations that take place on the tree. We describe the sub-tree transformations that we have developed for the language RPAL (a superset of the type-less lambda-calculus), and discuss our vision of a general tool for debugging functional programs such as LISP, Scheme, and ML.

138 **Chromatic Polynomials for Reducible and Fully 2 contractable Graphs**

M. S. Krishnamoorthy* and B. Ramamurthy.
Rensselaer Polytechnic Institute, Troy, NY 12180.

In this paper, we describe a method to obtain a closed form expression for calculating chromatic polynomials for outer-planar, k-trees and 2 contractable graphs. We also give a heuristic method for calculating the chromatic polynomials for graphs specifically suited for hand calculations. We use this method in calculating chromatic polynomials of all graphs which has 6 vertices. In the process, we uncover some of the errors in the previously published results.

139 Hamiltonian Properties of Graphs and Double Vertex Graphs

Yousef Alavi*, Western Michigan University

Don R. Lick, Eastern Michigan University

Jiuqiang Liu, Eastern Michigan University

Let G be a (V, E) graph of order at least 2. The **double vertex graph** $U_2(G)$ of G is the graph whose vertex set consists of all 2-element subsets of V such that two vertices {u, v} and {x, y} are adjacent if and only if the two subsets have a common element and the other two elements are adjacent in G. In this paper we will give an extension to Dirac's sufficient condition for a graph to be hamiltonian and then use this result to give an improved sufficient condition for the double vertex graph of a graph to be hamiltonian.

140 **Domination Problems in Directed Path Graphs**

Mark Keil, University of Saskatchewan
Renu Laskar*, Clemson University
Paul Manuel, University of Newcastle

In this paper we study the cycle and clique domination problems in directed path graphs.

141 Minimum number of edges with at most s crossings in drawings of the complete graph

Heiko Harborth, Techn. Univ. Braunschweig, Germany

The minimum number $h_s(n)$ of edges with at most s crossings in any drawing of the complete graph K_n is discussed. Especially, it is asked for n_0 such that $h_s(n)=0$ for all $n \geq n_0$, that means, for drawings where every edge is intersected more than s times.

142 NON-REVISITING CYCLES ON SURFACES

Hari Pulapaka
Department of Mathematics, University of Florida

Let M be a polyhedral map on a surface. A cycle C of M is said to be non-revisiting if it never returns to a face of M, once it leaves . Furthermore, C is said to be non-bounding if it does not bound a cell on the surface. It was shown by Barnette that any polyhedral map on the projective plane, torus, or Klein bottle has a non-bounding, non-revisiting cycle. The existence of such cycles on other surfaces is unknown. In this talk, we define the notion of a polygonal representation of a polyhedral map. A graph-colouring conjecture is proposed and it is shown that the conjecture is true for planar graphs. It is shown that any polyhedral map on a surface has a non-bounding, non-revisiting cycle if and only if the graph-colouring conjecture is true.

143 THE CHROMATIC NUMBER OF A FACTORIZATION OF A GRAPH

Gary Chartrand*, Western Michigan University
Héctor Hevia, Universidad Católica de Valparaíso
Ortrud R. Oellermann, Brandon University

The chromatic number of a factorization of a graph is defined. In the case of complete graphs, a related parameter is defined, verified to be well-defined, and studied.

144 TOTALLY SEGREGATED GRAPHS II

D. E. Jackson*, Eastern New Mexico University, Portales, NM 88130
and
Roger Entringer, University of New Mexico, Albuquerque, NM 87131

A graph is said to be totally segregated iff no two vertices of the same degree are adjacent. Previously, an upper bound for the number of edges in a totally segregated graph with order n and maximum degree at most d had been obtained, and shown to be sharp for $2d \leq n$. That result is used here to determine the maximum number of edges possible in a totally segregated graph of order n.

145 Associative Graph Products and their Independence, Domination and Coloring Numbers

Douglas Rall *, Furman University
Richard Nowakowski, Dalhousie University

Several well-known conjectures in graph theory (e.g. Vizing's conjecture on the domination number of cartesian products and Hedetniemi's coloring conjecture for categorical products) concern the way in which a graphical parameter behaves on graph products. We consider associative graph products and their multiplicative nature relative to the domination, total-domination, independence and irredundance numbers as well as to the partitioning parameters of chromatic, achromatic, domatic and adomatic numbers.

146

NEW RESULTS ON ORTHOGONAL DOUBLE COVERS OF GRAPHS

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An orthogonal double cover of the complete graph K_n is a collection of n spanning subgraphs $G_1, G_2, \ldots, G_n$ of the K_n such that
(i) every edge of the K_n belongs to exactly 2 of the G_i's and
(ii) any two different G_i's intersect in exactly one edge.

We continue the talk of last year's conference and present several new results, e.g. the following ones.

Theorem For all positive integers n there exists an orthogonal double cover of K_n, where all the G_i's have maximum degree 2. Moreover, all G_i's have a special structure. Almost all components are vertex disjoint K_3's, whereas the only remaining component is an isolated point, a path of length 1 or 2, or a C_4's (in case of $n = 9$ 2 C_4's and one isolated point).

This proves a conjecture of Chung and West, even in a stronger form.

Theorem Let k be an interger $k \geq 3$. Then there is an integer $n_0(k)$ such that for all $n \equiv 1 \bmod k$ and $n \geq n_0(k)$ there exists an orthogonal cover of K_n, where all the G_i's consist of an isolated point and $\frac{n-1}{k}$ vertex disjoint C_k's.

This solves a problem of Hering.

147

Tournaments for triads

W D Wallis, Southern Illinois University

We consider the design of tournaments with the following properties:
- in each match, three teams compete;
- the order of teams in matches is significant.

Various regularity conditions will be imposed. For example, one might require that every unordered triple occurs exactly once or that every ordered triple occurs exactly once; one might ask that every team plays exactly once (or at most once) in each of the three positions; and so on.

In an exciting climax, a **real-world** application of such designs will be revealed.

148

3-Coloring the vertices of an arrangement.

Anthony J. Macula, SUNY Geneseo, Geneseo, NY 14454
(macula@geneseo.bitnet)

An arrangement of lines is a finite collection of lines in the Euclidean plane. If we consider the points of intersection as vertices and line segments between these points as edges, then we have a graph. In this paper we consider conditions on the intersection properties of the lines that make the resulting graph 3 vertex colorable. We will also discuss a few conjectures.

149

INTEGER DISTANCE GRAPHS

P.S. Nair, Creighton University, Omaha, NE 68178

This paper presents an extended discussion on a family of graphs. The discussion includes the derivation of degree sequence, number of edges, number of components. A necessary and sufficient condition for the graph to have an euler path and euler circuit is also presented.

150

Chessboard Graphs and Association Schemes

R. Laskar and C. Wallis*, Clemson University

The rook's graph, in which the vertex set consists of the n^2 ordered pairs on n symbols and in which vertex (i,j) is adjacent to vertex (k,l) if and only if i=k or j=l, is one member of the family of chessboard graphs, graphs whose vertices represent positions of a chessboard and whose edges represent the permissible moves of a particular type of chess piece. Additionally, the rook's graph is identical to the combinatorial structure known as the L_2 design (or association scheme). By considering similar association schemes, variations of chessboard graphs are devised, and domination parameters of these chessboard graph variants are discussed.

Key Words: chessboard graph, association scheme, domination

151

Double cover with few cycles

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A cycle of a graph is a subgraph with even degree at every vertex.

THEOREM: Let G be a bridge-less cubic graph containing no subdivision of the Petersen graph. Then G has a cycle double cover consisting of at most $\sqrt{\frac{3}{2}|V(G)|}+1$ cycles.

152

Triangular Embeddings on Surfaces with Euler Characteristic 0

Linda Valdes, San Jose State University

All triangular embeddings on the surfaces S(1) and N(2) with p = 8 vertices are given. Some necessary conditions are presented for embeddings when p > 8, and all triangular embeddings of regular graphs on these two surfaces are found.

153

Chromatic Difference Sequences

Karen L. Collins, Wesleyan Univ., Middletown CT

Define a graph G to be stable if the normalized chromatic difference sequence of G is equal to the normalized chromatic difference sequence of G^2, the Cartesian product of G with itself. It is known that circulants and finite abelian Cayley graphs are stable. Let W(G) be the clique number and A(G) be the independence number of graph G. We show that the chromatic difference sequence of a stable graph G begins with W(G) terms equal to A(G). If G contains partitionable graph H with the same clique number, then the (W(G)+1)st term is at least A(G)/A(H). For stable graphs of large odd girth, this gives upper bounds on the independence ratio of the graph which agree with previously known lower bounds.

154

Some Distance Problems on Generalized Graphs

Weizhen Gu Southwest Texas State University

Let G be a connected graph with n vertices. Let α be a permutation in $S_{|V(G)|}$. The α-generalized graph over G, denoted by $P_\alpha(G)$, consists of two copies of G along with edges $v\alpha(v)$. In this paper we investigate the relations for diameter, radius, and mean distance between $P_\alpha(G)$ and G for any permutation $\alpha \in S_{|V(G)|}$. We also investigate some extremal graphs G whose α- generalized graphs reach maximum diameter.

155

Invertible families of sets of bounded degree

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Let $\mathcal{F} \subseteq 2^{[n]}$. $\mathcal{F}$ is invertible iff there exists a permutation π of $[n]$ such that for all $U \in \mathcal{F}$, $\pi(U) \cap U = \emptyset$ (define $\pi(U) = \{\pi(i) | i \in U\}$). The degree of $\mathcal{F}$ is the maximum size of $\{U \in \mathcal{F} | i \in U\}$. Let $f(d)$ be the smallest number such that the following statement holds: For all $\mathcal{F}$ of degree at most d, if every subfamily $\mathcal{G}$ of $\mathcal{F}$ with $|\mathcal{G}| \leq f(d)$ is invertible, then $\mathcal{F}$ is invertible.

Theorem: $f(d) \leq (d-1)\binom{2d-1}{d} + 1$.

The proof uses an analysis of families of subfamilies of $\mathcal{F}$. This leads to the following covering problem on bipartite graphs: Let G be a graph. A family $\mathcal{F} \subseteq 2^{V(G)}$ is an edge cover of G iff for every edge e of G, there is a $U \in \mathcal{F}$ which includes e. $\mathcal{F}$ is a minimal edge cover of G iff for $\mathcal{H} \subset \mathcal{F}$, $\mathcal{H}$ is not an edge cover of $\mathcal{G}$. Let $g(d)$ be the maximum cardinality of a minimal edge cover $\mathcal{F}$ of a bipartite graph where $\mathcal{F}$ has degree at most d.

Theorem: $f(d) \leq g(d) \leq (d-1)\binom{2d-1}{d} + 1$.

The proof of this result uses Sperner theory.

Keywords: Extremal set theory, covering problems, invertible families.

156

On a Conjecture of László Pyber

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László Pyber conjectured that every bridgeless graph G has a circuit cover such that every vertex of G is contained in at most $\Delta(G)$ circuits of the cover, where $\Delta(G)$ is the maximum degree of G. An equivalent statement of the Circuit Double Cover Conjecture is that every bridgeless graph G has a circuit cover such that each vertex v of G is contained in at most $d(v)$ circuits of the cover, where $d(v)$ is the degree of v. This paper affirms Pyber's conjecture by a stronger result, which states that every bridgeless graph G has a circuit cover such that each vertex v of G is contained in at most $d(v)$ circuits of the cover if $d(v) \geq 3$ and in at most three circuits otherwise. Our proofs rely on results on integer flows.

157

NONORIENTABLE QUADRANGULAR EMBEDDINGS OF COMPLETE MULTIPARTITE GRAPHS

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Nonorientable quadrangular embeddings are constructed for almost all complete multipartite graphs where $v - \frac{1}{2}\varepsilon$ is an integer. The polyhedra determined by certain of these are minimal quadrangulations of nonorientable surfaces.

158

Some Results from Discharging Techniques

Daniel P. Sanders, Ohio State Univ

Essential to the solution of the Four Color Problem was the discharging method. This method can be used to solve other problems as well, in coloring and in determining the local structure of graphs on surfaces. This talk shows a couple of examples. A Heawood- or diagonal-coloring of a graph on a surface requires that two vertices must have different colors if they are either adjacent or diagonal, that is that they are adjacent to two faces which share an edge. Together with Yue Zhao, the author showed that all plane triangulations have a diagonal-10-coloring. Let edges and triangles of a graph be called light if they are incident only to vertices of low degree. A light edge inequality is presented for plane graphs of minimum degree five which shows that such a graph has a large number of edges joining either two 5-vertices or a 5-vertex to a 6-vertex. A similar light triangle inequality is presented. Graphs are presented which show that these inequalities cannot be improved.

159

NEW RESULTS AND PROBLEMS ON THE LEVERAGE IN THE N-CUBE.

Paul Erdös (Hungarian Academy of Sciences),
Peter Hamburger*, Raymond E. Pippert, and William D. Weakley
(Indiana-Purdue University at Fort Wayne).

The leverage of a set S of elements (vertices and/or edges) of a graph G, with respect to a graphical parameter P, is the change induced in P by the removal of S. We consider some new results and open problems in which the graph G is a n-cube, P is the distance, the distance sum, or the diameter.

160

On Diameter of Permutation Extended Graphs

Xingde Jia, Southwest Texas State University

Key words: permutation graph, diameter, explicit construction.

Let G be a graph with vertex set V. Suppose that $\alpha \in S_n$ is a permutation over V. Let $\alpha \odot G$ denote the graph which consists of two copies of G as disjoint subgraphs and the following edges: $(x, \alpha(x))$ for all $x \in V$. $\alpha \odot G$ is called permutation α extended graph of G. Permutation extended graphs were introduced by Chartrand and Harary in 1967. Let n be a large integer. Define $a = \lceil \log n / \log\log n \rceil$. Suppose that G is a connected graph with n vertices. If G is $(c_1 a, a, c_2 a)$-separable, where c_1 and c_2 are constants, it is proved that there exists a permutation $\alpha \in S_n$ such that

$$\mathrm{diam}(\alpha \odot G) \leq c_3 \frac{(\log n)^3}{(\log\log n)^2},$$

where $c_3 = c_3(c_1, c_2)$ is a constant depending only on c_1, c_2. In particular, it is proved that $\mathrm{diam}(\alpha \odot C_n) = O((\log n)^3/(\log\log n)^2)$ for some permutation α, where C_n is the circle with n vertices. Some open problems are also discussed in this paper.

161 *Some Results on Linear-k-Arboricity*

Kuo-Ching Huang

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A *linear-k-forest* of an undirected graph G is a subgraph of G whose connected components are paths of length at most k. The *linear-k-arboricity* of G, denoted by $la_k(G)$, is the minimum number of *linear-k-forest* needed to partition the edge $E(G)$ of G. In this paper, we study $la_k(G)$ and some results are obtained, especially when G is the complete graph and balanced complete bipartite graph.

162 **Domination From Afar**

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In a graph $G = (V, E)$, a set $S \subset V$ is a dominating set if each vertex of $V - S$ is adjacent to at least one vertex in S. The domination number $\gamma(G)$ is the cardinality of a smallest dominating set. Extending this concept to distance domination, we investigate three distance domination parameters. The first, $\gamma_{\leq n}(G)$, is the order of a smallest set $S \subset V$ such that each vertex in $V - S$ is within a distance n of some vertex in S. This parameter has been studied recently by several other authors. We expand this idea by introducing additional distance domination invariants: $\gamma_{=n}(G)$ is the cardinality of a smallest set $S \subset V$ for which each vertex of $V - S$ is distance n from a vertex in S, and $\gamma_{\exists n}(G)$ is the order of a smallest set $S \subset V$ with the property that there exists a path of length n between each vertex in $V - S$ and some vertex in S. Given a graph G, we relate these domination parameters to the n^{th} power, the n-step, and the distance-n graph of G. Bounds are determined for these parameters and exact values are obtained for some families of graphs.

Key words: Domination number, distance domination, n^{th} power of G, n-step graphs, distance-n graphs.

163 **NETWORK RELIABILITY FUNCTIONS CAN CROSS MANY TIMES**

A.K. Kelmans, University of Puerto Rico

The network reliability is investigated as a function of the edge reliabilities. In 1967 we showed that the network reliability functions can cross for equisize networks.In 1975 we showed that the same is true even if these networks have the maximum reliabilities for some edge reliabilities. A fascinating question that has been debated in many papers since then is how many crossings can occur among the reliability fuhctions of equisize networks. Recently, Colbourn, Harms and Myrvold found a single pair of equisize graphs whose reliability functions cross twice.In this talk we give an answer to this question by providing a construction which show that for every N there is a pair of equisize graphs whose reliability functions cross at least N times. By using this construction, we (together with J. Brown) give (with the aid of the computer program Maple) examples of equisize pairs whose reliability functions cross two and four times.

164 **A BIJECTION FOR SPANNING TREES OF COMPLETE MULTIPARTITE GRAPHS**

OMER EGECIOGLU*, Department of Computer Science, University of California Santa Barbara, Santa Barbara CA 93106
JEFFREY REMMEL, Department of Mathematics, University of California San Diego, La Jolla, CA 92093

We construct a bijective proof for the number of spanning trees of complete multipartite graphs. The weight preserving properties of our bijection yields a 6-variate weight generating function which keeps track of various statistics on spanning trees. This bijection allows for the ranking and unranking of the spanning trees of an n-vertex complete multipartite graph in O(n) time. As a further application, we compute the asymptotic distribution of leaves in these families of spanning trees.

165 **Recent Results Involving the Rank of the Adjacency Matrix of a Graph**

Jean H. Bevis, Kevin K. Blount, George J. Davis,
Gayla S. Domke*, Judy M. Lalani, and Valerie A. Miller
Georgia State University

The rank of a graph G, denoted r(G), is defined to be the rank of the associated adjacency matrix. This paper contains results about the rank of graphs which are the Cartesian product of complete graphs, cycles, and paths (including hypercubes). The paper also establishes the rank of graphs with a single cut vertex or a single cut edge whose blocks are complete graphs or cycles. We also study the rank of classes of graphs such as bipartite graphs and regular graphs. A Nordhaus-Gaddum type result involving the rank of a graph and its complement is also included.

166 **On the structure of self complementary graphs**

Robert Molina, Alma College

Every self complementary graph (s.c. graph) contains a bipartite self complementary graph (b.s.c. graph) as a subgraph. (Here s.c. graph and b.s.c graph are defined differently). We use this fact to understand the structure of s.c. graphs and also to help enumerate s.c. graphs. The b.s.c graphs of order up to 12 are presented.

167 **Core of a Block Graph**

Russell Martin* and Renu Laskar
Dept of Mathematical Sciences, Clemson University

A core of a graph, G, is a path, P, that is central with respect to minimizing $d(P) = \Sigma_{v \in V(G)} d(v, P)$ where d(v, P) is the distance of v to the path P. An algorithm for finding a core of a block graph (a graph whose maximal 2-connected subgraphs are complete graphs) is presented. This algorithm is a modification of one for finding a core of a tree, presented in a paper by Christine Morgan and Peter Slater.

168 **Reliability of Three-Regular Graphs**

Jacqueline Nadon *, Cornelia von Schellwitz, and Wendy Myrvold
University of Victoria

Circulant graphs have been proposed as a network topology because amongst all other (n, m)-graphs, they maximize the minimum cutset size and minimize the number of minimum cutsets. Such a graph is known as a super-λ graph. We compare 3-regular circulants to the other 3-regular super-λ graphs and provide evidence that they are perhaps the worst super-λ topology to choose. A family of generalized Petersen graphs is proposed as an alternate choice - while not always the most reliable graphs, they tend to be near the top of the list.

169 **Disjunctive Products and Antimatroids**

Robert E. Jamison*, Clemson University, and Cornell University,
Selma Strahringer, Telenet AG, Darmstadt, Germany

Antimatroids are closure systems which satisfy an "anti-exchange" law. They model any situation in which the relevant structure is given by "shellings" -- e.g., assembly or disassembly of machinery. The disjunctive product, introduced by Strahringer and Wille, for relational structures, was the first product of closures which preserved the antimatroid property.

The disjunctive product of lines provides a notion of "betweenness" for ordinal data in several variables. This notion of betweenness leads to a new and rather strange notion of convexity on euclidean space. The embedding of graphs into this structure represents an attempt to capture the 1-skeleta of polytopes in this onvexity. I will discuss several Steinitz type theorems about the kinds of graphs which can occur as polyhedral 1-skeleta.

170 **The Half-Half Case of the Problem of Zarankiewicz**

Jerrold R. Griggs and Jianxin Ouyang*
University of South Carolina, Columbia, SC 29208

Let $f(m,n)$ be the minimum number of 0's in any $2m \times 2n$ (0,1)-matrix with the property that there is no $m \times n$ submatrix consisting only of 1's. Equivalently, we want the maximum size of a bipartite graph (A, B) with parts $|A| = 2m$ and $|B| = 2n$ that contains no complete bipartite subgraph covering half of the vertices on each side. This is an interesting special case of the famous problem of Zarankiewicz. We prove that $f(m,n) \geq 2n + m + 1$ for $n \geq m$, and for each m equality holds for almost all n.

WEDNESDAY, MARCH 9, 1994
5:00 p.m.

171 **On the Decomposition of Complete Bipartite Graphs into n-cubes**

S. I. El-Zanati*, C. L. Vanden Eynden, Illinois State University

We investigate the necessary conditions for the existence of n-cube decompositions of complete bipartite (and of complete) graphs. We prove these necessary conditions sufficient in some cases. We also investigate n-cube factorizations of complete bipartite graphs.

172 **Externally Redundant Sets in Graphs**

S.T. Hedetniemi, R.C. Laskar, A.A. McRae*, and C.W. Wallis,
Clemson University

A vertex set S in a graph G = (V,E) is an externally redundant set if for every v in V-S, at least one of the the following two conditions holds:

(1) Vertex v would have no private neighbor in S union v, i.e. N[v] - N[S] is empty.

(2) There is a vertex w in S, such that w has at least one private neighbor with respect to S, but has no private neighbor with respect to S union v.

Let er(G) equal the minimum cardinality of an externally redundant set in G and ER(G) equal the maximum cardinality of a minimal externally redundant set in G. We relate these two new parameters to irredundance, domination, and independence.

173 **Maximizing Spanning Trees in Almost Complete Graphs**

Bryan Gilbert*, Wendy Myrvold, and Vassilios Dimakopoulos
University of Victoria

Shier showed that the almost complete (n,m)-graphs, for $m = 0$ up to $\lfloor n/2 \rfloor$ that have the most spanning trees are those whose complement is a matching. Our goal is to characterize the graphs with the most trees for $m = \lfloor n/2 \rfloor + 1$ to n. We prove that when $\overline{G}$ is a collection of paths, the number of trees in G is maximized when the path lengths are as even as possible. Also, if $\overline{G}$ is a cycle of order c and $\overline{H}$ consists of two cycles of order k and $c-k$, $k \le c-k$, then G has more spanning trees than H if k is even and fewer if k is odd.

One application is synthesis of reliable networks under the all-terminal reliability model. It is well known that the most reliable network must have the maximum number of spanning trees when edges are sufficiently unreliable.

174 **Orders and hypergraph representations of cwatsets.**

Julie Kerr, University of Michigan.

A subset C of $\mathbf{Z}_2^d$ is a cwatset if, for each element $\mathbf{b}$ of C, there exists a permutation σ in S_d such that $(C+\mathbf{b})^\sigma = C$. Here σ acts on an element of $\mathbf{Z}_2^d$ by permuting its components. The weight of the i-th column of C is the number of elements of C having a 1 in the i-th component. A (d,m)-cwatset is a cwatset in $\mathbf{Z}_2^d$ having m columns of weight k and $(d-m)$ columns of weight $(n-k)$, where n is the order of the cwatset and $0 < k < n$.

It is known that the order of a cwatset divides $2^d d!$ In this paper, we find additional constraints on the orders of cwatsets.

i) A cwatset of odd order has order at most $\binom{d}{(d-1)/2}$ if d is odd or $\binom{d}{(d-2)/2}$ if d is even.

ii) Let m be an integer, $0 < m < d$, such that m is even if d is odd and m is either odd or even if d is even. If $m > (d-m-1)(d-m-2)$ and if $(d-m)$ does not divide d then there is no $(d,2)$-cwatset of order $md/2$.

iii) Any (d,m)-cwatset has order which is at most $\binom{d}{m}$ and is divisible by $d/\gcd(d,m)$.

We also show that a (d,m)-cwatset can be represented in terms of a hypergraph with highly symmetrical properties.

175 **Dominating Matrices**

Klaus G. Fischer (*) & Jay Shapiro
George Mason University

An (r x n) matrix M with integer coefficients will be called mixed if every row has a positive and negative entry. Such a matrix will furthermore be called dominating if it contains no square mixed minor. If u is an integral vector, then denote by u^+ the vector whose i^{th} coordinate is defined by $[u^+]_i = [u]_i$ if $[u]_i$ is positive and $[u^+]_i = 0$ otherwise. Likewise, $u^- = (-u)^+$. It is shown that a mixed matrix is dominating iff for any selection of k rows, $u_1,\dots,u_k$ and any choice of $v_i = \pm u_i$, $(v_1 + \cdots + v_k)^+ \geq v_i^+$ for some i. A connection is made to the Minkowski ring of polytopes and polynomial rings. In particular, for an integral vector u let $f_u = X^{u^+} - X^{u^-}$. It is shown that M is dominating precisley when for any integral vector w, $f_w \in \langle f_{u_1},\dots,f_{u_r} \rangle$ iff w is in the integer span of the vectors $u_1,\dots,u_r$.

176 THE EDGE ARBORICITY OF A RANDOM GRAPH

LANE CLARK, SOUTHERN ILLINOIS UNIVERSITY AT CARBONDALE

The edge arboricity $a(G)$ of a graph G is the minimum number of acyclic subgraphs of G whose union covers the edges of G. P.A. Catlin and Z.-H. Chen [*Proceedings of the Second International Conference in Graph Theory, Combinatorics, Algorithms and Applications* (Y. Alavi etal., eds.) SIAM (1991) 119–124] showed that almost every $G \in \mathcal{G}(n,p)$ satisfies $a(G) = e(G)/(n-1)$ provided p is constant where $e(G)$ denotes the number of edges in G. P.A. Catlin, Z.-H. Chen and E. M. Palmer [*Ars Combinatoria* 35A (1993) 129–134] showed that almost every $G \in \mathcal{G}(n,p)$ satisfies $a(G) = e(G)/(n-1)$ provided $p^2n = c\log n$ for constant $c \geq 28$ and conjectured that the result is correct for much lower edge probabilities. We verify their conjecture by showing the following.

Theorem. *Let $432\log n/n^{1/2} < p = p(n) < 1/2$. Then almost every $G \in \mathcal{G}(n,p)$ satisfies $a(G) = e(G)/(n-1)$.*

177 CONVEXITY OF MINIMAL TOTAL DOMINATING FUNCTIONS IN GRAPHS

BO YU, *Dept of C & O, University of Waterloo, Ont., Canada*

A *total dominating function* (TDF) of a graph $G = (V,E)$ is a function $f : V \to [0,1]$ such that for each $v \in V$, the sum of f values over all neighbours of v (i.e., all vertices adjacent to v) is at least one. Integer-valued TDFs are precisely the characteristic functions of total dominating sets of G. A *minimal* TDF (MTDF) is one such that decreasing any value of it makes it non-TDF. The *boundary* of a TDF f is the set of vertices v such that the sum of f values over all neighbours of v is exactly one. Vertex v is a *dominating* vertex if its neighbour set strictly contains the neighbour set of some other vertex. Vertex v is a *high* vertex if there exists an MTDF f such that for every neighbour w of v, if w is in the boundary of f, then w is a dominating vertex. An MTDF f is called *universal* if convex combinations of f and any other MTDF are minimal. We give a sufficient condition for an MTDF to be universal: An MTDF g is a universal MTDF if all non-dominating vertices are in the boundary of g, and $g(v) = 0$ for every high vertex v. This generalizes previous results and has two corollaries: (1) Complete n-partite graphs have a universal MTDF. (2) A regular graph has a universal MTDF if it has no high vertices. Define a *splitting operation* on a graph G as follows: take any vertex v in G and a vertex w not in G and join w with all the neighbours of v. A graph G has a universal MTDF if and only if the graph obtained by splitting G has a universal MTDF. A corollary is that graphs obtained by the operation from paths, cycles, complete graphs, wheels, and caterpillar graphs have a universal MTDF.

178 **A Census of Closed Binary Labellings of Small Trees**

Robert. E. Jamison, Clemson University & Cornell University and Deda Zheng*, South Carolina State University

A binary labelling of a tree T is an assignment of binary vectors to the vertices of T. Such a labelling induces an edge-labelling by taking the label on an edge to be the sum of the labels on its endvertices. If both the vertex and edge labellings are one-to-one and every edge label also occurs as a vertex label, then the labelling is called closed. Closed labellings give rise to totally multicolored embeddings of families of trees into complete graphs with the Boolean edge coloring.

179 **Computer construction of universal finitely presented groups**

Prof. Natasa Bozovic, Department of Mathematics and Computer Science, San Jose State University, San Jose, California, USA

The problem of explicit construction of a universal finitely presented (fp) group, whose existence was proved in 1961 (G. Higman), is discussed.This problem of the combinatorial group theory, posed originally in 1969, is to find an example of a universal fp group with as small number of defining relations as possible. Using our previous results about universal groups, it follows that for an fp universal group U, the following of its subgroups are universal as well: the commutant K and every finitely generated (fg) subgroup which contains K, every subgroup of finite index, every normal fg subgroup N such that the quotient U/N is solvable, or nilpotent, or torsion free, etc. That means that starting with known examples of universal groups, one could find finite presentations of the corresponding subgroups. There are already some elaborated methods and computer programs for finding such presentations. Further development of such programs and some other related computational topics will be discussed.

180 **Hermite normal form computation for integer matrices**

George Havas* and Bohdan S. Majewski, University of Queensland

We consider algorithms for computing the Hermite normal form of integer matrices. Various different strategies have been proposed, primarily trying to avoid the major obstacle that occurs in such computations:
explosive growth in size of intermediate entries. We present a new algorithm with excellent performance. We investigate the complexity of such computations, indicating relationships with NP-complete problems.
We also describe new heuristics which perform well in practice. We present experimental evidence which shows our algorithm outperforming the previous methods.

181

Embedding m-cycle systems

Erin R. Spicer Auburn University

Bryant and Rodger have shown that for all odd m, for all $u \equiv 1$ or $m \pmod{2m}$ and for all $v \equiv 1$ or $m \pmod{2m}$ an m-cycle system of order u can be embedded in an m-cycle system of order v if and only if $v > \frac{(m+1)u}{m-1}$, except possibly for the smallest such value of v in the case where $u \equiv v \equiv m \pmod{2m}$. Results will be presented which settle the exceptional case when $m \leq 13$.

182

Ring Extensions and Chiral Polytopes

Barbara Nostrand - York University

Abstract polytopes are partially ordered structures which generalize the notion of polyhedra in a combinatorial sense. There are abstract polytopes which correspond to all of the classical regular polytopes and many other well-known structures. Chiral polytopes are repetitive structures with maximal rotational symmetry which lack reflexive symmetry. While much is known about regular polytopes, little is yet known about chiral polytopes. The simplest chiral polytopes are all twisted tessellations of tori. While these chiral tori can be used to construct locally toroidal chiral polytopes of rank 4, we can also construct locally spherical polytopes. We use hyperbolic honeycombs to construct the symmetry groups of abstract polytopes over finite rings. The corresponding polytopes belong to families with related local symmetry. Ring extensions allow us to construct additional members of these families and to alter global structure.

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Key words and phrases. Regular Polytopes, Chirality, Rotational Symmetry, Projective Linear Groups, Hyperbolic Honeycombs.

183

Restricted Choice Numbers

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Suppose G is a simple graph, and $k \geq \chi(G)$ is an integer. The restricted choice number $c_k(G)$ is defined in the same way as the choice number $c(G)$ is, except that there are only k colors (or symbols) from which to form the lists on the vertices of G.

We knock off the easy observations, pose the hard questions as problems, and render two elementary services: (1) estimation of $c_k(K_{m,n})$ for various values of k, m, and n, especially when n is much larger than m; (2) characterization, for each $k \geq 2$, of those G such that $c_k(G) = 2$.

184

A spanning tree of the 2^m-dimensional hypercube with maximum number of degree-preserving vertices

by
Sul-young Choi*, LeMoyne College
Puhua Guan, University of Puerto Rico

A degree-preserving vertex v in a spanning tree T of a graph G is a vertex satisfying $deg_T(\mathrm{v}) = deg_G(\mathrm{v})$. For an n-dimensional hypercube Q_n, the maximum number of degree-preserving vertices in a spanning tree is $2^n/n$ if $n = 2^m$ for an integer m. (If $n \neq 2^m$, then the maximum number of degree-preserving vertices in a spanning tree is less than $2^n/n$.) We also construct a spanning tree of Q_{2^m} with maximum number of degree-preserving vertices.

185

On a Conjecture of Ryser

P.E. Haxell, University of Waterloo

A special case of a conjecture of Ryser states that if a 3-partite 3-uniform hypergraph has at most n pairwise disjoint edges then there is a set of vertices of cardinality at most n meeting all edges of the hypergraph. The best known upper bound for the size of such a set is (8/3)n, given by Tuza. Here we improve this to 5/2)n.

186 **Degree-Preserving Graph Transformations.**

Todd G. Will*, Davidson College, Davidson NC; Douglas B. West, UIUC, Urbana IL

If two edges ab, cd of a simple graph are deleted and replaced with two missing edges ac, bd then one obtains a new simple graph with the same degree sequence as the first. Such a transformation is called a two-switch since it switches two edges for two non-edges. Berge showed that if two labeled graphs G and H have the same degree sequence then G can be transformed into H by a sequence of two-switches. We show that computing the minimum length of such a transforming sequence is an NP-complete problem. In addition, based on an initial conjecture by Gimbel, we report on attempts to find a strict digraph analogue for Berge's original result.

187 **SEPARATING POINTS WITH PARALLEL CLASSES OF HYPERPLANES**

W. Edwin Clark, Department of Mathematics, University of South Florida, Tampa, FL 33620-5700 email: eclark@math.usf.edu

Let V = V(n,F) be an n-dimensional vector space over a field F and let V(n,q) = V(n,GF(q)). A *hyperplane* in V is a coset of an (n-1)-dimensional subspace W of V. The set H of all cosets of such a W is a *parallel class of hyperplanes*. A parallel class H of hyperplanes is said to *separate* a subset S of V if each hyperplane in H meets S in at most one point. A family P of parallel classes of hyperplanes is said to be *k-point separating* if for every k-subset S of V there is a parallel class in P that separates S. The goal is to determine the cardinality $LS_k(V,F)$ of a smallest k-point separating family of parallel classes of the hyperplanes of V. In previous work it was shown that if F is infinite and if k > 1 and n > 1 then $LS_k(V, F) = n(k-1)$. Here we study the case where F is finite. We prove that if q is sufficiently large relative to n and k then $LS_k(V(n,q), GF(q)) = n(k-1)$, however, for small q the problem seems to be very difficult -- even when n = 2. Aside from some inconclusive bounds, complete answers are obtained when n = 2 and k = 2, 3, and 4 for all q.

188 **Reconstructing Systems of Distinct Representatives in Rectangular Arrays, the Choice Number of Graphs, and the List Coloring Conjecture**

Hunter S. Snevily, University of Idaho

We discuss some recent approaches to the list coloring conjecture and some related (new) problems.

189 **Multiply Twisted Hyper-E-cube: A Low Diameter Interconnection Network**

Sirisha R. Medidi* and Muralidhar Medidi
University of Central Florida

We present the *Multiply Twisted Hyper-E-cube*, denoted $EQ(n)$-network, which has low diameter and hence is a suitable candidate for interconnection networks. We describe a recursive method for generating the topology of an n-dimensional $EQ(n)$-network from lower dimensional $EQ(n)$ networks. The network preserves many of the desirable properties of a good interconnection network such as regular structure, small number of links, fault-tolerance (up to one vertex failure and two link failures), efficient routing *etc*. An $EQ(n)$-network of $(n+1)2^n$ nodes has a diameter of $2\lceil (n+1)/2 \rceil + 2$, which is significantly low, and hence incurs less communication cost as an interconnection network. The EQ_n-network compares favorably with other interconnection networks like Barrel Shifter, Illiac Mesh, G-network and Gg-network presented earlier in the literature.

190 **Degree Sequences with Single Repetitions**

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A graphic sequence is said to contain a single repetition j of length k if all values of the sequence are distinct except the value j that is repeated k times. Let s(n,k) and S(n,k), where s(n,k)+S(n,k)=n-1, denote the functions such that for any pair of integers n and k with n>k+1 there exists a graphic sequence (d1,...,dn) with a single repetition j of length k if and only if s(n,k)<j<S(n,k). There is known a formula for s(n,k) if k=2 or k=3. We give the formula for an arbitrary k.

191

Alley CATs in search of good homes

FRANK RUSKEY(*), *University of Victoria, Canada*
ROBERT COHEN, PETER EADES, AND AARON SCOTT, *The University of Newcastle, Australia*

A degree sequence of a graph is a monotonically non-increasing sequence of the degrees of its vertices. We develop an algorithm that generates all degree sequences of given length and prove that the algorithm does only constant computation per sequence produced. Such algorithms are said to be CAT (for Constant Amortized Time). The algorithm can be modified to produce degree sequences of connected and biconnected graphs and still be CAT. Similarly, a score sequence of a tournament is a monotonically non-decreasing sequence of the out-degrees of its vertices. We also develop an algorithm that generates all score sequences and appears to be CAT.

192

Hyperplane Counts of Quota Voting Schemes

Aaron Meyerowitz, F.A.U.

A **voting scheme** for n voters is a method of combining their n individual votes for one of two candidates to choose a winner. Thus it is selection of 2^{n-1} winning coalitions, one from each complementary pair of subsets of voters. We require monotonicity in that a winning coalition stays winning if it is enlarged. A **quota scheme** is one obtained by giving each voter a non-negative weight and declaring the winner to be the candidate whose supporters have the greater total weight. Not all voting schemes are quota schemes although all schemes for 5 voters or less are. The number of quota (and non-quota) voting schemes have been enumerated up to $n = 7$.

A non-negative weight function will give a quota scheme unless it lies on one of $2^{n-1} - 1$ hyperplanes (corresponding to two complementary coalitions having the same total weight). These hyperplanes, along with n others reflecting the requirements that each weight be non-negative, divide $\mathbf{R}^n$ into cells and weight functions in the same cell give the same voting scheme.

The number of cells can be computed by determining the lattice of intersections of the $2^{n-1} + n - 1$ hyperplanes, computing its Möbius function, and forming the **Whitney polynomial** $p(x) = \sum \mu(t)x^{\dim(t)}$. Then the number of cells is $|p(1)|$ and the number of bounded cells is $|p(-1)|$. It is not clear that this method is easier than enumerating all the quota schemes directly. However, the polynomial may yield more insight than its value at 1. We report on a calculation of the polynomials for $n \leq 6$.

193

A Characterization of Digraphs with Interval and Chordal Competition Graphs

J. Richard Lundgren* and Sarah K. Merz
University of Colorado at Denver

Competition graphs have appeared in a variety of applications from food webs to communication networks to energy models. The competition graph, $C(D)$, of a digraph D has the same vertex set as D and (x, y) is an edge in $C(D)$ if and only if there is a vertex z such that (x, z) and (y, z) are arcs in D. It has been observed that most actual food webs have interval competition graphs. Subsequent research investigating this phenomenon led to the problem of characterizing acyclic digraphs which have interval competition graphs since food web models are generally acyclic. This evolved into the problem of characterizing digraphs with interval or chordal competition graphs. The problem is difficult because forbidden subgraph characterizations generally don't work. Here we use an elimination ordering approach which leads to useful characterizations which can be generalized to determine which graphs have interval or chordal squares or two-step graphs.

key words: competition graphs, interval graph, chordal graph, elimination ordering, square, two-step graph

194

Consistent Recoverable Tree Embeddings in Balanced Hypercubes

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As a multicomputer structure, balanced hypercube is a variant of the standard hypercube structure for multicomputers, with desirable properties of strong connectivity, regularity, and symmetry. This structure is a special type of load balanced graph designed to tolerate processor failure. In balanced hypercubes, each processor has a backup (matching) processor such that they share the same set of neighboring nodes. Therefore, tasks that run on a faulty processor can be reactivated in the backup processor to provide efficient system reconfiguration. This paper studies the recoverable embedding of the tree structure in balanced hypercubes. Fault-tolerant ability of such embedding scheme is also discussed.

195

Crossings in the Complete Graph

Barry Piazza, University of Southern Mississippi
Rich Ringeisen, Old Dominion University
Sam Stueckle*, Northeastern University

In this talk we study the number of crossings possible in a drawing of the complete graph. Building on work of Eggleton, Guy, and Harborth we find gaps in the number of crossings possible in drawings of the complete graph besides the parity gaps found by Archdeacon and Richter. We then give possible directions in which to extend this work to find further gaps.

THURSDAY, MARCH 10, 1994
10:50 a.m.

196 An Analytic Result Regarding the Asymptotic Behavior of Level Graph Algorithms

Jacob Shapiro, Baruch College; Jerry Waxman*, Queens College; CUNY

Let G be a level graph, and let v1 and v2 be points in G. Let p represent the shortest path between v1 and v2 and let p' be the path between those same points produced by LGS. Let l(p) be the length of path p. The main result of this paper is to show analytically that, as the distance between v1 and v2 increases, the ratio l(p')/l(p) is bounded asymptotically by 1.25. This confirms previous experimental results of the authors.

197 **Point-halfspace graphs**

Edward Scheinerman, Johns Hopkins University; Ann Trenk, Wellesley College; and Daniel Ullman,* George Washington University

A graph G is called a point-halfspace graph in R^k if one can assign a point p in R^k to each vertex v and a closed hyperplane H in R^k to each edge e so that v is incident with e if and only if p is in H. For each k, we give complete structural and forbidden subgraph characterizations of the point-halfspace graphs in R^k. Surprisingly, these classes are closed under taking minors and we give forbidden minor characterizations as well.

198 RESULTS RELATED TO CHROMATIC NUMBER OF THE PLANE

(Last year's essays published in GEOMBINATORICS)

by Alexander Soifer
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The chromatic number of the plane χ_* in case of open monochromatic subsets is equal to 6 or 7. This and other related results by N. Brown, N. Dunfield and G. Perry will be discussed.

A new 6-coloring of the plane found by I. Hoffman and the presenter will be shown. It has type $(1, 1, 1, 1, 1, \sqrt{2}-1)$, i.e., the all colors but one forbid the distance one, and the last color forbids the uistance $\sqrt{2}-1$. The presenter's previous example had the type $(1, 1, 1, 1, 1, \frac{1}{\sqrt{5}})$.

199 **ON SENSE OF DIRECTION IN PRODUCT GRAPHS**

P. Flocchini (1,2) and Nicola Santoro* (2)
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(2) Carleton University, Ottawa, Canada

Sense of Direction refers to the existence of some global properties of the local labeling of the communication links in a distributed system. The presence of such properties is known to have an impact on the communication complexity of distributed problems (e.g., election, broadcast, etc.). In this talk, we describe a natural labeling for some classes of product graphs, prove that it forms a sense of direction, and study the election problem in such graphs. This investigation xtends and generalizes the existing results for specific topologies (e.g., hypercubes, rings, circulant graphs).

200 **An Algorithm for Constructing an Optimal Functionally-Connected Graph**

Donald L. Goldsmith, Troy State University

The splitting of a connected graph G by the removal of edges is often regarded as an undesirable occurrence, since then vertices in one component are not connected to those in another component. We will adopt a somewhat different approach, based on the idea that the vertices in one component may not *need* to be connected to those in another component; rather, the components may (in whatever sense the application demands) be self-contained.

As an illustration, consider an electrical power distribution network. Nodes in the network will be either producers or consumers. Splitting the network into two components causes no problem as long as each component contains at least one producer node.

We will call a graph G a ***supply graph*** if its vertex set $V(G)$ is partitioned into two disjoint, nonempty subsets, called the sets of ***producer*** and ***consumer*** vertices, respectively. A supply graph G is said to be ***functionally connected*** if every consumer vertex is connected by a path in G to at least one producer vertex.

Suppose now that G is a functionally-connected, edge-weighted, supply graph. One may ask for a functionally-connected spanning subgraph H of G with minimum total weight. Such a subgraph will be called an ***optimal*** subgraph of G.

We will first list some of the properties of an optimal subgraph. We will then describe an algorithm which produces an optimal subgraph in a functionally-connected, edge-weighted, supply graph, and we will prove that the graph produced by the algorithm is in fact optimal. Finally, we will give a detailed implementation of the algorithm.

201 POLYGONAL AND NEAR POLYGONAL GRAPHS

Manley Perkel, Department of Mathematics and Statistics, Wright State University, Dayton, OH 45435

A simple graph Γ is called a ***near-polygonal graph*** if it is triangle free, connected and contains a set Π of m gons (simple cycles of length m) with the property that every path of length 2 is contained in a unique member of Π. If, in addition, the girth of Γ is also m, then Γ is called a ***polygonal graph*** (or a ***strict*** polygonal graph if Π contains all the m-gons of Γ). We are interested in the case where Γ has a large automorphism group which fixes the set Π.

In this talk we will review the current knowledge of these graphs and mention some recently constructed infinite families of polygonal graphs with $m = 6$. We will also discuss constructions of these graphs from groups. Using these constructions and the group theory language CAYLEY, we have recently discovered the first known examples of polygonal graphs with highly transitive automorphism groups and having girth > 7.

202 Coloring Faces of Infinite Plane Triangulations

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We discuss the problem of coloring the faces of an infinite plane triangulation G so that the same color is never assigned to two faces that share a vertex (but not necessarily an edge, as in the conventional map-coloring problem). This is inspired by a recent problem posed in the *Amer. Math. Monthly* by D. Gale and R. M. Robinson, who asked how many colors are needed for a tiling of the hyperbolic plane into equilateral triangles with seven triangles meeting at each vertex. We describe a large class of triangulations G, including their example, for which we have an algorithm to color the faces in this sense with just $\Delta := \max_v deg(v)$ colors.

203 **Geometric Graphs of Chromatic Number Four**

Paul O'Donnell, Rutgers University

The chromatic number of the plane is the smallest number of colors needed to color the following graph G. The points in the plane are the vertices of G, with an edge between two points if their Euclidean distance is one. It is known that the chromatic number of the plane is between four and seven. A five-chromatic subgraph would raise the lower bound. If I discover such a subgraph, I will present it. Otherwise I will show some interesting four-chromatic subgraphs, including a relatively small one with girth four.

204 **Computing Prefix Sums in Extended Fibonacci Cubes**

Hari M. Pendyala* and Jie Wu, Computer Science and Engineering, Florida Atlantic University Boca Raton, FL-33431 jie@cse.fau.edu

The extended Fibonacci Cube is an novel interconnection network which is an extension of Fibonacci Cube. The Fibonacci Cube possesses, attractive recurrent structure and many desirable properties that can be applied to our advantage in many ways. Many of the hypercube algorithms can be emulated efficiently by the Fibonacci Cube and it uses fewer links than comparable hypercube while its size doesn't increase as fast as hypercube. We present parallel algorithms for computing the prefix sums on the extended Fibonacci Cube in O(logN) steps with the assumption that O(logN) data items are on each of N processors. The results further justify the usefulness of extended Fibonacci Cubes as a special type of injured hypercubes.

205 **The Computational Complexity of Ordered Subgraph Recognition**

Mark Ginn (with Dwight Duffus and Vojtech Rödl)
Emory University

For a fixed ordered graph $(G,<)$ we define the decision problem $(G,<)$ORD to have as an instance an unordered graph Γ and as a question whether or not there is an order $<$ on $V(\Gamma)$ such that $(G,<) \not\leq (\Gamma,<)$. In this paper we investigate how the computational complexity of this decision problem changes for different choices of $(G,<)$. First, we give several examples of small ordered graphs for which $(G,<)$ORD is in P. Most of these give characterizations of subclasses of perfect graphs and their complexity is well known. Then, for the majority of the paper, we look at ordered graphs for which $(G,<)$ORD is NP-complete. This leads us to make the conjecture that for any 2-connected ordered graph $(G,<)$, $(G,<)$ORD is NP-complete. We are able to answer this conjecture affirmatively for nearly all such ordered graph, and give an example that shows that this condition is not a characterization.

206 **Sylow's Theorem and Parallel Computation**

Peter Mark*, U Seattle; Eugene Luks, William Kantor, U Oregon

We present parallel algorithms for constructive versions of Sylow's theorem, including:

Given: a permutation group G and a prime p,

Find: a Sylow p-subgroup of G.

These results parallelize W. Kantor's polynomial-time sequential algorithms for Sylow's theorem. They add to a growing repertoire of efficient parallel algorithms developed by L. Babai, W. Kantor, E. Luks, A. Seress, and others to solve a variety of problems for permutation groups including membership testing, finding orders, normal closures, centers, and composition factors. If G is solvable, the problems are effectively reduced to linear algebra. This talk concentrates on the nonsolvable case. For such groups, a constructive version of the Frattini argument reduces the problem to the case where G is simple. To solve the problem for simple groups, one proceeds by case analysis as dictated by the classification of finite simple groups. The classification actually arises in a variety of essential ways in the machinery for parallel computation in permutation groups.

207 **Finding the Recurrence Relation for Tiling 2xn Rectangles**

Robert Brigham, University of Central Florida
* Phyllis Chinn, Humboldt State University
Linda Holt, California State University, San Marcos
Steve Wilson, Sonoma State University

Cuisenaire rods ("c-rods") are a set of rectangular solids with cross-section of 1 cm by 1 cm squares, color-coded by length, and varying from 1 cm long white rods and 2 cm long red rods to 10 cm long orange rods. A variety of number-theoretic and combinatorial geometry problems can be modeled using the c-rods. In this presentation three derivations will be given for a recurrence relation yielding the number of ways of tiling a 2 x n rectangle with 1 x 1 and 1 x 2 c-rods.

208 **A note on largest k-colorable subgraphs**

Joseph R. Barr
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It is a well known fact that every graph $G=(V,E)$ contains a k-colorable spanning subgraph with at least $\frac{k-1}{k}|E|$ of the edges. In particular, the case $k=2$ says that every graph contains a bipartite subgraph with at least $\frac{1}{2}|E|$ of the edges. In this short note, a probabilistic proof of that statement is given. The proof is an adaptation of the proof for the case $k=2$, theorem 2.1 on page 16 of the book by Alon and Spencer, *The probabilistic Method*.

209 **Embedding of Meshes of Trees into Hypercubes**

Haifeng Qian* and Jie Wu, Department of Computer Science and Engineering, Florida Atlantic University

We study two embedding schemes of meshes of trees into hypercubes presented by T. Leighton. The first embedding which is applicable to general meshes of trees has load 1, dilation 2, and congestion 2, but large expansion. The second embedding which is applicable to a restricted type of meshes of trees has dilation 1, congestion 1, and small expansion, but large load. This latter has a delay factor of r since it uses a single tree squashing scheme and the load of each node in the hypercube is non-balanced. We enhance this embedding method using dual tree squashing to reduce the delay factor from r to r/2.

210 **Conditional Graph Completion Classes**

Craig W. Rasmussen, Naval Postgraduate School

If $G=(V,E)$ is a simple graph of order p and size q, and if P is a property held by G, we say that G is *P-completable* if there is an ordering $e_1, e_2, \ldots, e_{\binom{p}{2}-q}$ of the edges of $K_p - E$ such that $G_k = \left(V, E + \bigcup_{i=1}^{k} e_i\right)$ has property P for each $k = 1, 2, \ldots, \binom{p}{2} - q$. The sequence $\{G_k\}$ is called a *P-completion sequence*. If all graphs with property P are P-completable, we say that P is a *completable property* and that the class Π of graphs with property P is a *completion class*. Of interest are *conditional completion classes*, i.e., classes for which not all orderings lead to completion sequences. We show that several familiar classes of graphs are conditional completion classes, and in several cases provide algorithms for construction of completion sequences.

Keywords: Chordal graphs, perfect graphs, matrix completions.

THURSDAY, MARCH 10, 1994
11:50 a.m.

211 **Geometrical Representations of Maximal Subgroups of the Gosset Group**

Edward Pervin, Software Compositions

This paper will describe the seventeen known classes of maximal subgroups of the Gosset simple group of order 48 x 10! as stabilizers of geometric features of 8-dimensional uniform polytopes.

212 **An Algorithm for Tiling mxn Rectangles with Cuisenaire Rods**

Eleanor Hare, Clemson University

Cuisenaire rods ("c-rods") are a set of rectangular solids with square cross-section 1 cm. x 1 cm., color-coded by length, and varying from 1 cm. long (white) rods and 2 cm. long (red) rods to 10 cm. long (orange) rods. An algorithm is presented for finding the number of ways to tile an mxn rectangle with one layer of c-rods where the lengths of the rods are at most k, for some fixed positive integer, k.

213 **A Special k-Coloring for a Connected k-Chromatic Graph**

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For each positive integer k we consider the smallest positive integer $f(k)$ (dependent only on k) such that the following condition holds. Each connected graph G with chromatic number $\chi(G) = k$ can be properly vertex colored by k colors so that for each pair of vertices x_0 and x_p in any color class there exist vertices $x_1, x_2, \cdots, x_{p-1}$ of the same class with $dist(x_i, x_i + 1) \leq f(k)$ for each i, $0 \leq i \leq p-1$. Thus the graph is k-colorable with the vertices of each color class placed throughout the graph so that no subset of the class is at a distance $> f(k)$ from the remainder of the class. We prove that $f(k) < 12k$ when the order of the graph is $\geq k(k-2)+1$.

214 **An Edge-Neighbor-Connectedness Network Reliability Model**

Z.S. Chen and C.L. Suffel* Stevens Institute of Technology

Let G be a simple and undirected graph with points and e edges. An edge in graph G is said to be subverted if its endpoints are deleted from G. The edge-neighbor-connectivity of G is defined to be the minimum number of edges to be subverted to disconnect G or to leave G empty or trivial. We consider a network reliability model where edges in G are assigned equal subversion probabilities and are independent of each other. The edge-neighbor-connectedness-reliability R(G) is defined to be the probability that surviving induced subgraph is connected. We discuss basic aspects of the model; in particular properties of the associated reliability polynomial, computational complexity of determining the reliability and design issues of reliable networks.

215 **CONSTRUCTING CLIQUES USING RESTRICTED BACKTRACKING**

Mark K. Goldberg, Reid D. Rivenburgh, Rensselaer

The problem of constructing a clique of the maximal size in a given graph was one of the first problems proven to be NP-hard. Moreover, it was recently proved that even the approximate version of the problem is NP-hard. Still, these results provide no clear indication as to how difficult, if at all, the problem is for graphs of bounded sizes. We present the results of computational experiments with a clique algorithm built on the idea of restricted backtracking. We introduce the notion of backtracking coordinates and use them to describe the locations of maximum cliques in graphs of different types, in particular, randomly generated graphs. Our experiments show interesting and somewhat unexpected patterns in the distribution of the backtracking coordinates that correspond to maximal size cliques in these graphs.

THURSDAY, MARCH 10, 1994
12:10 p.m.

216 **Classification of Semifields of Order 625 with Kernel GF(25)**

John G. Ramirez --- University of Puerto Rico

In 1960 Erwin Kleinfeld classified all semifields of order 16. Victoria Lee Boerner, in her Ph.D. thesis, classified all semifields of order 81 with kernel of order 9. In this talk we present unique representatives of the semifields of order 625 with kernel of order 25. We obtained 1,500 equivalence classes from the isomorphism relation and 13 equivalence classes from the isotopism relation. This result was achieved by making a reduction on the representation scheme for the semifields, and by the use of the computer.

217 **On the Mathematics of Flat Origamis**

Thomas Hull, University of Rhode Island

Origami is the art of folding pieces of paper into works of sculpture without the aid of scissors or glue. Modern advancements in the complexity of origami (e.g., the work of Montroll and Maekawa) reveal a rich geometric structure governing the possibilities of paperfolding. In this paper we initiate a mathematical investigation of this "origami geometry" and explore the possibilities of a graph theoretic model. In particular, we study properties of origami models which fold flat (i.e., can be pressed in a book without crumpling). Necessary and sufficient conditions are given for an origami model to locally fold flat, and the problems encountered in trying to extend these results gloabally are discussed.

218 Achromatic Numbers of Some Uniform Hypergraphs

Nam-Po Chiang, Tatung Institute of Technology

Let $H = (V, E)$ be a simple hypergraph. A proper s-coloring of a hypergraph H is a partition $V = X_1 \bigcup X_2 \bigcup \cdots \bigcup X_s$ such that $|X_i| > 0$, $i = 1, 2, \ldots, s$, and for any i, $1 \le i \le s$, there is no edge $e \in E$ such that $e \subseteq X_i$. Further, if $V = X_1 \bigcup X_2 \bigcup \cdots \bigcup X_s$ is a proper s-coloring of H and for each pair (i, j) there is an edge e such that $e \subseteq X_i \bigcup X_j$, then we call this coloring a complete s-coloring. The chromatic number $\chi(H)$ is the minimal integer s such that H admits a complete s-coloring and the achromatic number $\Psi(H)$ is the maximal integer s' such that H admits a complete s'-coloring.

In this paper, we consider the upper bound of the achromatic number of a simple hypergraph and determine the achromatic numbers of the complete k-uniform hypergraphs, the complete t-partite hypergraphs, and the projective planes of order q.

219 **Limited Penalty Immune Networks**

Arthur M. Farley and Andrzej Proskurowski, University of Oregon

A connected graph G is *immune* to a class of vertex failures if it remains connected after removal of the set of vertices corresponding to any element of the class of failures. With a graph's immunity to a class of failures, we can associate a penalty measure, being the maximum additive increase in pairwise distance between remaining vertices of the graph over all vertex failure sets of the class. Specifically, we define the set of graphs LPI(f, p) to be those graphs immune to the failure of any set of f vertices with penalty equal to p.

We generalize our earlier results on self-repairing graphs (i.e., graphs in LPI(1,0)) by characterizing infinite, minimal classes of graphs in LPI(f,p) for $f \geq 1$, $p=0$ and for $f=1$, $p \geq 0$. Our results are based upon two different generalizations of twin graphs. A *twin graph* is either a 4-cycle or a twin graph to which a new vertex has been added by connecting it to a pair of twins (i.e., two vertices having identical open neighborhoods in G). Twin graphs are minimum self-repairing graphs. Viewing graphs as models of the topological aspect of communication networks, we discuss automatic routing of messages that realize the immune behavior desired.

220 **On A Conjecture of T-Colorings**

Daphne Der-Fen Liu

California State University, Los Angeles

In T-colorings or the channel assignment problem, T is a finite set of nonnegative integers containing 0. A T-coloring of a graph G is a function, $f: V(G) \rightarrow Z^+ \cup \{0\}$, such that the difference of colors assigned to two adjacent vertices cannot fall in T. The span of f is the difference of the largest and smallest colors in f(V). The T-span of G, denoted by $sp_T(G)$, is the minimum span among all T-colorings of G. Let g represent the collection of sets T that greedy algorithm calculates all T-spans of complete graphs, and let e represent the collection of sets T that $sp_T(G)=sp_T(K_{\chi(G)})$ for all graphs G, where $\chi(G)$ is the chromatic number of G. Many sets T of g are obtained (Cozzens and Roberts[1982],Raychaudhuri[1989],Tesman[1988],etc.) While only few members of e are found (Cozzens and Roberts [1982], Raychaudhuri[1989], Liu[1992].) Liu[1993] showed the first known families of g - e (*i.e.* sets T in g but not in e.) We will explore the relationship between g and e. Different families of sets T in g - e are obtained. Before this article, all known families of e are also members of g. This motivated us to study the conjecture of Liu[1992]: "$e \subset g$." We find a counterexample of this. A set T in e but not in g is demonstrated.

221 **Cyclic Difference Schemes**

C.J. Colbourn, University of Waterloo
D.L. Kreher*, Michigan Technological University

An r by c array D with entries from an Abelian group A is called a difference scheme based on (A,+) if it has the property that the vector difference between any pair of columns contains every element of A equally often. If |A| = s, we denote such an array by D(r,c,s). A difference scheme D(r,c,s) can be used to construct an orthogonal array OA(rs,c+1,s,2). A difference scheme is cyclic if the cyclic shift of any row of the scheme is still a row of the scheme. We construct an infinite family of cyclic difference scheme and improve the lower bound on the number of columns that a difference scheme or orthogonal array can have.

222 **LATTICE PATHS WITH OR WITHOUT HORIZONTAL STEPS**

by Jacques Labelle, Université du Québec à Montréal

Given U (an alphabet) and $U \xrightarrow{h} \mathbb{Z} = \{\cdots,-2,-1,0,1,2,\cdots\}$, the Dyck language is $\{w \in U^* \mid h(w) = 0$ and $w = w_1w_2 \Rightarrow h(w_1) \geq 0\}$. Consider the operator $\mathcal{D} : \mathbb{N}[x,x^{-1}] \to \mathbb{N}[[t]]$, from Laurent polynomials to generating functions, defined by $\mathcal{D}\{U(x)\} = \mathcal{D}_U(t)$ where $U(x) = \sum_{u \in U} x^{h(u)}$ and $\mathcal{D}_U(t)$ is the generating function of the number of Dyck words (or paths) of length n. The object of this paper is to study properties of the operator $\mathcal{D}$, and two similar operators $\mathcal{J}$ (for two-sided Dyck paths) and L (left-factor of Dyck words). We consider the effect on $\mathcal{D}_U(t)$ of adding k different new horizontal steps (or letters) to U. Closed forms are obtained in the following cases : $U(x) = x^m + x^{-1}$, $m \geq 1$, $U(x) = x^2 + x + x^{-1} + x^{-2}$; $U(x) = x + x^{-1}$ (classical Dyck), $U(x) = x + 1 + x^{-1}$ (classical Motzkin).

223 **Minimal alternating cycles in tripartite graphs.**

Donald Y. Goldberg* (Occidental College), Albin Lee Jones (Occidental College and University of Toronto), Daniel S. Wilkerson (University of California at Berkeley).

The 1977 József Kürschák Competition in Hungary included this problem: "There are three disjoint schools, each with n students. Each student is acquainted with a total of $n+1$ students in the other two schools. Show there are three students, one from each school, who are mutually acquainted." Two elegant proofs are given in J. Surányi, *Matematikai versenytételek* (Budapest, 1992).

We prove the following generalization: Let G be a tripartite graph. A vertex in G is considered *biased* if its adjacency set is a subset of one of the three parts. A cycle in G is said to be *alternating* if any three consecutive vertices represent all three parts. Suppose that each of the three parts in G has n vertices, that no vertex is biased, and that the degree of each vertex is at least k. Then for some $r \leq n+2-k$, there is an alternating cycle in G of length $3r$.

224 **On The Criticality of Graphs Labelled With a Condition at Distance Two**

John Georges and David Mauro, Trinity College, Hartford, CT 06106

An assignment of non-negative integers to the vertices of graph G is called a labelling with a condition at distance two if and only if the labels of adjacent vertices in G differ by at least two, and the labels of vertices which are distance two apart in G are different. The lambda-number of G is the smallest integer k for which there exists a labelling of G with a condition at distance two into {0,1,2, ..., k}. We define a graph H to be lambda-edge critical if for every edge e in E(H), the lambda-number of H-e is less than the lambda-number of H. In this paper we investigate the properties of lambda-edge critical graphs. We characterize the connected lambda-edge critical graphs with lambda-number less than 5 and give examples of infinite families of connected lambda-edge critical graphs with lambda-number 5. For the cases where the lambda-number of G is greater than or equal to the order of G, we use the relationship between the lambda-number of G and the path covering number of the complement of G to obtain classes of lambda-edge critical graphs. In particular, we show that the complete multipartite graphs are lambda-edge critical.

225 **Cell Selection for Dags with Indegree and Outdegree Bounded by Two Is Strongly NP-complete.**

Wing Ning Li
A. Gregory Starling
Department of Computer Science
University of Arkansas

The cell selection problem arises in VLSI circuit design and is a very important problem. Aside from its physical and electrical characteristics, the basic optimization problem is the following. Given a dag (directed acyclic graph), a set of pairs of numbers L_i for each vertex v_i, and two numbers K_1 and K_2, select a pair from L_i for each v_i such that in this selection the sum of the first components of the selected pairs is no more than K_1 and, using the second component of each selected pair as the weight of the corresponding vertex, the path with the largest sum of these weights is no more than K_2. In this paper, we prove that, with the restriction that the indegree and the outdegree of each vertex are bounded by 2, the cell selection problem is NP-complete even when the first component of each pair in L_i takes a value of either 1 or 2 and the second component takes a value of either 1 or 4.

THURSDAY, MARCH 10, 1994
4:40 p.m.

226 **Character Sums and Z-cyclic Whist Tournaments**

I. Anderson, N.J. Finizio*, and R.W.K. Odoni

Construction of Z-cyclic whist tournaments for v = st^2 + 1 players is discussed. Here s,t denote primes, each congruent to 3 (mod 4), s,t > 3. Sufficient conditions for existence are introduced. Using character sums and specific solutions, satisfaction of the sufficient conditons is discussed.

227 **Pairs of weighted nonintersecting lattice paths**

Heinrich Niederhausen, Florida Atlantic University

A lattice path makes a *left turn* if a vertical step follows a horizontal step: $\rightarrow\!\uparrow$. We give the weight $\mu p^i q^j$ to a left turn at the lattice point (i,j). The product of the weights at left turns is the weight of the path. Analogously, a path can have each *right* turn at (i,j) weighted by $\nu q^i p^j$. We consider pairs of paths where the 'upper' path has weighted left turns and the 'lower' path has weighted right turns. The total weight of nonintersecting pairs of paths has been derived by Krattenthaler and Sulanke (1993), using the 'rotational method'. We show how the result fits in the q-Umbral Calculus and can be phrased and solved as a bivariate initial value problem.

228 Finding a bound on the size of P_j-free graphs

Mark R. Dillon, University of Colorado at Denver

Finding an upper bound on the number of edges in an F-free graph, where F is some arbitrary graph, can be difficult. Specifically, we examine this problem when F is a path on j vertices. We present exact upper bounds on the size of P_j-free graphs when their order is small and when j is small.

229 **Irregular assignments producing consecutive labelings for graphs**

Mike Jacobson, Ewa Kubicka*, Grzegorz Kubicki
Department of Mathematics, University of Louisville

We say that a graph G is consecutive if it is possible to assign integer weights to its edges so that all vertex labels (sum of weights of all edges incident to a vertex) are consecutive numbers (almost consecutive for orders congruent to 2 modulo 4). We show that all graphs without isolated vertices or isolated edges are consecutive. Moreover, the produced labeling is symmetrical with respect to zero.

230 **A new algorithm for computing convex hull**

Stephen V. Rice* and Laxmi P Gewali
University of Nevada, Las Vegas

Computing the convex hull of a set of points in two dimensions is a well explored problem and several algorithms of optimal time complexity have been reported in the literature. We present a new algorithm to compute the convex hull of n points in two dimensions in O(nlogn) time, which is optimal within a constant factor. The algorithm first converts points into n lines in the dual plane defined by the coordinates of the points. We show that the intersection of lower half planes I(l) (respectively, upper half planes I(u)) induced by lines can be computed in O(nlogn) time by a careful application of binary search. The lines on the boundary of I(l) and I(u) exactly corresponds to the vertices of the convex hull in the primal plane. The algorithm is simple and easy to implement.

THURSDAY, MARCH 10, 1994
5:00 p.m.

231

Subset sums of sets of reisdues

E.Lipkin, School of Math Sciences, Tel Aviv University

We study multisets A_q of nonzero residues modulo q. Let A_q be an r-multiset, i.e. A_q contains at least r elements $\not\equiv 0(\mod q')$ for any divisor q' of q. Let A_q^* be a set of sums of subsets of A_q. We obtain results describing the structure of sets A_q^* and A_q by using certain properties of set A_q^*. Also we prove G.Diderrich conjecture concerning conditions when $A_q^* = \mathbb{Z}_q$, a complete system of residues modulo q.

232

Determinants, Power Series, Partitions

Zoltan Reti, University of Florida

Newton's formula, which establishes the relationship between the elementary symmetric functions and the power sum symmetric functions is just one of the very many examples of convolution-type identities.
The solution of such a system of equations lead to the examination of determinants of Hessenberg matrices in a natural way. We will show how to specialize the matrix to obtain important combinatorial numbers: multinomial coefficients, Stirling numbers of either kind, Lah numbers.
We will also use this representation to carry out operations with formal power series elegantly and hope to rescue from oblivion some beautiful formulas of the last century too.

233

Computing Cycle Equivalent Vertices in Linear Time

Robert R. Crawford, Ali A. Kooshesh*, Western Kentucky University
Ksheerabdhi Krishna, University of New Mexico

The notion of control dependence is central to compilers that attempt to extract parallelism from a given program. Consequently, computing the control dependence relation accurately and efficiently is an important aspect of most parallelizing, optimizing compilers.
In many applications, however, the complete control dependence relation is not needed. It suffices instead to compute the control dependence equivalence relation, i.e., a partition of the nodes of the control flow graph wherein two nodes are related if they have the same set of control dependences. It is been shown that computing the control dependence equivalence can be reduced to computing a simple graph property called cycle equivalence. Two vertices of a strongly connected directed graph are cycle equivalent iff for all cycles C, either C contains both or neither vertices. In this talk, we present a method for partitioning the vertex set of a directed graph into cycle equivalence classes that requires linear time with respect to the number of edges in the input graph.

234

Preservers of a fixed degree sequence

LeRoy B. Beasley, Utah State University, Logan, Utah 84322

We investigate the linear operators on the sets of undirected, directed and bipartite graphs that preserve a fixed degree sequence. This is related to studying assignment functions, the simplest of which is the permanent function when considering the adjacency matrix of the graph as a real (0,1)-matrix.

235

An Improved Algorithm for Finding the Compact Sets in a Graph

Muralidhar Medidi
University of Central Florida

A compact set in a weighted graph $G = (V, E)$ is defined to be the vertex set C of a connected subgraph of G such that the weight of any edge between any pair of vertices within this subgraph is smaller than that of any edge with one end-vertex in C and the other in $V - C$. Compact sets can be used for clustering and other partitioning problems encountered in computer systems, networks, VLSI design, information retrieval, *etc.*

Recently, two $O(n^2)$-time algorithms were independently presented to find all the compact sets in a graph with n vertices. We present an $O(m + n \log n)$-time algorithm for finding all the compact sets in a graph with n vertices and m edges; thus, our algorithm is an improvement over earlier algorithms for sparse graphs and works equally well for dense graphs.

236 **Generalized Court-Balanced Tournament Designs**

Peter Rodney - University of Vermont

A generalized court-balanced tournament design, GCBTD(v, c, 1; k), is an arrangement of the blocks of a balanced incomplete block design, BIBD(v, k, 1), into a c-row array, such that no element appears more than once per column and every element appears the same number of times in each row. We show that for any positive c and k, there exists a GCBTD(v, c, 1; k) for sufficiently large v. We will also discuss the relationship between GCBTDs and other related designs.

237 **Vector Versions of q-Identities**

James Haglund, Kennesaw State College

A new statistic on vector partitions is introduced. Several q-identities involving partitions are then extended to partitions over vectors. In particular, vector versions of the MACKOH identity, and of a classic identity due to MacMahon involving inversions of permutations, are obtained. Other applications include q-versions of identities due to MacMahon involving compositions of vectors and Simon Newcomb's Problem. This is a summary of the authors dissertation, University of Georgia, December, 1993.

238 **The Complexity of Integral Network Flow with (1,2) Multipliers in a Unit Capacity Network**

Wing Ning Li

Department of Computer Science
University of Arkansas

We look at one of the simplest versions of network flow with multipliers problem. In this case, the multiplier associated with each node is either 1 or 2, i.e. the flow through a node either remains the same or is doubled, and the capacity of each edge is 1. We prove that this simple version of the integral network flow with multipliers problem is strongly *NP*-hard, and for any k the k-absolute approximation of this problem remains to be strongly *NP*-hard.

239 **Minimal Imbeddings of Some Graph Products**

Ghidewon Abay Asmerom, Virginia Commonwealth University

In this paper we consider the genus of some graph products of H and G. These graph products are those that can be regarded as covering spaces of a voltage graphs H^* obtained by modifying H according to the configuration of G, a Cayley graph. This always starts with a suitable imbedding of H in some orientable surface followed by the modification of the edges of H to get H^*. The sufficient conditions are put on H and G so that the imbedding of the covering graph of H^* is a minimal. The imbedding technique used here involves both surgery and voltage graph theory. Sample results involving the tensor, the augmented tensor, and the lexicographic products will be presented.

240 **ALGORITHMS CONCERNING SOFIC SHIFTS**

Natasa Jonoska*, University of South Florida
Tom Head, State University of New York - Binghamton

A sofic shift is a set of bi-infinite sequences representable as the set of all labels of bi-infinite paths in a finite edge-labeled digraph. A topologically continuous shift commuting map from one sofic shift onto another sofic shift can also be represented by means of a finite labeled digraph. Such maps are topological conjugacies if they are injective. We provide an elementary polynomial time algorithm for deciding the injectivity of such maps. We also provide a quadratic time algorithm for deciding the equality of pairs of irreducible sofic shifts when these shifts are given by irreducible deterministic representations.

THURSDAY, MARCH 10, 1994
5:40 p.m.

241

ADDITIVE PERMUTATIONS OF CARDINALITY EIGHT PART II: PERMUTATIONS WITH FOUR POSITIVE AND FOUR NEGATIVE ELEMENTS

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An ordered set $X = \{x_1, x_2, ..., x_n\}$ of relatively prime integers $(x_1 < x_2 < ... < x_n)$ is called a basis of additive permutations (an A-basis) if there exists a permutation $Y = (y_1, y_2, ..., y_n)$ of X such that the vector sum $X+Y = (x_1+y_1, x_2+y_2, ..., x_n+y_n)$ is again a permutation of X. Y is then called an additive permutation of X. All A-bases of cardinality 7 and less have been known for some time. All A-bases of cardinality 8 with at most three negative elements, and their additive permutations, have been enumerated in 1992-93. The purpose of this paper is to present all remaining 18 A-bases of cardinality 8 and their additive permutations.

242

How wide can you spread your chop sticks?

Moshe Rosenfeld, PACIFIC LUTHERAN UNIVERSITY

Three chopsticks can be arranged so that the smallest angle among any two is 90 degrees.How about 4 ? 5 ? or n chopsticks ? A set L, of n lines through the origin in 3-space determines n(n-1) angles. Let a(L) denote the smallest angle determined by the lines in L. In this talk we derive an upper bound for a(L). For 4 lines we show that the largest possible angle is arccos(1/3), for 6 arccos(1/sqrt[5]) and for 5, . . . arccos(I don't know yet).

243

Generalized Degrees and Even Cycles in Graphs Ron Gould, Emory University, Atlanta, GA. Debra Knisley *, East Tennessee State Univ., Johnson City

A graph G of order n satisfies a minimum generalized degree condition $\delta_k \geq f(n)$ if every collection of k independent vertices is collectively adjacent to at least $f(n)$ of the vertices in $V(G)$. We consider a generalized degree analogue of a result due to Bondy and Simonivits (1974) on even cycles. We show that if $\delta_k(G) \geq cn^{1/p}$ for some real number c, then for sufficiently large n, G must contain a C_{2p} . Keywords: generalized degrees, neighborhood unions, even cycles.

244

How Far From Cordial Can a Graph Be?

Mark S. Anderson, Rollins College

A partition of the set V_G of vertices of a graph G into two subsets V_0 and V_1 induces a partition of the set E_G of edges of G into two subsets E_0 and E_1, the first being the set of edges whose vertices belong to the same class. We can measure the *cordiality* of a graph G by finding the minimum over all such partitions of the value

$$c = \left\lfloor \frac{|V_0 - V_1|}{2} \right\rfloor + \left\lfloor \frac{|E_0 - E_1|}{2} \right\rfloor .$$

If c(G)=0, we say that the graph G is cordial. In this paper we present bounds for the value c.

245

On Extremal Sort Sequences

Minyoung Kim* ,The Alabama A & M University
Ashok T. Amin, The University of Alabama in Huntsville

Let I(n) denote the set of integers {1,2,...,n} and let X = {x1, x2, ..., xn} denote a set whose elements are to be sorted. A sort sequence S(n) is a sequence of all unordered pairs of elements of I(n). With a sort sequence S(n), we can associate a sorting algorithm A(S(n)) as follows. An execution of the algorithm performs pairwise comprisons of elements in the input set X as defined by the sort sequence S(n), except that the comparisons whose outcomes can be inferred from the previous comparisons are not performed and such outcomes are said to be actively predicted. Let w(S(n)) denote the expected number of active predictions for the sort sequence S(n) assuming all input orderings are equally likely. Let w*(n) and w0(n) denote the maximum and minimum values, respectively, of w(S(n)) over all sort sequences S(n). Using a graph theoretic approach, we obtain bounds on w*(n), w0(n), and show that w*(n) is of order nlogn.

246

$(f,2)$–Rotational Steiner Triple Systems

Zhike Jiang *University of Waterloo*

A Steiner triple system of order v is called (f,k)-rotational if it admits an automorphism consisting of f fixed points and k cycles of length $(v-f)/k$. When $k=2, f=1$ such a system is known as a 2-rotational Steiner triple system and it is known that a 2-rotational Steiner triple system exists if and only if $v \equiv 1,3,7,9,15$ or $19 \pmod{24}$. In this paper we deal with the existence of $(f,2)$–rotational Steiner triple systems, with $f>1$ and show that such a system of order v exists if and only if (i) $v \equiv 1,3 \pmod 6$, (ii) $f \equiv 1,3 \pmod 6$, (iii) if $v-f \equiv 2 \pmod 4$ then $v \geq 3f$; if $v-f \equiv 0 \pmod 4$ then $v=f$ or $v \geq 3f-2$, and (iv) $v \neq 3f-2$ when $f \equiv 1 \pmod{12}$, $v \neq 21$ when $f=3$, $v \neq 3f-2, 3f+10$ when $f \equiv 15 \pmod{24}$, $v \neq 3f+4$ when $f \equiv 3 \pmod{12}$, $v \neq 25$ when $f=7$, $v \neq 3f-2$ when $f \equiv 7 \pmod{12}$, $v \neq 27$ when $f=9$, $v \neq 3f-2$ when $f \equiv 21 \pmod{24}$, $v \neq 3f+10$ when $f \equiv 9 \pmod{24}$ and $v \neq 3f+4$ when $f \equiv 9 \pmod{12}$.

Key Words: Steiner triple system, difference triple, Skolem sequeince, orbit of permutation groups.

247

REVISITING EUCLID'S ALGORITHM: A GEOMETRIC POINT OF VIEW

Vanessa Collazo, Arlene Correa and Pablo M. Salzberg(*), University of Puerto Rico, Rio Piedras Campus, P.O. Box 23355, Puerto Rico 00931

The intersection of a line of rational slope a/b with any circular cross section of the 2-dimensional torus $T_2 = R^2/Z^2$ are equally spaced, and the number of intersections is inversely related to the *gcd*(a,b). We use these facts to derive a version of Euclid's algorithm for numbers. This approach suggests a bi-residual variant of the *least absolute value* algorithm. Computational evidence shows that an extension of this variant to Harris'algorithm improves significantly its efficiency. This framework also seems appropriate to deal with discrete lines (computer graphics), where some new results were obtained.

248

The Software GADAR and its Application to Extremal Graph Theory

Alexander Schliep, University of Delaware

For the class of algebraically defined graphs — i.e., graphs in which adjacency between vertices is defined by some algebraic relations and hence, during the calculation of invariants, neighborhoods can be *computed* instead of being *looked up* in adjacency lists — existing packages turned out to be too inefficient.

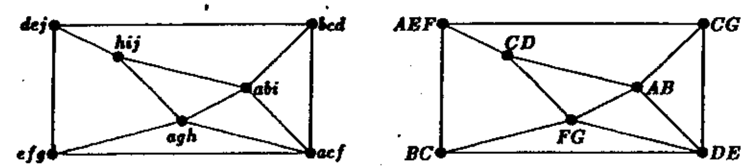
GADAR (short for "Graphs with Algebraically Defined Adjacency Relations") is a package based on Mathematica. For its design and implementation techniques of object-oriented programming were used. The package is applied to investigation of the infinite family of graphs $D(k,q)$ for which some non-intuitive conjectures were obtained. As Lazebnik, Ustimenko and Woldar could prove, the family $D(k,q)$ is an example of a family of graphs which for given order and girth have the largest size (with a finite number of exceptions).

249

FRACTIONAL COLORINGS WITH LARGE DENOMINATORS

David C. Fisher – University of Colorado at Denver

The left graph shows 7 committees with edges between committees with common members. *Avoiding scheduling conflicts, how long will it take to have each committee meet for 1 hour?* The graph can be minimally colored with 4 colors giving a 4 hour schedule. However, the right graph shows a minimum fractional coloring with $3\frac{1}{3}$ colors where A, C, E, F are $\frac{1}{3}$ colors, and B, D, G are $\frac{2}{3}$ colors. Notice the colors on a node sum to 1, and adjacent nodes do not share colors. Scheduling A's for the first $\frac{1}{3}$ hour, B's for the next $\frac{2}{3}$ hour, C's for the next $\frac{1}{3}$ hour, etc. gives a $3\frac{1}{3}$ hour schedule.



In the above example, an hour is divided into 3 subintervals because the least common denominator of a minimum fractional coloring is 3. Erdős asked for an n node graph: *Is this denominator always less than n?* Chvátal, Garey and Johnson gave a negative answer by finding examples whose denominators grow as $\sim e^{\sqrt{n \ln(n)}/2}$ (subexponential). They also showed the denominator is at most $n^{n/2}$ (superexponential). This talk gives examples whose denominators grow as $\sim 1.346193^n$ showing the denominator can grow exponentially. These examples use a result of Larsen, Propp, and Ullman on the fractional coloring of graphs formed by a construction of Mycielski.

250

On 0-1 Approximations

Jacek Ossowski

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Given any numbers $x_1, \ldots, x_n \in (0,1]$ and a permutation $\sigma \in S_n$ the task is to approximate the sums $x_1,\ x_1+x_2,\ \ldots\ , x_1+\cdots+x_n$ and $x_{\sigma(1)},\ x_{\sigma(1)}+x_{\sigma(2)},\ \ldots,\ x_{\sigma(1)}+\cdots+x_{\sigma(n)}$ by respectively $\epsilon_1,\ \epsilon_1+\epsilon_2,\ \ldots,\ \epsilon_1+\cdots+\epsilon_n$ and $\epsilon_{\sigma(1)},\ \epsilon_{\sigma(1)}+\epsilon_{\sigma(2)},\ \ldots\ ,\ \epsilon_{\sigma(1)}+\cdots+\epsilon_{\sigma(n)}$, where $\epsilon_1, \ldots, \epsilon_n \in \{0,1\}$. How accurate the approximations can be? The answer is provided by the following theorem:
For any $x_1, \ldots, x_n \in (0,1]$ and $\sigma \in S_n$ there exist numbers $\epsilon_1, \ldots, \epsilon_n \in \{0,1\}$ such that for all $k = 1, \ldots, n$

$$|\sum_{i=1}^{k} x_i - \sum_{i=1}^{k} \epsilon_i| \leq n/n+1$$

and

$$|\sum_{j=1}^{k} x_{\sigma(j)} - \sum_{j=1}^{k} \epsilon_{\sigma(j)}| \leq n/n+1 \ .$$

The number $n/n+1$ is the best possible (the best previously known upper bound was $1-2^{-2^n}$). In this paper we present a proof obtained independently of Donald E. Knuth who solved the problem shortly before the author. In our approach we use theorems of Ford and Fulkerson on common systems of distinct representatives.

251

On the Spectrum of OCDD

Xiaojun Zhu, University of Waterloo

Keywords: *orthogonal steiner triple system, group divisible design, orthogonal group divisible design*

Two {3}–GDD of the same type $(V,\mathcal{G},\mathcal{B}_1)$ and $(V,\mathcal{G},\mathcal{B}_2)$ are orthogonal if they satisfy two conditions:
(O1) If $\{x,y,a\} \in \mathcal{B}_1$ and $\{x,y,b\} \in \mathcal{B}_2$, then a,b are not in the same group.
(O2) For any two distinct intersecting triples $\{x,y,z\}$ and $\{u,v,z\}$ of $\mathcal{B}_1$, the two triples $\{x,y,a\}$ and $\{u,v,b\} \in \mathcal{B}_2$ satisfy $a \neq b$.
Such a pair is denoted by OGDD—orthogonal group divisible designs.
The special case when all groups of of size 1 corresponds to the OSTS(orthognal steiner triple system). The problem of constructing OSTS was raised in 1968 by O'shaughnessy as a method to construct Room squares, it was not until 1992 the spectrum of OSTS was completely determined. The general case has been studied extensively since then. While not all cases have been solved yet, much progress has been made on the following result:
For any $g > 1$ and n which satisfies the necessay conditions, there exists an OGDD of type g^n except possibly for finitely many cases.

252

THE FASTEST EUCLID'S ALGORITHMS IN TOWN

Rodney Cruz (*) and Pablo M. Salzberg, University of Puerto Rico, Rio Piedras Campus, P.O. Box 23355, Rio Piedras, Puerto Rico 00931.

We introduce a family of versions of Euclid's algorithm to find the greatest common divisor of two integers whose speeds depend on memory availability. These algorithms can be implemented in any numerical basis.

253

To Break a Graph Into Pieces

Robert R. Goldberg* and Jerry Waxman
Queens College of CUNY, Flushing, N.Y. 11367

Let G be an undirected connected graph and v, a vertex of G. This paper describes a fast parallel method to determine when v is an articulation point of G. The method incorporates an extension of a previous result of the authors for determining, in parallel, which vertices of a graph are k-Reachable from a given node of the graph. A combinatorial circuit was shown to implement k-Reachability in parallel for O(k) steps. Based on this, we show that all the articulation points of a graph can be achieved in O(d) time, where d is the diameter of G.

254

New Classes of *p*-Competition Graphs and ϕ-Tolerance Competition Graphs

Charles A. Anderson, Larry J. Langley, J. Richard Lundgren, Patricia A. McKenna* , Sarah K. Merz
University of Colorado at Denver

The concept of the competition graph has been extensively studied and generalized since its introduction in the late sixties. We will consider the generalizations known as the *p*-competition graph and the ϕ-tolerance competition graph. The *p-competition graph* of a digraph D=(V,A) has vertex set V and an edge between *x* and *y* if and only if there are at least *p* distinct vertices for which *x* and *y* "compete" in D. A graph G is a *ϕ-tolerance competition graph* if there is a digraph D=(V,A) and an assignment of tolerances t_i to the vertices v_i such that for $i \neq j$, $[v_i,v_j] \in E(G)$ if and only if $|O(v_i) \cap O(v_j)| \geq \phi(t_i,t_j)$, where ϕ is a symmetric function from N×N to N such as min, max, or sum ($N=\{n \in Z \ni n \geq 0\}$). Using a construction of Myceilski as well as other methods, we will generate large classes of graphs which fall into one category but not another. For example, we will construct large classes of graphs which are not competition graphs which are min-tolerance competition graphs. We will also present an algorithm for generating graphs which are competition graphs but not 2-competition graphs. KEY WORDS: *p-competition graph, ϕ-tolerance competition graph*

256

The splitting formula for computing the Tutte polynomial of a binary 3-connected matroid

ARTUR ANDRZEJAK   VANDERBILT UNIVERSITY, DEPT. OF MATHEMATICS

A splitting formula for computing the Tutte polynomial $t(M)$ of a 3-connected binary matroid M will be presented. Specializations of the Tutte polynomial include: the Jones polynomial of an alternating knot; the weight enumerator of a linear code; the reliability of a network; the number of colorings and flows in graphs. Accordingly, computational methods for finding the Tutte polynomial of a matroid (in general a #P-hard problem) are of practical and theoretical importance.

The formula uses notion of a bipointed Tutte polynomial, an extension of the (single) pointed Tutte polynomial introduced by Thomas Brylawski. Applying this formula we can compute $t(M)$ from Tutte polynomials of some minors obtained by representing M as a 3-sum. A 3-sum of a binary 3-connected matroid can be found in polynomial time using the exact 3-separation (S_1,S_2) of M. Our splitting formula requires the calculation of Tutte polynomials of matroids on no more than $\max(|S_1,S_2|)$ elements. Thus by choosing an exact 3-separation of M with minimal $\max(|S_1,S_2|)$ and by applying the splitting formula we can drastically reduce the computation time of $t(M)$.

FRIDAY, MARCH 11, 1994
9:10 a.m.

256

SELF-ORTHOGONAL MENDELSOHN TRIPLE SYSTEMS

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L. Zhu, Suzhou University

A Mendelsohn triple system of order v, briefly MTS(v), is a pair (X, 𝔹) where X is a v-set (of points) and 𝔹 is a collection of cyclic triples on X such that every ordered pair of distinct points from X appears in exactly one cyclic triple of 𝔹. The cyclic triple (a,b,c) contains the ordered pairs (a,b), (b,c), and (c,a). An MTS(v) corresponds to an idempotent semisymmetric Latin square (quasigroup) of order v. An MTS(v) is called self-orthogonal, denoted briefly by SOMTS(v), if its associated semisymmetric Latin square is self-orthogonal. It is well-known that an MTS(v) exists if and only if $v \equiv 0$ or 1 (mod 3) and $v \neq 6$. It is also known that a SOMTS(v) exists for all $v \equiv 1$ (mod 3) except $v = 10$ and that a SOMTS(v) does not exist for $v = 3, 6, 9$ and 12. In this paper it is shown that a SOMTS(v) exists for $v \in \{15, 24, 42, 48, 51, 57, 60, 69\}$ and for all $v \geq 75$ where $v \equiv 0$ (mod 3).

257

An Algorithm for the Enumeration of Costas Sequences and Similar Permutations

John G. Ramirez and Oscar Moreno*, Gauss Research Laboratory
University of Puerto Rico, Rio Piedras

We have developed an algorithm that performs backtracking adding terms to both sides of the sequence. Using the symmetries among the permutations this algorithm will reduce efficiently the computation time by not repeating a searched pattern as subpattern in a different branch of the search. This was very efficiently adapted for working in a parallel computer. This algorithm was useful for having a parallel program that enumerates all Costas sequences of size n in a very efficient manner. Costas arrays are special permutations which are important in sonar and radar applications. We obtained all Costas sequences of size 19 in one week (this task would had taken Dr. Silverman, a previous researcher from the ARMY Research Office, more than one year) and the set of all Costas sequences of size 20 in one month. These techniques also led P. Pei (a former researcher from our laboratory) to obtain more specialized Costas sequences for even larger sizes.

258

A Matrix Analysis of Carrier Posets of Biconnected Graphs

Peter W. Stephens, Department of Mathematics, U. C. L. A.

We define the *Carrier Poset*, $(\mathcal{P}_G, \leq_{\mathcal{P}_G})$, for a biconnected simple graph G. The incidence algebra, $\mathcal{A}(\mathcal{P}_G)$, for this poset is considered by studying a certain class of functions, f, called *broken cycle content functions*. We describe matrices, M_f, of these functions, as elements of $\mathcal{A}(\mathcal{P}_G)$, with respect to a particular linear order on $\mathcal{P}_G$. For any such *broken cycle content matrix* for a *full span chain*, with the *pre-cut* and *post-cut* properties, in $(\mathcal{P}_G, \leq_{\mathcal{P}_G})$, we show that there is an associated submatrix of M_f which is upper triangular, weakly decreasing in rows, and weakly increasing in columns. We study *full span bicomponent trees* for biconnected simple graphs and interpret our results on chains in terms of these structures. These results suggest a general mathematical view of why certain classical optimal graph algorithms always seem to work in regions of $\mathcal{P}_G$ associated with such full span chains.

259

p-Competition Graphs of Strongly Connected and Hamiltonian Digraphs

Larry Langley, J.Richard Lundgren, Patricia A. McKenna,
Sarah K. Merz*, University of Colorado at Denver

Craig W. Rasmussen, Naval Postgraduate School

Competition graphs were first introduced in the late sixties by Joel Cohen in the study of food webs and have been extensively studied since 1978. We will examine one generalization of competition graphs, the p-competition graph. Graphs which are the competition graph of a strongly connected or Hamiltonian digraph are of particular interest in applications to communication networks. It has been previously established that every graph such that the size of the minimum edge clique cover is less than or equal to the number of vertices in the graph minus the number of isolated vertices (except K_2) is the competition graph of a strongly connected loopless digraph. We establish an analogous result for p-competition graphs. Furthermore, we establish some large classes of graphs, including trees, which are the p-competition graph of a loopless Hamiltonian digraph. For $p = 2$, we find that an interval graph on $n \geq 4$ vertices is the competition graph of a loopless Hamiltonian digraph. The difficulties in generalising results for competition graphs to p-competition graphs will be discussed.

key words: competition graph, p-competition graph, strongly connected, Hamiltonian, trees, interval graph

260

A Feasible Set Expansion for the Two-Variable Greedoid Polynomial

Elizabeth McMahon* and Gary Gordon, Lafayette College

The two-variable greedoid polynomial is a generalization of the Tutte polynomial of a matroid and the one-variable greedoid polynomial. We give an expansion for the polynomial in terms of feasible sets which is a consequence of a partition of the Boolean lattice of subsets of the ground set into intervals based on the feasible sets of the greedoid. As an application, we show that when G is a rooted digraph, the subdegree of one of the variables is the minimum number of edges whose removal leaves an acyclic digraph. This expansion generalizes Whitney's expansion of the chromatic polynomial of a graph.

261 Some terraces with difference properties:
On a conjecture of B.A. Anderson.

Stephen D. Cohen and Philip A. Leonard,* Arizsona State Univ

In his 1990 paper on quasi-2-complete Latin squares, Bruce Anderson made a conjecture, for primes p congruent to 5 modulo 8, regarding a particular construction of 2-sequencings (or terraces) for the additive group of residues modulo p. We have investigated and proven this conjecture for "virtually all" such primes by a "sieve plus character sum" argument. The verifications needed for small primes indicate that the construction yields an abundance of terraces having the two-occurrence property for an additional row of differences.

262 **Visualizing Subset Sums: Theorems and Conjectures**

Stan Wagon*, Macalester College, and Herb Wilf, University of Pennsylvania

Lots of *Mathematica*-generated graphics led us to a concise theorem about subset sums. A triple (n, t, m) of positive integers is *uniform* if the size-t subsets of $\{1, 2, \ldots, n\}$ are equidistributed with respect to their mod-m sums; $\text{mod}(a, b)$ means the least nonnegative residue of a modulo b. We use $[n]$ to denote $\{1, 2, \ldots, n\}$.

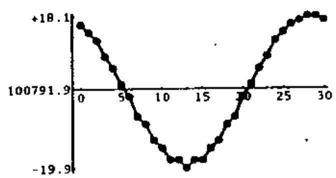
THEOREM. The following are equivalent:

1. (n, t, m) is uniform.
2. For all $d > 1$ that divide m, $\text{mod}(t, d) > \text{mod}(n, d)$.
3. The Gaussian polynomial $\binom{m}{1}_q$ divides $\binom{n}{t}_q$.

We have two approaches: one uses Gaussian polynomials; the other uses induction and is bijective. More interesting is the case of nonuniformity, where graphics computations lead to many conjectures. Prime moduli, for example, yield especially neat patterns.

CONJECTURE. *If p is prime then the sequence of frequencies of mod-p sums of t-subsets of $[n]$ is unimodular, meaning that, starting at the minimum residue residue, the sequence is nondecreasing to its maximum and then nonincreasing back to the minimum (see diagram).*

This assertion is equivalent to a mod-p conjecture about the coefficients of Gaussian polynomials that strengthens the famous theorem about unimodularity of these coefficients.



The distribution of the 3,124,550 mod-31 sums of 9-tuples from $\{1, 2, \ldots, 26\}$

263 **A linear algorithm for the planarity of 2-complexes**

R. Ayala (1), J. Caceres (2), A. Marquez*(3) and A. Quintero (1)
(1) Facultad de Matematicas. Universidad de Sevilla
(2) Facultad de Ciencias. Universidad de Almeria.
(3) Facultad de Informatica. Universidad de Sevilla.

We give in this communication a more general characterization of when a 2-complex has a plane embedding than the characterization gave by Gross and Rosen (Colloq. Math 44(1981) 241-247), obtaining in such a way a linear time planarity algorithm for 2-complexes with better perfomance than the algorithm given by the same authors (Jour. of ACM 20(1979) 611-617).

264 **A Subclass of Multiple Interval Graphs**

Nishit Kumar and Narsingh Deo, UCF, Orlando

Trotter and Harary define a *multiple interval graph* G, of interval number t as one for which, there exists a function f which assigns to each vertex u of G, a subset $f(u)$ of the real line, the subset being the union of t (not necessarily disjoint) closed intervals ; and distinct vertices u, v of G are adjacent if and only if $f(u) \cap f(v) \neq 0$. Several authors have studied these graphs with the motivation of recognizing interval numbers for different families of graphs. It has been shown that recognizing these graphs with fixed interval numbers is NP-Complete, and hence it is unlikely that there would be a neat characterization for multiple interval graphs.

We are motivated to define a new subclass of multiple interval graphs, called *multi-dimensional interval graphs*, and study from the point of view of characterizing them. In a multi-dimensional interval graph G, with *dimensionality* (similar to the notion of interval number) t, the function f assigns to each vertex u of G, an ordered set of closed intervals, $f_1(u), f_2(u), \ldots, f_t(u)$; and distinct vertices u and v are adjacent if and only if $f_i(u) \cap f_i(v) \neq 0$, for some i, $1 \leq i \leq t$. These graphs are a subclass of multiple interval graphs and a superclass containing the generalized interval graphs considered by Roberts.

Here we show that trees are a proper subclass of multi-dimensional interval graphs with dimensionality 2. For a complete bipartite graph $K_{m,n}$, a lower bound of $\lceil mn/(m+n-1) \rceil$ on the dimensionality has been proved. The lower bound has been shown to be tight for $m = n$ and $m > 3n$.

265 **Ramsey and Extremal Theory for Matroids**

Fair Hurst and Talmage Reid*, Dept. of Math., The University of Mississippi, University, MS 38677.

Ramsey numbers for matroids, which mimic properties of Ramsey numbers for graphs, have been defined as follows. Let k and l be positive integers. Then $n(k, l)$ is the least positive integer n such that every connected matroid with n elements contains either a circuit with at least k elements or a cocircuit with at least l elements. We determine the largest value of these numbers known and find extremal matroids with small circuits and cocircuits. We also use these numbers to determine size functions of classes of matroids with small cocircuits. Results on matroid connectivity, geometry, and extremal matroid theory are used here.

266 **NEW DIFFERENCE FAMILIES AND NEW OPTIMAL OPTICAL ORTHOGONAL CODES FROM FINITE FIELDS**

MARCO BURATTI
Facoltà di Ingegneria de L'Aquila, 67040 Poggio di Roio, L'Aquila, Italy

Proceeding in the way indicated by R. M. Wilson in his classic "cyclotomic paper", the author presents a very effective method for constructing simple difference families from finite fields and hence Steiner 2-designs of order a prime power.

This method also unifies all the before known direct thecniques based on Galois fields (by R. C. Bose in 1939, R. M. Wilson in '72 and the author himself in '93).

Finally, this method has been more generally adapted for obtaining optimal optical orthogonal codes.

267 **The Kolakosky Sequence is Cube-Free**

S. C. Cater and R. W. Robinson*, Computer Science Dept., University of Georgia

The Kolakosky sequence is an ω-word over $\{1,2\}$ which is self-generating. It has long been suspected that the Kolakosky sequence is cube-free. Here it is proved to be cube-free, and also that a word xx can only occur as a consecutive subsequence if $|x| = 1, 2, 3, 9$, or 27. The proof proceeds by establishing a close relationship between subsequences of the Kolakosky sequence and certain periodic ω-words.

268 **On Maximum Sum Problems in Graphs**

A. T. Amin* and P. J. Slater
University of Alabama in Huntsville

Given a graph G in which each edge and/or vertex is assigned a real number called its weight; We consider the problem of determining the maximum sum of edge and/or vertex weights among all (induced) subgraphs of G. It is shown that when G can be an arbitrary graph the corresponding decision problems are NP-Complete. Linear algorithms are given for the case when G is restricted to be a series parallel graph.

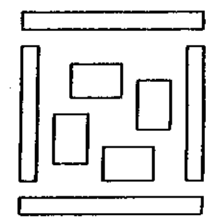
269 **On Rectangle Visibility Graphs**

Joan Hutchinson*, Macalester College, Tom Shermer, Simon Fraser University, and Andy Vince, University of Florida

A *rectangle visibility graph* is one whose vertices can be represented by disjoint rectangles in the plane with two vertices adjacent if and only if there exists an unobstructed horizontal or vertical band of positive width joining one rectangle to the other. A rectangle visibility representation of K_8 is shown below.

THEOREM. A rectangle visibility graph with $n \ge 8$ vertices has at most $6n - 20$ edges, and for each $n \ge 8$ there is a rectangle visibility graph with $6n - 20$ edges.

These graphs include all planar graphs and have thickness at most 2; by the theorem they do not include all thickness-2 graphs. A complete characterization for these is not known, although such a characterization is known for the 1-dimensional analogue, horizontal line-segment visibility graphs with vertical visibility determining edges. Define the *degree* of a graph to be the number of edges divided by the number of vertices. We show that for each integer i, $0 \le i \le 6$, there is an infinite family of rectangle visibility graphs and an infinite family of nonrectangle visibility graphs, each of which has degree asymptotically equal to i.



270 **Binary 2-paving matroids**

Sanjay Rajpal, Dartmouth College

A rank r matroid is called k-paving if it has no circuits of size less than r-k+1. Binary 1-paving matroids have been determined by Acketa. We determine all binary 2-paving matroids.

271

Transversals in Turn-squares

Wendy Myrvold*, Univ. of Victoria
Mark Ellingham, Vanderbilt University

While searching for an orthogonal triple of Latin squares of order 10, Parker came across a remarkable Latin square which has 5504 transversals and more than a million orthogonal mates. This square is an example of a turn-square.

A *turn-square of order* $2n$ is a Latin square which is constructed from an n by n Latin square L and bit matrix B as follows:

$$\begin{matrix} L+n*B & L+n*(J-B) \\ L+n*(J-B) & L+n*B \end{matrix}$$

(J is the all-ones matrix). For turn-squares of order 10, we show that each duplex of the starting matrix L having odd parity with respect to B corresponds to 64 transversals in the constructed turn-square, and that this completely characterizes the transversals of the turn-square.

272

Products of Sequences with Zero Autocorrelation

R. Craigen
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Let sequences $s_1, s_2, \ldots$ be the lists of coefficients of polynomials $p_1, p_2, \ldots$ The *autocorrelation* of this set of sequences is the list of coefficients of terms of positive degree in the Laurent polynomial

$$p_1(x)p_1(\frac{1}{x}) + p_2(x)p_2(\frac{1}{x}) + \cdots$$

Sets of two and four sequences with entries $0, \pm 1$ having zero autocorrelation have found uses in optics, in the transmission of signals and in the construction of certain special matrices. More recently, the author has shown how an arbitrary number of them can be used to construct Hadamard matrices; this has led, among other things, to a great increase in known admissible orders.

The fewer sequences used at the start, the better the results obtained; sets of fewer sequences are rare, but they get more abundant as the number of sequences increase. Most new sequences are short ones found by computer search; we rely on recursive constructions for larger lengths. It has long been known how to multiply the lengths of pairs of sequences, but these are quite rare. C. H. Yang has given a number of ways to multiply the lengths of sets of four sequences, which are somewhat less rare. For more than four sequences the known products increase the number of resulting sequences to such an extent that they are not very interesting. We show how to get around this problem to multiply larger sets of sequences with relatively little loss.

273

Decompositions of complete multipartite graphs into selfcomplementary factors with given diameters

D. Fronček*, McMaster University, J. Širáň, University of Vermont

A multipartite graph $K_{m_1,m_2,\ldots,m_r}$ is (t,d)-*isodecomposable* if it can be decomposed into t isomorphic factors with the diameter d. It is known that if a bipartite graph $K_{n,m}$ is $(2,d)$- isodecomposable, then $d = 3,4,5,6$ or ∞; and if an r-partite graph $K_{m_1,m_2,\ldots,m_r}$ with $r > 2$ is $(2,d)$-isodecomposable, then $d = 2,3,4,5$ or ∞.

We completely determine the spectrum of all bipartite and tripartite $(2,d)$-isodecomposable graphs. For complete four-partite graphs we determine the spectrum of $(2,5)$-isodecomposable graphs; for $d = 2,3,4$ we solve the spectrum problem completely for $(2,d)$-isodecomposable graphs with at most one odd part and for the graphs $K_{n,n,n,m}$ and $K_{n,n,m,m}$.

For all $r \geq 5$ we determine the smallest $(2,d)$-isodecomposable r-partite graphs for all possible diameters. We also show that if p_0 is the order of a smallest $(2,d)$-isodecomposable complete r-partite graph, then such a graph exists for each order $p > p_0$.

274

RECTANGULAR VISIBILITY REPRESENTATIONS OF BIPARTITE GRAPHS

Alice M. Dean*, Skidmore College, and Joan P. Hutchinson, Macalester College

A *rectangular visibility graph* is a graph whose vertices are rectangles in the plane, with adjacency determined by horizontal and vertical visibility. We show that $K_{p,q}$ has a representation with no rectangles having collinear sides if and only if $p \leq 3$ and $q \leq 4$. More generally, we show that $K_{p,q}$ is a rectangular visibility graph if and only if $p \leq 4$ or $q \leq 4$.

275

DISJOINT SPANNING TREES IN COUNTABLE GRAPHS AND INFINITE MATROIDS

Jerzy Wojciechowski
West Virginia University

Tutte and Nash-Williams independently proved that a finite graph G has k edge-disjoint spanning trees if and only if for every partition $\mathcal{P}$ of the set of vertices of G the number of edges with endpoints in distinct sets of $\mathcal{P}$ is at least $k(|\mathcal{P}|-1)$. Later, Edmonds generalized this result by proving a necessary and sufficient conditon for a finite family of finite matroids on the same set to have a system of disjoint bases.

Let $\mathcal{M} = (M_r)_{r\in R}$ be a system of possibly infinite matroids on a set S. Following the ideas of Nash-Williams, for every transfinite sequence f of distinct elements of S, we define a number $\eta(f)$. We prove that the condition that $\eta(f) \geq 0$ for every possible choice of f is necessary for $\mathcal{M}$ to have a system of mutually disjoint bases. Further, we show that this condition is sufficient if R is countable and M_r is a rank-finite matroid for every $r \in R$. We also conjecture a necessary and sufficient condition for the existence of k edge-disjoint spanning trees in a countable graph.

276 **Single Change Neighbor Designs**

RL Constable, University of St Andrews,
DL Preece, University of Canterbury,
NCK Phillips*, TD Porter and WD Wallis,
Southern Illinois University

A SCN(v,k) design is an ordered list of cycles of length k chosen from a complete graph on v vertices with the properties (1) every edge occurs in at least one cycle and (2) for each cycle C in the list the next listed cycle is obtained from C by replacing one vertex. We have bounds on the size of such a design, examples with k<v<11, an extension theorem and some open questions.

277 SET REPRESENTATIONS OF GRAPHS - AN ALGORITHM

Nancy Eaton, University of Rhode Island

Let U be any set and let L be a subset of the natural numbers. An L-SET REPRESENTATION of a graph G is a function F mapping the vertices of G into the power set of U so that two vertices u and v form an edge in G if and only if the size of the intersection of F(u) and F(v) lies in L. We call the set L the RULE. If L = {p, p+1, ...}, the set representation is commonly called a p-INTERSECTION REPRESENTATION. For a particular graph G and a given rule L, we are interested in the size of the smallest possible universal set U for which there exists an L-set representation of G. We also consider the question: for a given graph G, what is the size of the smallest possible universal set over all possible rules L.

Now consider L to be the set of positive odd integers. We present an algorithm which gives an L-set representation for any graph using a universe of size at most n-1. Also, some related results are mentioned and open problems are given.

278 **Tree Automata for Cutwidth Recognition**

Nancy G. Kinnersley* and William M. Kinnersley
Dept. of Computer Science, University of Kansas

If a minor- or immersion-closed family of graphs has bounded treewidth, a finite-state tree automaton can be constructed that recognizes its members in linear time. The automaton may also be used to enumerate the elements of its obstruction set. In this paper, a Myhill-Nerode approach is used to construct tree automata to recognize graphs with fixed cutwidth k. The automaton can also be used to produce a cost k layout when one exists. We explicitly construct the automata for two families of graphs---trees and graphs whose only cycles have length three. The automata are used to identify all graphs having this structure in the cutwidth three obstruction set.

279 **Loop and Cyclic Niche Numbers of Novas and Wheels**

Steve Bowser* and Charles A. Cable, Allegheny College

For a class of digraphs, D, a graph, G, and a natural number m, let D(G,m) be the set of digraphs in D having niche graph equal to the graph G extended by m isolated vertices. Define n(D,G) to be either infinity if D(G,m) is empty for every natural number m, or the minimum m such that D(G,m) is non-empty, otherwise. For D = the set of all digraphs, n(D,G) is called the loop niche number of G; for D = the set of all loopless digraphs, n(D,G) is called the cyclic niche number of G; and for D = the set of all acyclic digraphs, n(D,G) is called the (original) niche number of G. Various results are given concerning the cyclic and loop niche numbers of certain classes of graphs. In particular, we give exhaustive results for novas, wheels, and complete bipartite graphs.

281 **On the Existence of Some Combinatorial Arrays**

D.V. Chopra, Wichita State University

An array T with m constraints (rows), N runs (treatment - combinations or columns), and with s levels is merely a matrix of size (m x N) with s symbols (say, 0,1,2,..., $s-1$). For $s = 2$ the array T is called a binary array. Imposing some combinatorial constraints on T leads us to some interesting and useful structures which are valuable in applications to statistics and combinatorics. In this paper we consider and study binary arrays T with some specified combinatorial structure(s). We also discuss the use of such arrays to statistical design of experiments (in particular to fractional factorial designs).

Key Words: array, constraints, runs, fractional factorial designs, levels of an array.

282 **GENERATING PRIMES USING TABLES OF THE GENERALIZED FIBONACCI POWER SERIES**

by Joseph Arkin*, David C. Arney, Frank R. Giordano and Rickey A. Kolb, United States Military Academy, West Point, NY 10996-1786.

In this paper we discuss the generation of primes using tables made up of k rows of coefficients in the expansions of the generalized Fibonacci power series for the generating function

$$(1 - ax - bx^2)^{-k}$$

where a,b are positive integers and k = 1,2,3,... . Simple manipulations and additions are used to produce a prime number sieve for the tables.

283 **MIXED DOMINATION IN TREES: A PARALLEL ALGORITHM**

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A set of vertices S in a graph $G = (V, E)$ is called a *dominating set* of G if every vertex in the set $(V \setminus S)$ is adjacent to some vertex in the set S. For arbitrary graphs, the problem of computing smallest dominating set is NP-complete. A more general version of this problem is called "mixed domination" problem.

In this paper we present new parallel NC algorithm to find smallest mixed dominating set in trees. The model of parallel computation used is the CRCW P-RAM (Concurrent Read Concurrent Write Parallel RAM), where more than one processor can concurrently read from or write into the same memory location during the same memory cycle. Writing conflicts are resolved in a non-deterministic fashion. The algorithm requires $O(n)$ processors and runs in $O(\log n)$ time on a CRCW P-RAM.

Keywords: NC algorithm, domination, tree, graph

284 **Bipartite Interval Tolerance Graphs**

K. P. Bogart, Dartmouth College, *Larry Langley, University of Colorado, Denver

We show that any bipartite graph that is a cocomparability graph is an interval tolerance graph. This completely characterizes bipartite bounded tolerance graphs. We give a method of constructing a representation for all such graphs.

Interval tolerance graphs were introduced in two papers by Golumbic and Monma, and Golumbic, Monma and Trotter. There is no known characterization of interval tolerance graphs that gives an efficient means of recognition. We address the problem by considering classes of graphs, in this case bipartite graphs.

The method used to construct a representation for bipartite interval graphs involves considering the associated interval tolerance order. In the case of bipartite graphs the associated order is width two, and this leads to an interval representation.

285 **Crapo's beta invariant for greedoids**

Gary Gordon, Lafayette College

The beta invariant is a well-studied and important integer invariant for matroids. We define a beta invariant for greedoids and compute it for several greedoid classes. These classes include trees, rooted graphs, rooted digraphs, posets, triangluated graphs and convex subsets of a point set. When the greedoid is an antimatroid, the beta invarint can be determined from the semi-lattice of free convex sets.

286

Graph-Matching Neural Networks for Automated Fingerprint Identification

Carol G. Crawford, U. S. Naval Academy

This talk presents results of a collaborative research effort with Eric Mjolsness, Center for Theoretical and Applied Neural Science at Yale University, to design algorithms for searching and matching fingerprints in a large data base. Specifically, the approach makes essential use of inexact graph-matching formulations and neural networks for handling matching and classification of fingerprints. Given a sparse set of minutia from a fingerprint image, together with their locations in the plane and other labels such as ridge counts to nearby minutia, the first goal is to construct a graph-like representation of the minutia map and then to design matching algorithms which can be implemented as neural networks. An overview of the general approach , including relevant graph representations, will be presented.

This ongoing research program is an outgrowth of an initial investigation for the Federal Bureau of Investigation and has received funding from the FBI and from the Office of Naval Research.

287

ON A PARTITIONING PROPERTY OF MAXIMAL ANTICHAINS

Niall Graham, University of Alabama in Huntsville

In a boolean lattice B, it is always possible to partition any maximal antichain A into two subsets X and Y such that the union of the upset of X with the downset of Y yields the entire lattice B. An elementary proof of this result is presented. For arbitrary posets, this partitioning property is found to be NP-hard, so a nice characterization theorem is unlikely. Finally, these results inspire a new sufficient condition for the satisfiability of conjunctive boolean formulas.

288

Constant-Time Computation of Minimum Dominating Sets

Marilynn L. Livingston* and Quentin F. Stout
Southern Illinois U-Edwardsville & U Michigan

Let G be a graph and let P(n) denote an element from a one-parameter family of graphs, such as a path of length n, a cycle of length n, or a complete binary tree of height n. We are concerned with determining minimum dominating sets of graphs of the form G x P(n). Using finite state automata and dynamic programming, we show a constant time algorithm to produce a minimum dominating set of G x P(n), for fixed G and all n, for the one-parameter families mentioned. Previous researchers had used similar techniques but obtained only linear-time algorithms. We also show how a closed form expression can be obtained for the minimum domination number of G x P(n).

Results of the implementation of the algorithm are given. We discuss extensions of the algorithm to the determination of all minimum dominating sets for G x P(n), and to the related problems of coverings, packings, and codes. In addition, we discuss algorithm extensions to perfect domination and to several different types of domination.

289

A Kind of Domain Decomposition Method for the Fluid Mechanics

Wu Yunhai, Zhongshan University, Guangzhou, P.R.C.

This paper considers two-dimension steady Navier-Stokes equations with Direchlet boundary condition for viscous incompressible flows on a bounded region Ω, which can be written as

$$\begin{cases} -\nu\Delta U + (U \cdot \nabla)U + \nabla p = f & \text{in } \Omega \\ \text{div } U = 0 & \text{in } \Omega \\ U = g & \text{on } \partial\Omega \end{cases}$$

It presents a kind of domain decomposition algorithm which has much better parallel character and can be used on MIND computer.

We divide Ω into more than two parts and set up a series of subproblems with equations in each subdomain. The communications among subdomains are going on in the overlapping areas. The difficulty of the compatibility condition, which arises when subdomain problems are not solved exactly at each iteration, is overcome by adding artificial compressibility constants to the continuity equation. The convergence under certain sense is proved and the relation between overlapping domains and convergent rate is discussed. The efficiency of the algorithm has been shown by the numerical experiment for the flows in a channel with a facing step.

290

Recursive E-Graphs

Robert Hon* and Gregory A. Schaper
East Tennessee State University

E-Graph construction allows for invariants of large graphs to be determined from the values of the invariants of their smaller component graphs. In this paper the construction of E-Graphs is generalized by Recursive E-Graphs (RE-Graphs). The chromatic number, clique number, vertex independence number, and vertex cover number of RE-Graphs are determined. A class of RE-Graphs which may be suitable for use as communications networks is identified.

Keywords: E-Graph, graph invariants, networks.

291

The Analysis of Neural Network Structure

Stan Klasa*, Yin Hongfeng, Tao Li
Concordia University

Several kinds of evolution of neural network structure are analyzed in this paper. First, the approach of pruning connection from concurrent network is provided. Then, an improved back-propagation algorithm is proposed in this paper. By using the algorithm, hidden neurons in the network can be removed both statically and dynamically. An adding neuron method is also given for nonlinearly separable problems. How to construct a global neural network from several local networks is discussed. The relationship between the neural network structure and knowledge structure is analyzed. Numerical examples are given to illustrate the utility of the proposed methods. The algorithms and methods proposed in the paper are applicable for solving problems such as pattern recognition, function approximation, image compression, knowledge representation, etc.

292

On the Structure of Maximum 2-part Sperner Families

Shahriar Shahriari, Pomona College, Claremont, CA 91711

Color the elements of a finite set S with two colors. A collection of subsets of S is called a 2-part Sperner family if whenever for two distinct sets A and B in this collection we have A contained in B then B-A has elements of S of both colors. All 2-part Sperner families of maximum size were characterized by P. L. Erd"os and G. O. H. Katona in 1986. In this paper we provide a different, and quite elementary proof of the structure and number of all maximum 2-part Sperner families, using only some elementary properties of symmetric chain decompositions of the poset of all subsets of a finite set.

293

Vizing's Conjecture on Domination and Related Problems

B.Hartnell*, Saint Mary's University,Nova Scotia
D.Rall, Furman University, South Carolina

A conjecture (1963) of Vizing states that the cardinality of every dominating set of the Cartesian product G X H is at least as large as the product of the domination numbers of the graphs G and H. Besides being fascinating in its own right, there are a number of related questions that arise in attempting to resolve the conjecture. This talk will indicate several of these and their relationship to the authors' attempts to settle this problem.

294

ALGORITHM AND COMPUTATIONS IN FREE LIE SUPERALGEBRAS

Alexander A. Mikhalev, Andrej A. Zolotykh,
Department of Mechanics and Mathematics, Moscow State University.

We present a number of results in combinatorial theory of Lie superalgebras: bases, subalgebras, composition technique automorphisms, primitive elements, Fox differential calculus and others. These results include a series of effective algorithms for solving combinatorial problems in Lie superalgebras. We have a package of programs with computer realizations of these algorithms. Some of the results were published in mathematical journals, some were included in the monograph "Infinite dimensional Lie superalgebras" by Yu.A.Bahturin, A.A.Mikhalev, V.M.Petrogradsky and M.V.Zaicev, Walter de Gruyter Publ., 1992; some of new results and algorithms will be included in a new book "Combinatorial aspects of Lie superalgebras" by A.A.Mikhalev and A.A.Zolotykh, CRC PRESS, Boca Raton, FL, in preparation.

295

Transition-Restricted Walks and a Generalization of Menger's Theorem

Kenneth A. Berman* and Jerome L. Paul
Department of Computer Science, University of Cincinnati, Cincinnati, OH

Let $G = (V,E)$ be a multi-digraph with a set S of sink vertices and a set T of source vertices. A *transition-restricting function of depth d* is a mapping ρ from $E^1 \cup E^2 \cup \ldots \cup E^d$ to 2^E. A walk $W = se_1u_1e_2u_2 \ldots u_{p-1}e_pt$ is ρ-*restricted* if $e_i \in \rho(e_{i-j}, e_{i-j+1}, \ldots, e_{i-1})$, $2 \le i \le p$, $1 \le j \le \min\{d, i-1\}$. Let $\mu_k(\rho)$ denote the maximum number of ρ-restricted walks from S to T such that no two walks have a subwalk of length k in common. Let $\beta_k(\rho)$ denote the minimum cardinality of a set B of walks of length k such that every ρ-restricted walk from S to T contains as a subwalk at least one walk from B. Clearly, $\mu_k(\rho) \le \beta_k(\rho)$. We show that $\mu_k(\rho) = \beta_k(\rho)$ when $k \ge d$. However, $\mu_k(\rho)$ and $\beta_k(\rho)$ are not necessarily equal when $k < d$. We present an algorithm that is polynomial in the size of G for computing $\mu_k(\rho)$ for k, d fixed and $k \ge d$. On the other hand, we show that the problem of computing $\mu_k(\rho)$ is NP-hard, even for fixed k and d, when $k < d$.

296 **Determination of 3- and 4-colored Ramsey numbers by simulated annealing**

Luc T. Wille, Department of Physics, Florida Atlantic University

Simulated annealing has been employed to construct triangle-free colorings of complete graphs. Starting from a randomly colored complete graph on n vertices, K_n, a randomly selected edge is given a new color arbitrarily selected from those available. Using the number of triangles as the cost function, the new graph is accepted with a temperature-dependent probability. If a triangle-free coloring of K_n with m colors is found, n constitutes a lower bound on the m-color Ramsey number R_m(3). For m = 3 this algorithm rapidly finds a coloring of K_{16}, but fails to converge for n = 17, in agreement with the known fact that R_3(3) = 17. For m = 4 all solutions found are considerably below the established lower bound of 51. This failure of the annealing approach is rather unexpected. Inspection of the simulations shows that the system ends up in a situation in which all remaining triangles are of the same color, a configuration from which the algorithm is unable to make further progress. This behavior is interpreted in terms of the theory of critical phenomena from statistical physics.

297 **Fibres in width 3 ordered sets**

Zbigniew Lonc, Warsaw University of Technology
Warsaw, Poland

A fibre in an ordered set is its subset intersecting every maximal nontrivial antichain. Let, for an ordered set P, m(P) denote the ratio: the cardinality of the smallest fibre in P over the cardinality of P. It is an open problem to establish how large m(P) can be. It is known that m(P) can not exceed 2/3. In this paper we improve this bound to 11/18 for ordered sets of width 3.

298 **On Graphs With A Unique Gamma Set**

G. Gunther*, Sir Wilfred Grenfell College, Nfld.
B. Hartnell, Saint Mary's University, N.S.
L. Markus & D. Rall, Furman University, S.C.

We present some preliminary investigations into properties of graphs having exactly one minimum dominating set. In addition, an efficient algorithm to determine if a tree has a unique gamma set will be outlined.

299 **Computing the Average Distance of a Distance Hereditary Graph in Linear Time**

Ortrud R Oellermann, Brandon University, Brandon, Manitoba, Canada

A connected graph G = (V, E) is distance hereditary if for every induced connected subgraph H of G and every pair u,v of vertices of H the distance between u and v in H is the same as the distance between u and v in G. The average distance of a graph is the average distance between all pairs of vertices in G. A O(|E|) algorithm for computing the average distance of a distance hereditary graph is developed.

Key Words: Distance hereditary, average distance of a graph, linear time.

300 **SENSITIVITY ANALYSIS FOR THE COMPLETE BIPARTITE WEIGHTED MATCHING PROBLEM**

H. Arsham, University of Baltimore, Baltimore, MD 21201-5779

A complete graph G(N, EN) is said to be bipartite if its node V can be partitioned into two disjoint subsets N1 and N2 such that each of its edges e ∈ EN has one endpoint in N1 and the other in N2. When a non-negative weight Ce is assigned to each edge e, the result is a complete bipartite weighted matching (CBWM) graph. An interesting problem in CBWM is to find a perfect match i.e, a match with optimal sum of its weights. The existing sensitivity analysis for CBWM problem is limited to one-change-at a-time in any weights. This paper sets forth a general approach for constructing sensitivity range for all weights which allows simultaneous (dependent, or independent) variations in all or some of the weights while preserving the current perfect match. The proposed stability analysis is based on a linear program formulation and an efficient solution algorithm. The results are encouraging and of immediate relevance, since they can be put to use by a practitioner who has to deal with uncertainties in the estimated weights.

FRIDAY, MARCH 11, 1994
4:20 p.m.

301 **Efficient algorithms for some one-dimensional peg solitaire problems**

B. Ravikumar, University of Rhode Island, Kingston, RI 02881

Peg solitaire is played on a board containing holes which can hold some pegs. If three holes on a line have pegs on the first two, then a move can remove them and introduce one on the third. The goal is to reduce a given starting position to one in which only one peg is left. If such a move sequence exists, we say that the starting position is winnable. We consider the following basic questions on a one-dimensional board: (i) How many winnable positions are there on a board of size n? (ii) Given two positions p and q, is there an efficient (e.g. polynomial time bounded) algorithm to determine if q can be reached from t? We consider several related problems and present complete or partial solutions. A basic idea behind our solutions is the use of a special kind of Turing machine (called a Hennie machine).

302 **Scheduling Rooted Forests with Communication Delays**

Garth Isaak, Lehigh University

We consider minimizing makespan when scheduling unit time tasks on two machines with unit communication delays. When the precedence constraints are a rooted forest, a greedy algorithm (which can also be viewed as a list scheduling algorithm), is optimal. The makespan for forest F is (F+Q)/2 where Q maximizes over non-leaf elements v, the number of elements incomparable to v plus one minus the path length from the root to v. Possible extensions will also be discussed.

303 BEYOND CHORDAL GRAPHS

Terry McKee, Wright State University, Dayton Ohio 45435

Much as chordal graphs are precisely the intersection graphs with *tree* hosts, "ekachordal" graphs are the intersection graphs with *chordal* hosts. Continuing inductively (i.e., next using ekachordal hosts) builds up a family of "chordal-type" graphs, all of whom share certain combinatorial features. Ekachordal graphs, the first step beyond chordal graphs, already (rather trivially) include all graphs that have a dominating vertex. Ekachordal graphs have a recursive definition analogous to perfect elimination orderings and even appear in the statistical and database applications of chordal graphs.

304 **SOME CATERPILLARS ARE 2-SPLITTABLE**

Gary E. Stevens, Hartwick College, Oneonta, NY

The Partitioned Graph Isomorphism Problem, which asks if it is possible to color the edges of a graph red and green so that the subgraphs induced by the coloring are isomorphic, is known to be NP-complete. It is just as difficult when restricted to trees. Graphs which can be colored in this way are called 2-splittable. It has been shown that for some classes of trees, namely Fibonacci trees, binary heaps, and spiders, the question is decidable. The question has not been answered for graphs known as caterpillars. This paper defines a subclass of caterpillars, called shoelaces, which have all their "legs" at the front and rear and then shows that the Partitioned Graph Isomorphism Problem is decidable for this class. First the set of all shoelaces is partitioned into ten subsets. Five of these subsets contain laces which are 2-splittable and specific coloring schemes are described. The other five subsets are shown to contain only shoelaces which cannot be split.

305 **On the total One-factorization of Graphs**

Guo-Hui Zhang
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A *total one-factor* of a simple graph G is a matching M in G, together with an independent set I of vertices in G, so that each vertex not in I is incident with an edge in M. A *total one-factorization(TOF)* of G is a partition of $E(G) \cup V(G)$ into total one-factors. Clearly, for a graph to have TOF, it is necessary that G is regular. In this talk, we investigate the total one-factorization of a *(k, n)-graph*, which represents a k-regular graph of order n.

306

The Four Post Cyclic Tower Problem

Paul K. Stockmeyer, College of William and Mary

Among the games introduced in [1] is a variation of the Tower of Hanoi puzzle in which there are four posts, labeled a, b, c, and d. The only allowable moves for the disks are from post a to post b, from b to c, from c to d, and from d to a. Also, as usual, a disk can never be places on top of a smaller disk. The problem is to move a tower of n disks from post a to post c.

The algorithm proposed in [1] makes 3^n-1 moves. While this algorithm does in fact move the tower as required, it does not do so in the minimum number of moves. In this paper we present a new algorithm for this problem, and conjecture that it is optimal.

[1] Scorer, R. S., P. M. Grundy, and C. A. B. Smith, "Some Binary Games," **The Mathematical Gazette 280** (1944), p. 96-103.

307

Large Antichains in the Partition Lattice.

E. R. Canfield(*), U. of Georgia, and L. H. Harper, UCR.

The collection of partitions of an n-element set forms a poset under the relation of refinement. The subcollection of partitions having exactly k blocks constitutes an antichain, that is a collection no two of whose elements are related by refinement. The size of this antichain is S(n,k), a Stirling number of the second kind. In this paper we prove that, as a function of n, the ratio of the largest antichain in the partition lattice to the largest Stirling number S(n,K_n) is unbounded. Keywords: Sperner theory, partition lattice, asymptotic normality.

308

SPHERE OF INFLUENCE GRAPHS

T.S. Michael*, U.S. Naval Academy & Thomas Quint, Yale

Let $\mathfrak{X} = \{X_1, \ldots, X_n\}$ be a set of points in the metric space M. Let r_i denote the minimum distance between X_i and any other point in $\mathfrak{X}$. The sphere of influence B_i is the open ball with center X_i and radius r_i. The sphere of influence graph $\mathrm{SIG}(M, \mathfrak{X})$ has vertex set $\mathfrak{X}$ with distinct vertices X_i and X_j adjacent provided $B_i \cap B_j \neq \emptyset$. A graph G is an M-SIG provided it is isomorphic to $\mathrm{SIG}(M, \mathfrak{X})$ for some $\mathfrak{X}$. The class of M-CSIGs is defined similarly using closed balls. We work in the general metric space M and obtain results that are surprisingly stronger than those of Harary, Jacobson, Lipman, and McMorris, who assumed M to be the Euclidean plane. For instance, every M-CSIG has a $\{K_2, K_3\}$-factor. Also, if the metric is induced by a norm on $\mathbf{R}^2$, then every non-planar M-SIG or -CSIG contains a triangle. Moreover, there are connections to Ramsey theory.

309

Graph Algorithms Experimentation Facility

J. Abello* and D. Sonom, Texas A&M University

We present a software facility to experiment with Graph Algorithms. The facility is implemented as a client to XAGE, a software environment developed under the direccion of James Abello. XAGE allows a user to animate and record algorithmic actions. Diverse graph generation and representation methods are offered. Several mechanisms allow a user to configure graph experiments. These include heuristics for NP-complete problems and recognition algorithms. We will describe our experiences with GEF and similar software tools.

310

On a Conjecture concerning Graphical Partitions

Firasath Ali* Cecil Rousseau
Memphis State University

For n an even positive integer, denote by $g(n)$ the number of graphical partitions of n. It has been conjectured that $\lim_{n\to\infty} \frac{g(n)}{p(n)} = 0$, where $p(n)$ is the total number of partitions of n. Let $c_k = \lim_{n\to\infty} \frac{g_k(n)}{p(n)}$, where $g_k(n)$ denotes the number of partitions of n that satisfy the first k Erdős-Gallai conditions. Erdős and Richmond have shown that c_k can be computed from a certain $(2k)$-dimensional integral. It is easy to see that $c_1 = \frac{1}{2}$. We obtain exact values for c_2 and c_3, and prove that $\lim_{k\to\infty} c_k \leq \frac{1}{4}$. Let $a_k = \lim_{n\to\infty} \frac{h_k(n)}{p(n)}$, where $h_k(n)$ denotes the number of partitions on n in which the first k successive ranks are all negative. We prove that $a_k = \binom{2k}{k}/2^{2k}$ for each k.