Invited Instructional Lecturers

Monday, February 12, 1990

Professor Eugene Lawler, University of California at Berkeley, will speak on *Ultrametrics and Evolutionary Trees*, at 9:30 AM, and on *Sequence Analysis in the Human Genome Project*, at 2:00 PM.

Tuesday, February 13, 1990

Jeffrey Lagarias, AT&T Bell Laboratories, will speak on *Interior Point Methods in Linear Programming*, at 9:30 AM, and on *Tiling Regions with Polyominoes*, at 2:00 PM.

Wednesday, February 14, 1990

Professor John H. Conway, Princeton University, will speak on *Lexicodes--Strange Codes from Game Theory*, at 9:30 AM and at 2:00 PM.

Thursday, February 15, 1990

Professor Peter Winkler, Emory University and Bellcore, will speak on *Bonbs, Stars and Lollipops: What We Know and Don't Know about Random Walks on Graphs, Part 1*, at 9:30 AM.

Professor A. K. Dewdney, University of Western Ontario, will speak on *A Mathematician Looks at the Stock Market*, at 4:30 PM. Professor Dewdney's talk is a Florida Atlantic University Twenty–Fifth Anniversary Event.

Friday, February 16, 1990

Professor Peter Winkler, Emory University and Bellcore, will speak on *Bombs, Stars and Lollipops, Part 2*, at 9:30 AM.

Professor Paul Erdös, Hungarian Academy of Sciences, will speak on *Some Coloring Problems on Graphs*, at 10:45 AM.

MONDAY, FEBRUARY 12, 1990

REGISTRATION begins at 8:00 A.M. in the downstairs lobby of the University Center. GCN (left or front) and GCS are the two halves of the Gold Coast Room. FAU Rooms A and C are reached through the second floor Lounge, where COFFEE will be served.

	GCN	GCS	FAU A	FAU C
9:00 AM	OPENING and WELCOME Vice President BERRY Dean CARRAHER			
9:30	LAWLER			
10:30	COFFEE			
10:50	1 EALY	2 SLATER	3 CABANISS	4 ELLINGHAM
11:10	5 ATKINSON	6 KIRCHHERR	7 ABRHAM	8 KUBICKI
11:30	9 MENDELSOHN	10 MCKEE	11 SEAH	12 DE JACKSON
11:50	13 PELIN	14 MAJOR	15 LEE	16 BEASLEY
12:10 PM	17 SEGUEL	18 CAI	19 BAGGA	20 SHIER
12:30	LUNCH (On your own -- Cafeteria and Rathskellar open; there are many nearby restaurants)			
2:00	LAWLER			
3:00	COFFEE			
3:20	21 PEDERSEN	22 NIEDERHAUSEN	23 DOMKE	24 KOOSHESH
3:40	25 SPRAGUE	26 HU	27 SUN	28 GEWALI
4:00	29 EL-REWINI	30 ROSENFELD	31 GRAULICH	32 BRIGHAM
4:20	33 TU	34 GRIGGS	35 CHINN	36 ULLMAN
4:40	37 CHAUNDY	38 TROTTER	39 KENDRICK	40 KLERLEIN
5:00	41 NOBERT	42 SIMION	43 LASKAR	44 CHANDRASEKHARAN
5:20	45 SHERWANI	46 GRIMALDI	47 MCKAY	48 KLINGSBERG
6:15	CONFERENCE RECEPTION in the BOARD of REGENTS ROOM on the THIRD floor of the ADMINISTRATION BUILDING.			

There will be Conference transportation back to the motels at 5:50 PM, returning to the reception about 6:30. There will be transportation from the reception back to the motels.

TUESDAY, FEBRUARY 13, 1990

REGISTRATION HOURS (second floor LOUNGE) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. FAU Rooms A and C are reached through the second floor Lounge, where COFFEE will be served. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	FAU-A	FAU-C
8:40 AM	49 GRINSTEAD	50 GEORGES	51 GUPTA	52 CELIS
9:00	53 ZITO	54 HEDETNIEMI	55 GUAN	56 SOUMYANATH
9:30	LAGARIAS			
10:30	COFFEE			
10:50	57 SANDERS	58 MAJUMDAR	59 LATIFI	60 ELMALLAH
11:10	61 G GORDON	62 LIVINGSTON	63 MILLER	64 MCINTYRE
11:30	65 HIGGINS	66 MYNHARDT	67 PIPPERT	68 DEOGUN
11:50	69 ASHLOCK	70 SCHELP	71 STOUT	72 DEWDNEY
12:10 PM	73 EXOO	74 WOJCICKA	75 G PURDY	76 BIENSTOCK
12:30	LUNCH BREAK			
2:00	LAGARIAS			
3:00	COFFEE			
3:20	77 SRIVASTAVA	78 E HARE	79 SCHMIDT	80 CACERES
3:40	81 CHATTERJEE	82 RAYCHAUDHURI	83 RUSKEY	84 SIMMONS
4:00	85 LONGSTAFF	86 LAWSON	87 SCHMEICHEL	88 CHEROWITZO
4:20	89 JACOBS	90 ISAAK	91 HORAK	92 MAGLIVERAS
4:40	93 FAJTLOWICZ	94 MAULDIN	95 ROYLE	96 TAPIA R.
5:00	97 CRUZ	98 WANG	99 PRITIKIN	100 MORENO
5:20	101 BASTIDA	102 BETTAYEB	103 FRICKE	104 ORAL

6:00 CONFERENCE PARTY at the home of JACK FREEMAN : 741 AZALEA ST, (but park on AURELIA ST) 395 - 7921.

CONFERENCE TRANSPORTATION will leave for the motels at 5:45. There will be transportation from the UNIVERSITY CENTER to the party at about 5:50, and from the motels at about 6:15. There will be transportation from the parties back to the motels. As always, we urge car-pooling. It is a pleasant walk to the Freeman home, should you be adventurous.

WEDNESDAY, FEBRUARY 14, 1990

REGISTRATION HOURS (second floor LOUNGE) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. FAU Rooms A and C are reached through the second floor Lounge, where COFFEE will be served. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	FAU-A	FAU-C
8:40 AM	105 MAKKI	106 CANFIELD	107 BENNETT	108 ALI
9:00	109 GOWRISANKARAN	110 GUBSER	111 SCHONHEIM	112 LEISS
9:30	CONWAY			
10:30	COFFEE			
10:50	113 BAUER	114 CATLIN	115 JW BROWN	116 SCHWEIZER
11:10	117 MEYEROWITZ	118 DJIDJEV	119 MILLS	120 EMANY-K
11:30	121 POWELL	122 BELL	123 LAYWINE	124 BERMAN
11:50	125 SAXTON	126 KEIL	127 GREIG	128 BOALS
12:15 PM	**CONFERENCE PHOTOGRAPH** at the OUTDOOR STAGE. We will lead you from the lobby, if you can't find it on your own, but PLEASE PARTICIPATE!			
12:30	LUNCH BREAK			
2:00	CONWAY			
3:00	COFFEE			
3:20	129 SEN	130 C PURDY	131 HOBBS	132 SARVATE
3:40	133 STEWART	134 HARTSFIELD	135 HOELZEMAN	136 CLARK
4:00	137 LIN	138 RICHTER	139 TONG	140 VAN REES
4:20	141 HANSEN	142 HARBORTH	143 MYRVOLD	144
4:40	145 JAMISON	146 CHEN	147 FULLER	148 GOLDBERG
5:00	149 OPATRNY	150 BABAI	151 ALAVI	
5:20	153 EGGEN		155 FARHAT	

The CONFERENCE BANQUET will be held in the new Cafeteria Building at 7:30 PM (seating at 7:15). There will be a cash bar (beer and wine only) from 6:30 to 7:30. Conference transportation will be available to the motels at 5:45. There will be transportation from the motels to the University at approximately 6:25. There will be transportation back to the motels after the banquet.

THURSDAY, FEBRUARY 15, 1990

REGISTRATION HOURS (second floor LOUNGE) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. FAU Rooms A and C are reached through the second floor Lounge, where COFFEE will be served. There will be book exhibits in Room 232 from 9:00 to 4:30.

	GCN	GCS	FAU-A	FAU-C
8:40 AM	157 BRAND	158 LIMAdeSA	159 ADHAR	160 LENGYEL
9:00	161 GOLDWASSER	162 TESMAN	163 GREGORY	164 JOHNSON
9:30	WINKLER			
10:30	COFFEE			
10:50	165 BOHUS	166 KUBICKA	167 FABRIKANT	168 LUNDGREN
11:10	169 VERDE-STAR	170 HIND	171 OLARIU	172 TIAN
11:30	173 SHAPIRO	174 PENRICE	175 NEMHAUSER	176 RALL
11:50	177 HILGERS	178 KUPLINSKY	179 DAS	180 HEVIA
12:10 PM	181 ANDERSON	182 GUNDERSON	183 FREDRICKSEN	184 JARRETT
12:30	LUNCH BREAK			
2:00	185 BATTEN	186 ARASMITH	187 LAI	188 FAUDREE
2:20	189 FUGLISTER	190 GASARCH	191 LINDQUESTER	192 FISHER
2:40	193 ROGERS	194 FIGUEROA-CENTENO	195 SZEKELY	196 J RYAN
3:00	197 PAYNE	198 BROERE	199 GOULD	200 HATTINGH
3:20	201 C GORDON	202 DEAN	203 GIBI	204 J ZOU
4:00			207 TRENK	208 Q YU
4:30	DEWDNEY			

There will be an informal CONFERENCE PARTY 6:30-7:30 in the Cafeteria Patio area--to be moved indoors if weather dictates. There will be Conference transportation back to the motels at 5:45 PM and back to the party at 6:15. There will be transportation back to the motels after the party.

FRIDAY, FEBRUARY 16, 1990

REGISTRATION HOURS (second floor LOUNGE) 8:15-11:00 A.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. FAU Rooms A and C are reached through the second floor Lounge, where COFFEE will be served. There will be book exhibits in Room 232 from 9:00 to 11:30.

	GCN	GCS	FAU-A	FAU-C
8:40 AM	209 YUCAS	210 M-L YU	211 FOLDES	
9:00	213 J WU	214 MCCANNA	215 KEMNITZ	216 ZHENG
9:30	WINKLER			
10:30	COFFEE			
10:45	ERDOS			
11:50	217 WALLER	218 VANDERJAGT	219 WALLIS	220 BOLAND
12:10 PM	221 BOWSER	222 KONG	223 WISEMAN	224 SNYDER
12:30	LUNCH			
2:00	225 W GU	226 NEWMAN-WOLF	227 CT RYAN	228 THOMAS & Seymour
2:20	229 BAREFOOT	230 QUACKENBUSH	231 BW JACKSON	232 HELL
2:40	233 PROPP	234 HURLBERT	235 SARKAR	236 HAMBURGER
3:00	237 HEMMETER	238 NEL		

There will be transportation back to the motels following the last talks.

THANKS FOR COMING!!

WE'LL SEE YOU AT LSU FOR THE TWENTY-SECOND SOUTHEASTERN INTERNATIONAL CONFERENCE ON COMBINATORICS, GRAPH THEORY AND COMPUTING,

***** FEBRUARY 11-15, 1991 *****

(MARDI GRAS WEEK, OF COURSE).

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MONDAY, FEBRUARY 12, 1990   10:50 A.M.

1

Computable Groups and Computable Permutation Structures

Clifton E. Ealy Jr.
Department of Mathematics and Statistics, Western Michigan Univ
Kalamazoo, Michigan 49008-5152

In this paper we will study groups and permutation structures with the additional assumption that a "Turing Machine" resides in the background on which we do our computing.

2

Path covers of E(G) and island covers of V(G)

Peter J. Slater, The University of Alabama in Huntsville

Let the path number pn(G) of a graph G be the minimum number of edge-disjoint paths required to cover the edges in E(G), and let the island number is(G) of a graph G be the minimum number of vertex-disjoint paths required to cover V(G). A discussion of theoretical and computational questions will be followed by "interpolation" theorems and conjectures.

3.

On Edge-graceful Regular Graphs and Trees

Sharon Cabaniss*, Richard Low, and John Mitchem, San Jose State University

Let G be a simple graph with vertex set V(G) and edge set E(G). Let $p = |V(G)|$ and $q = |E(G)|$. In 1985, Lo defined the edge-graceful labeling of G where $l: E(G) \to \{1, 2, \ldots, q\}$ is one-to-one and l induces a label on the vertices defined by $l^*(v) = \sum_{uv \in E(G)} l(uv) \pmod p$.

The labeling is edge-graceful if all vertex labels are distinct modulo p, in which case G is called an edge-graceful graph. For positive integers n and t, C_n denotes the cycle graph on n vertices, and C_n^k denotes the kth power of C_n. That is, $V(C_n^k) = V(C_n)$ and $uv \in E(C_n^k)$ if and only if $d(u,v) \leq k$ where $d(u,v)$ denotes the distance from vertex u to vertex v in C_n. $C_{n,k}$ denotes the graph such that $V(C_{n,k}) = V(C_n)$ and $uv \in E(C_{n,k})$ if and only if $d(u,v) = 1$ or k.

Theorem: Let n be odd and $1 \leq k < \frac{1}{2}n$, then C_n^k is edge-graceful.
Corollary: K_n, C_n and $C_{n,k}$ are edge-graceful for odd n and k positive.

Theorem: Let T be an odd tree with a root of even degree. If T has no adjacent degree 2 vertices, no two degree 2 vertices with the same parent, and an even number of non-root degree 2 vertices, then T is edge-graceful.
Corollary 1: If T is an odd tree with at most one vertex of degree 2, then T is edge-graceful.
Corollary 2: For $n \geq 2$, any odd full n-ary tree is edge-graceful.
Corollary 3: If T is an odd tree with a root of even degree at least 4 such that no two degree 2 vertices are adjacent and no two degree 2 vertices have a common parent, then T is edge-graceful.

4

Basic subgraphs and adjacency matrix rank

M. N. Ellingham, Vanderbilt University

Let the *rank* of a graph be the rank of its adjacency matrix over the real numbers. We show that a graph G of rank r always has a *basic subgraph*, an induced subgraph of order r and rank r. Using basic subgraphs, we investigate when vertices can be added to a graph without increasing the rank. Many properties of graphs with rank r can be studied by examining a finite set of graphs known as the *maximal reduced graphs of rank r*. Our results on adding vertices without increasing the rank are used to determine all maximal reduced graphs of small rank, and to construct two infinite families of maximal reduced graphs.

5

Computing Sylow subgroups of permutation groups

M.D. Atkinson*, School of Computer Science, Carleton University, Ottawa, Canada K1S 5B6

Peter M. Neumann, The Queen's College, Oxford OX1 4AW, UK

An algorithm for computing a Sylow p-subgroup of a permutation group is described and justified. The new algorithm analyses the permutation action in terms of point stabilisers, orbits and block systems. It differs from previous algorithms by relying more on normaliser computations than centraliser computations. The algorithm, PERSYL, has been implemented in the group theory programming system Cayley. Sample timings are given to provide a comparison with the Sylow subgroup algorithm available in Cayley. The results indicate that the algorithm performs well for soluble groups and for other groups with large soluble sections.
Keywords: Permutation group, Sylow subgroup, algorithm.

6

On certain cordial graphs

W. W. Kirchherr, San Jose State University

A *labeling* of a graph $\mathcal{G} = (V, E)$ is a mapping $f : V \to \{0,1\}$. The mapping f induces an edge-labeling f^* on $\mathcal{G}$, $f^* : E \to \{0,1\}$, defined by $f^*((u,v)) = |f(u) - f(v)|$ for all $(u,v) \in E$. Let $v_f(0) = \{v \in V \mid f(v) = 0\}$, $v_f(1) = \{v \in V \mid f(v) = 1\}$, $e_{f^*}(0) = \{e \in E \mid f^*(e) = 0\}$, and $e_{f^*}(1) = \{e \in E \mid f^*(e) = 1\}$.

A labeling f of a graph $\mathcal{G}$ is *cordial* if $|v(0) - v(1)| \leq 1$ and $|e(0) - e(1)| \leq 1$.

A graph $\mathcal{G}$ is cordial if it admits a cordial labeling.

In this paper three types of graphs are investigated with respect to cordiality, namely, graphs which are the complete product of two cordial graphs, graphs which are the subdivision graphs of cordial graphs, and cactus graphs. We give sufficient conditions for the cordiality of graphs of the first two types and show that a cactus graph is cordial if and only if the cardinality of its edge set is not congruent to 2 (mod 4).

7

ESTIMATES OF THE NUMBER OF GRACEFUL VALUATIONS OF CYCLES

Jaromir Abrham*, Dept. of Industrial Engineering, University of Toronto and

Anton Kotzig, département de mathématiques et de statistique, Université de Montréal

The following two results are obtained: for n=0(mod 4), the number of α-valuations of C_n (the cycle on n vertices) grows exponentially with n. For n=3(mod 4), the number of graceful valuations of C_n grows exponentially with n. These results also permit to improve the lower bounds on the number of Skolem sequences of order n, obtained earlier by one of the authors.

Key words: Graceful valuation; cycles

8

Greatest Common Subgraph Index of Graphs

Grzegorz Kubicki, Emory University

A greatest common subgraph (gcs) of a family of graphs, all of the same size, is a graph of maximum size that is a common subgraph of every graph in this family. A gcs index of a graph G is the largest integer q such that there exists a family of graphs (all of size q) with the property that G is its unique gcs. The lower bounds for gcs index are given. The sufficient condition for a graph to have infinite gcs index is presented. The modification of gcs index for connected graphs is discussed.

MONDAY, FEBRUARY 12, 1990 11:30 A.M.

9

Theorem proving with the Knuth-Bendix algorithm

N. S. Mendelsohn*, B. Wolk, R. Padmanabhan, Univ. of Manitoba.

The Knuth-Bendix Algorithm has been modified in such a way as to produce proofs of theorems involving as many as 600 identities. Examples are given involving graph theory or group theory. Some of these are a challenge to professional mathematicians.

Keywords: Universal algebra, directed graphs, groups.

10

Intersection Graphs and Cographs

Terry McKee, Wright State University

Complement-reducible graphs (a.k.a. cographs) form a widely studied and used class of perfect graphs. Intersection graphs (and multigraphs) provide a common framework for many concepts in graph theory.

We link intersection graphs with cographs by introducing "clique-reducible multigraphs" (multigraph analogs of cographs) and what we call "domination-reducible graphs" (a.k.a. diagonal graphs).

KEY WORDS: intersection graphs
cographs
complement-reducible graphs
chordal graphs
intersection multigraphs

11

On graceful complete tripartite graphs

Yong-Song Ho, National University of Singapore
Sin-Min Lee, San Jose State University and
Eric Seah*, University of Manitoba.

A graph G=(V,E) is graceful if there exists a vetex labeling F: V->{0,1, ,|E|} such that the induced edge labeling F*((u,v))= |F(u)-F(v)| is a bijection between E and {1,2,...,|E|}. In this note we show that for any increasing sequence a,b,c the complete tripartite graph K(a,b,c) is graceful for all a < 3, and K(a,b,c) is not graceful for (a,b,c) = (3,3,3), (3,3,4) and (3,3,5). We conjecture that K(a,b,c) is not graceful for all a > 2.

12

Totally Integrated Graphs

Roger C. Entringer, University of New Mexico, Albuquerque,NM 87131
Douglas E. Jackson*, Eastern New Mexico University, Portales, NM 88130

We determine the minimum possible number of edges in a graph in which every two vertices of the same degree are adjacent.

13

Exact Computation Sequences

Alex Pelin and Bill Kraynek
School of Computer Science; Florida International University

Knuth and Bendix developed a method for solving word problems in free algebras with equations by transforming equations into rewrite rules. This is done by defining an ordering on the set of terms of the algebra and orienting the equations according to that ordering. This way the equations become simplification rules that can be used to reduce the terms of the algebra by replacing each instance of the left-hand side of a rule by the coresponding instance of the right-hand side. The system of rules generated by the Knuth-Bendix procedure is terminating and confluent, i.e., every sequence of simplifications is finite and the order in which the simplifications are applied is irrelevant to the final outcome. In such a system each term has a unique normal form, which is a term that cannot be simplified. The Knuth-Bendix procedure produces a system which is complete for the given set of equations, i.e., two terms are equal if and only if they simplify to the same normal form. Their procedure keeps adding simplification rules to the set of simplifications obtained from the original set of equations until the set is complete. Pelin and Gallier give a completion method that is done in stages. This way the ordering on terms may be changed at each stage. They also handle conditional equations and they eliminate some other restrictions of the Knuth-Bendix procedure. We will also present computation and algebraic properties of the multistage completion procedures, also called exact computation sequences.

14

p-edge clique coverings of graphs

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An edge clique covering (ECC) of a graph is a collection of cliques which cover all the edges of the graph. For many classes of intersection graphs (e.g., competition graphs, neighborhood graphs, upper bound graphs), ECC's are used to give characterization theorems. In this note, we survey some recent resultson several types of p–intersection graphs (two sets must have at least p elements in common before an edge is introduced) where the characterizations use a natural generalization of ECC.

Key words: intersection graph, edge clique covering

15

On edge-magic graphs

Sin-Min Lee*, San Jose State University,
Eric Seah, University of Manitoba, and
S. K. Tan, National University of Singapore.

Sedlacek, Stewart, Doob and other had considered the magic graph G in the sense that there exists a bijection f: E -> {1,2,...,|E|} such that the induced mapping f*: V -> Z+ defined by f*(u) = sum {f(u,#): (u,#) in E} has the property that f*(u) = f*(v) for all u,v in V. We define a graph G is edge-magic if there exists a bijection f: E -> {1,2,...,|E|} such that the induced mapping f ^: V->Z+ defined by f^(u) = sum {f(u,#): (u,#) in E} (mod |V|) has the property that f^(u) = f^(v) for all u,v in V. We develop some general theory on edge-magic graphs.

16

LINEAR OPERATORS STRONGLY PRESERVING DIGRAPHS WHOSE MAXIMUM CYCLE LENGTH IS SMALL

LeRoy B. Beasley*, Utah State University and Norman J. Pullman, Queen's University, Ontario, Canada

We characterize those linear operators L such that for all digraphs, D, L(D) has maximal cycle length k if an only if D has maximum cycle length k.

We do this for k = 0 and k = 1, the cases k > 1 having been done previously.

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17

SYMMETRIC FFTs THROUGH CYCLIC CONVOLUTIONS

by

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Abstract. A symmetric FFT is a fast algorithm that takes advantage of certain redundancies in the input data to speed up the computation of the discrete Fourier transform of an N-periodic sequence. We consider here even, odd, quarter-even and quarter-odd sequences whose period is $N = 2^k$. The best known symmetric FFTs are due to Cooley-Lewis-Welch and to Swarztrauber. These algorithms are based on the regular FFT algorithm for non-symmetric sequences and can only be used with real data. The symmetric FFTs based on cyclic-convolutions mainly use Rader's Fourier transform algorithm and can be equally applied to real and complex sequences.

18

On i-perfect Graphs

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Dept. of Computer Science, Univ. of Toronto

A K_i-free colouring of a graph G is a partition $\{V_1, V_2, \ldots, V_k\}$ of the vertices of G such that each V_i induces a K_i-free graph. The K_i-free chromatic number of G, denoted $\chi(G: -K_i)$, is the least number k for which G has a K_i-free k-colouring. Let $\omega(G: -K_i) = \lceil \frac{\omega(G)}{i-1} \rceil$. We generalize the notion of perfect graph by defining a graph G to be i-perfect iff for each induced subgraph H of G, $\omega(H: -K_{i+1}) = \chi(H: -K_{i+1})$. (A 1-perfect graph is the standard notion of perfect graph.)

We study various aspects of i-perfection including minimal i-imperfect graphs, operations which preserve i-perfection, and families of i-perfect graphs. We also consider the possibility of using the notion of i-perfection to attack the strong perfect graph conjecture.

Key words: i-perfect graphs, K_i-free colouring, perfect graphs.

19

Chromatic Numbers in Tournaments

Kunwarjit S. Bagga* and Frank W. Owens, Ball State University

In the discussion of chromatic problems, the usual aim is to avoid a given monochromatic configuration in a graph. For example, the chromatic number is the minimum number of colors needed to color the nodes of a graph so that there is no monochromatic K_2, or that the nodes of every K_2 have *distinct* colors. Thus one can consider the following generalization of this problem. Given graphs G and H, color the nodes of G so that the nodes of every subgraph of G isomorphic to H are colored with distinct colors. In this paper we consider this problem for tournaments. For a tournament T_n on n nodes, we define $\chi_t(T_n)$ to be the smallest number of colors needed to color the nodes of T_n such that the nodes of every transitive triple in T_n have three distinct colors. $\chi_c(T_n)$ is similarly defined with the nodes of every 3-cycle getting three distict colors. We give several results on these measures.

20

Cones of Certain Nonnegative Matrices

R. Loewy, Technion-Israel Institute of Technology
D. R. Shier*, College of William and Mary

We consider the cone of all n × n nonnegative matrices having fixed vectors x and y^T as their right and left (Perron) eigenvectors. A cross section $\mathcal{P}$(x,y) of this cone is obtained by suitable normalization. For example the set of doubly stochastic matrices is obtained as a special case when x = y = $(1, 1, \ldots, 1)^T$. It is of interest to characterize the extreme points of $\mathcal{P}$(x,y), which is done by transforming the given problem into a special type of "transportation problem." We also study regions in which the number of extreme points is maximized.

Key words: extreme points, matrices, polytopes, trees

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21

Cellular Automata as Algebraic Systems

John Pedersen, University of South Florida, Tampa, FL 3362-5700

The behavior of one-dimensional cellular automata (CA) with even a small number of states can be exceedingly complex. It is known that a two-cell neighborhood is sufficient to simulate all other neighborhoods. In the two-cell case, the transition function is a binary operation (groupoid) on the state set. Fundamental algebraic operations on the groupoid have direct visual interpretation for the evolution of the CA, so algebraic structure theorems carry over meaningfully to CA. Periodic evolutionary behaviors of CA correspond to chains of groupoid varieties. In the three and four state cases it can be determined where these chains become stationary.

22

Colorful partitions, permutations, and trigonometric functions

Heinrich Niederhausen, Florida Atlantic University

What kind of enumeration scheme generates $(\arcsin t)^m$? What is the "inverse" problem, generating $(\sin t)^m$? We give two classes of counting problems involving colored partitions and permutations, which can be tuned to answer those questions (in terms of central factorial numbers) by choosing the right palette. Stirling numbers result from another extreme color choice, and there are many interesting other selections, giving rise to products of sine and cosine functions.

23

Th. Cosubchromatic Number of a Graph

Gayla S. Domke*, Georgia State University, and
Renu C. Laskar, Clemson University

The <u>cosubchromatic</u> <u>number</u> of a graph G, denoted $z_s(G)$, is the smallest order n of a partition $(V_1, V_2, \ldots, V_n)$ of the vertices of G such that the subgraph induced by the vertices in V_i forms a disjoint union of complete subgraphs in either G or the complement of G. This definition combines the ideas of the cochromatic number (Lesniak-Foster and Straight, 1977) and the subchromatic number (Albertson, Jamison, Hedetniemi, and Locke). This talk studies the properties of $z_s(G)$ and relates it to other graph parameters.

24

Improved bounds for the prison yard problem

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The prison yard problem is one of a family of guard problems, where one places guards at various points in or on a simple polygon (representing the walls of an enclosure) with the aim of covering every point of the interior or exterior regions with at least one guard. In the prison yard version, only vertex guards are used (i.e., guards placed on vertices of the simple polygon) and they are required to cover both the interior and the exterior of the polygon.
Joseph O'Rourke in his monograph, Art Gallery Problems and Theorems, conjectures that ceil(n/2) vertex guards are sufficient to cover the interior and exterior of a simple polygon of n vertices (the prison yard problem). The best bounds to date (floor(2n/3) vertex guards) were derived by O'Rourke in 1983 and have not been improved until now. We develop two improved upper bounds for the number of vertex guards needed to cover the interior and exterior of a simple polygon. The two bounds give rise to an overall upper bound of ceil(7n/12). (We have just learnt that D. Kleitman and Z. Furedi have proved the conjecture.)

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25

Parallel Algorithm to Construct a Dominance Graph on Nonoverlapping Rectangles

Alan P. Sprague, University of Alabama at Birmingham.

Given a set of iso-oriented nonoverlapping rectangles, the dominance graph is a directed graph having the rectangles as vertex set. It expresses the notion of "aboveness" on the rectangles: it contains an edge from a rectangle b to rectangle c iff c is immediately above b. (That is, c is above b and no other rectangle is between them.) The dominance graph is motivated by the constraint graph commonly used in compaction algorithms for VLSI circuits. We give a parallel algorithm to generate the dominance graph. The algorithm has time complexity $O(n/p \log n + \log p \log n)$, where p is the number of processors. When $p \leq n/\log n$, it is optimal in any model for which the complexity of sorting is $\Omega(n \log n)$. The algorithm takes the form of a tree of merges. An arc is drawn at a merge, based on locally-available information; if globally incorrect, the arc must be replaced or erased at a later merge or in a final cleanup stage.

26

On generalized partitions of an n-set

Zhu-Xin Hu, Dept Of Math, University Of Illinois At Champaign-Urbaba

In this paper we introduce the definition of a ageneralized partition of an n-set associated with a finite partial ordered set, and the definition of a generalized partition poset. A classic(non-generalized) partition of an n-set and a simple graph with n vertices are just two special cases of the generalized partitions; and some types of generalized partitions are special metric spaces. Using partitions as tools, we study the properties of finite posets. A series of interesting results are obtained in the following directions: (i) embedding one poset into another poset, subsets of posets; (ii) the representing diagrams of generalized partitions; (iii) relationships among generalized partitions, partition posets, and associating posets.

27

On chromatic coefficients of unlabelled graphs

Hugo Sun, California State University at Fresno.

Explicit expressions in terms of symmetries of vertex subsets of unlabelled graphs are given for the first 4 coefficients of the chromatic polynomials.

We request our talk can be arranged on 12, 13, 14 February for we have to fly back to California by 15th February. Thank you for all your helpful arrangements.

My EMAIL address: lee@mathcs.sjsu.edu

Your truly

Sin-Min Lee.

28

Covering an Orthogonal Polygon by Horizontal Guards

Laxmi P. Gewali, University of Nevada, Las Vegas

Simeon Ntafos, University of Texas, Dallas

Covering a simple polygon by the minimum number of point guards under standard visibility is known to be NP-hard. This problem is still open when the polygon is restricted to be orthogonal. In this paper we first present an O(n) time algorithm to cover a simple monotone orthogonal polygon by the minimum number of horizontal guards under orthogonal visibility. We then extend the method to include simple orthogonal polygon (not necessarily monotone) and present an O(n**2) time algorithm.

MONDAY, FEBRUARY 12, 1990 4:00 P.M.

29

Scheduling Loops onto Arbitrary Target Machines Using Simulated Annealing

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Scheduling is a computationally intensive problem but static scheduling of task graphs that contain loops is even harder, especially when the loop upper bound is not known before execution time. In this paper we introduce a loop unrolling technique that allows several iterations of a set of loops as well as tasks within the same iteration to overlap in execution in such a way that minimizes the loop execution time. We use simulated annealing technique to achieve near-optimal mapping of the tasks surrounded by a set of nested loops on an arbitrary target machine. Our goal is to find: 1) the best unrolling vector 2) the Gantt chart that indicates the allocation and the order of the tasks in the post-unrolled loop on the available processors.

30

Almost orthogonal lines in E^d

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A set L of lines through the origin in the Euclidean space $\mathbf{E^d}$ is called *almost orthogonal* if among any three members of L there is at least one orthogonal pair. Answering a question of Paul Erdos, we show that L cannot contain more than 2d lines.

31

On Some D(i,j)-Balanced Graphs

Michael F. Capobianco and Patricia A. Graulich*
St. John's University

A D(i,j)-balanced graph is one having the property that the number of pairs of its points distant i or less from each other is equal to the number of pairs distant j or more. This concept arose in connection with the solution of the graph equation, $G^{i*} = \bar{G}$. In this paper, we study D(i,j)-balanced cycles, wheels and some other special graphs. Among other results, we find that no even cycle is D(i,j)-balanced, and that W_6 is the only D(i,j)-balanced wheel.

32

Distances and Diameters in Steinhaus Graphs

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Bounds are given on the distance between any two nodes of a Steinhaus graph. This leads to an estimation of the diameter of such graphs, and it is proven that almost all Steinhaus graphs have diameter at most four. However, it is conjectured that almost all of these graphs have diameter 2, and evidence which supports this conjecture is presented.

33

Computing the Greatest Common Divisor in a "VLSI" Model of Computation

C. Purdy, G. Purdy, and N. Tu*, University of Cincinnati

Abstract. We consider the problem of computing the greatest common divisor (gcd) of two positive n-bit integers in a computational model chosen to represent a VLSI chip. We are interested in upper and lower bounds on area and/or time, in terms of n, for computing the gcd. Previously it has been shown that for this computational model a so-called "binary" algorithm is preferable to the standard Euclidean algorithm. We examine several binary algorithms, including a probabilistic version, and determine ranges of n for which each is optimal.

34

Cutsets of the Boolean Lattice

Jerrold R. Griggs, University of South Carolina

A family $\mathcal{C}$ of subsets of the n-set $[n] = \{1, \ldots, n\}$ is a *cutset* in the Boolean lattice $2^{[n]}$ if it meets every maximal chain. Ko-Wei Lih asked for the maximum size $c(n)$ of a cutset $\mathcal{C} \subseteq 2^{[n]}$ that is *minimal*, which means that for every $C \in \mathcal{C}$, some maximal chain avoids $\mathcal{C} \backslash C$. Although $c(n)/2^n = \frac{1}{2}$ for $n \leq 5$, Lih noted that $c(6) \geq 33 > \frac{1}{2}2^6$. In joint work with Z. Füredi and D.J. Kleitman, we construct minimal cutsets with almost all elements, i.e., we prove $c(n) \sim 2^n$ as $n \to \infty$. Another project considers a related notion. A family $\mathcal{K} \subseteq 2^{[n]}$ is a *cutset for A*, where A is some given element of $2^{[n]}$, if no element of $\mathcal{K}$ is comparable to A and if $\mathcal{K} \cup \{A\}$ is a cutset. Nowakowski determined the minimum size of a cutset for A. The key to his proof is an elaborate inductive construction for even $n \geq 8$ of certain disjoint saturated chains of subsets unrelated to $A = \{1, \ldots, \frac{n}{2}\}$. In joint work with D.J. Kleitman, we derive an inequality for cutsets for A similar to Lubell's inequality for antichains that provides a simpler proof of Nowakowski's result.

35

Decomposition of Graphs into Non-isomorphic Matchings

P. Z. Chinn, Humboldt State University, Arcata, CA.
R. B. Richter, Carleton University, Ottawa, Ontario

A decomposition into non-isomorphic matchings, or DINIM for short, is a partition of the edges of a graph G into matchings of different sizes. An alternate way to view this is a proper coloring of the edges of G in such a way that each color-class has a different size. We prove that every sufficiently large 2-connected 3-regular graph has a DINIM.

Key words: edge-decomposition, factorization, matching, edge-coloring

36

Matching Finite Subsets of Lattices in the Plane

Daniel Ullman, George Washington University

Suppose you own an orchard consisting of a tree planted at each of the integer lattice points in the plane that are within 100 units of the origin. Now suppose you learn that your orchard must be transplanted to a new configuration that is like the current one but rotated about the origin by 45 degrees. The difficulty is that no tree can be moved more than 2 units. Can you accomplish your task? The question amounts to finding a matching in a certain bipartite graph. The infinite analogue of this question has an affirmative answer, and this fact serves as a key lemma both here and in M. Laczkovich's recent much-noted work on squaring the circle.

Keywords: matching, bipartite Euclidean graph.

37

Rural School Bus Routing and Scheduling

Frank Fiala and Duncan Chaundy*, Carleton University

The rural school bus routing and scheduling problem is an example of the vehicle routing problem. We are given a number of stops where students are waiting to be picked up along with a network of roads where topographical features mean that not all neighbouring nodes are connected. There is a fleet of fixed capacity school buses to deliver the students to the school. There are also school board imposed time restrictions on travelling. We present a heuristic approach to this problem based on a shortest path tree rooted at the school. The algorithm generates feasible routes to deliver the students to the school using the smallest number of buses possible. We have implemented this algorithm along with a graphically interactive user-interface which allows the user to see the nodes and the network. The resulting routes are then displayed as buses moving on the network along with a clock to show the current time.

KEYWORDS: Vehicle Routing, Heuristic Algorithms, User Interface

38

AN APPLICATION OF RANDOM PARTIAL ORDERS

P. Erdös, H. A. Kierstead, and W. T. Trotter

For a poset (X, P) and a point $x \in X$, let $\deg(x)$ count the number of points comparable to x. Then let $\Delta(X, P) = \max\{\deg(x) : x \in X\}$. Alternately $\Delta(X, P)$ is the maximum degree in the comparability graph. V. Rödl and W. T. Trotter proved that the dimension of (X, P) is bounded in terms of $\Delta(X, P)$ by showing that if $\Delta(X, P) \leq k$, then $\dim(X, P) \leq 2k^2 + 2$. Using the Lovász Local Lemma, Z. Füredi and J. Kahn lowered the upper bound to $\dim(X, P) \leq 50k(\log k)^2$. We show that there exists an absolute constant $\varepsilon > 0$ so that for every $k \geq 1$, there exists a poset (X, P) with $\Delta(X, P) \leq k$ and $\dim(X, P) > \varepsilon k(\log k)$. Of course this leaves open the question as to the correct exponent on the log k term.

39

On Edge-Graceful Labeling of Complete Bipartite Graphs

Eleanor Lang Kendrick, San Jose City College

A (p,q)-graph G = (V,E), of p Vertices and q Edges, is said to be "EDGE-GRACEFUL" if there exists a bijection f: E -> { 1, 2, ...,q } such that the induced f+ : V -> { 0, 1, 2,..., p-1 }, defined by f+(V) = $\sum$ { f(u,v):(u,v) $\in$ E(G) } (mod p), is a bijection.

K(m,n) is a complete bipartite graph with (m + n) vertices.

Theorem: For any given m, there are a finite number of values of n for which it is possible for K(m,n) to be Edge-Graceful.

Both the number of values for n, and the values themselves are computable.

Examples of Edge-Graceful labeling for certain categories of K(m,n) graphs will be presented.

40

Hamiltonian Cycles in $C_nX_2C_m$

Joseph B. Klerlein*, Western Carolina University
Edward C. Carr, High Point College

In 1978 Trotter and Erdos gave necessary and sufficient conditions for the direct product, C_nXC_m, of two directed cycles to be hamiltonian. In this paper we give some sufficient and some necessary conditions for hamiltonian cycles in $C_nX_2C_m$, the directed graph obtained from C_nXC_m by squaring a single m-cycle.

41

Solving Exactly the mixed Postman Problem

Yves Nobert*, Jean-Claude Picard, University of Quebec at Montreal

In February 1989 we presented at the Southeastern Information Conference a new model and associated with it a strong relaxation of the postman problem with both directed (arcs) and undirected (edges) links. We also explained then how this relaxation can lead to an optimal algorithm for this problem. This year we present the algorithm in full details. The computerised tests executed on randomly generated problems show the surprising efficiency of the method. The simplex method is used to find the optimal solution satisfying a set of constraints associated with the "eveness condition" and the "balanced sets" condition. If other constraints are find violated, they are added to the model and we reportimize using the dual simplex method. Very few interations of this process are needed to obtain the global optimal.

42

Combinatorial statistics on non-crossing partitions

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Several statistics on set partitions were introduced and studied previously, giving rise to q-analogues of the Stirling numbers of the second kind. For example, interesting such results appear in work by Gould, Garsia and Remmel, and Wachs and White. Here we investigate partition statistics restricted to the class of non-crossing partitions, developing q-analogues of the Runyon numbers $W(n,k) = \frac{1}{n}\binom{n}{k}\binom{n}{k-1}$.

Our results include the following refinement of the rank symmetry of the lattice of non-crossing partitions (i.e., $W(n,k) = W(n,n+1-k)$):

The q-analogue $W_q(n,k)$ of $W(n,k)$ derived from the ls statistic satisfies the (external) symmetry property

$$q^{-\binom{k}{2}}W_q(n,k) = q^{-\binom{n+1-k}{2}}W_q(n,n+1-k).$$

The proof is combinatorial. based on an involution whose fixed points are characterized and enumerated.

43

Grundy Colorings of Graphs

S. T. Hedetniemi, R. Laskar *, Clemson University
G. Domke, Georgia State University

For a graph $G = (V, E)$ a coloring $\propto$ is a partition of $V(G)$ into sets $V_1, V_2, \ldots V_k$ such that, each induced subgraph $<V_i>$ of V_i is independent. A coloring $\propto$ is a Grundy-Coloring if for every $v \in V_i$, $i > 1$ there exist vertices $w_j \in V_j$, $j = 1, 2, \ldots i-1$ such that $v\, w_j \in E(G)$. We study variations of Grundy Colorings and their relations with other coloring concepts.

Key words: partition, coloring.

44

Enumeration Techniques for Certain Classes of k-Terminal Graphs

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Orlando, FL	Clemson, SC	Clarion, PA

Using principles from Wimer's methodology for designing linear-time algorithms on partial k-trees, we present techniques to obtain difference equations for the number of vertex/edge subsets on certain k-terminal classes of graphs. We do this by developing necessary and sufficient conditions for counting the

(i) vertex subsets for graphs composed by adding edges only, and

(ii) vertex/edge subsets for graphs composed by identifying terminals only. We illustrate our techniques by deriving difference equations for the number of dominating sets in paths, full binary trees, star graphs, and linear 2-trees and the number of matchings in a full binary tree. We note that, to the best of our knowledge, the methods presented here are the first for obtaining difference equations for the number of solutions to various subset properties for certain classes of graphs.

45

Mis-aligned Single Row Routing Problems: A Graph Theoretic Approach

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Single Row Routing problem (SRRP) is one of the important subproblems in the layout design of multi-layer printed circuit boards. This routing style has also been applied to routing of micro-wave circuits and other routing problems. It has been extensively studied and several algorithms have been proposed. We concentrate on a special single row routing problem in which certain terminals are allowed to be mis-aligned (MSRRP).

In this paper we develop a graph theoretic approach to investigate the properties and algorithms for the MSRRP. We investigate the problem of routing an MSRRP with K tracks and restricted number of doglegs per net. We prove a necessary and sufficient condition for routing with K tracks if no doglegs are allowed. We also present an algorithm for optimal routing if only one dogleg is allowed per net.

46

COMPOSITIONS OF INTEGERS

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Rose-Hulman Institute of Technology

For any integer $n \geq 1$, the number of compositions, or order partitions, of n using only 1's and 2's as summands, is counted by F_{n+1} (the $(n+1)$-st Fibonacci number, where $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$, for all $n \geq 2$). In this paper we examine the numbers $F(n,k)$, where $n, k \in \mathbf{Z}^+$ with $n \geq 1$ and $k \geq 2$, and where $F(n,k)$ counts of number of compositions for n that use the integers $1, 2, 3, \ldots, k$, as summands. The results obtained will deal primarily with

(1) $F(n,k)$ for k fixed and n variable; as well as,

(2) how the $F(n,k)$'s are related for different values of n and k.

47

On G_2 Graphs

Mark McKay *, Mount Vernon Nazarene College
Renu Laskar, Clemson University

For any graphs G its G_2 Graph is defined to be a graph having the same set of vertices as G with an edge between two vertices u, v if and only if the distance in G, d(u, v) = 2.
A characterization of G_2 graphs along with other results relating G_2 graphs are given.

Key words: Squared graph, neighborhood graph.

48

A Family of Constructive Bijections Involving Stirling Permutations

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A permutation of the multiset $\{1^n, 2^n, \ldots, k^n\}$ is a <u>Stirling Permutation</u> iff between any two copies of i, $1 \leq i \leq k$, no integers smaller than i appear; the set of all such permutations will be denoted Stir(n,k). Since any element of Stir(n,k) can be constructed uniquely by choosing an element of Stir(n,k-1) and inserting a block of n k's somewhere in it,

$$|\mathrm{Stir}(n,k)| = (n(k-1)+1)|\mathrm{Stir}(n,k-1)| = \prod_{i=1}^{k} (n(i-1)+1).$$

A similar construction can build ordinary permutations of $\{1^n, \ldots k^n\}$, the set of which we call Ord(n,k). Each can be constructed uniquely by choosing an element of Ord(n,k-1) and inserting n k's in arbitrary positions, i.e. in any n-multiset of the n(k-1)+1 postions. This gives

$$|\mathrm{Ord}(n,k)| = \binom{n\cdot k}{n} |\mathrm{Ord}(n,k-1)| = \prod_{i=1}^{k} \binom{n\cdot i}{n}.$$ (The obvious formula $|\mathrm{Ord}(n,k)| = (n\cdot k)!/(n!)^k$ is equivalent to this one.)

We present here a construction that associates to each element σ of Ord(n,k) an element $\tau = f(\sigma)$ of Stir(n,k) in such a way that the preimage sets $f^{-1}(\{\tau\})$ are all equal in size. The theorem that permits this correspondence can be interpreted as an assertion about the number of partitions of 0 into t parts modulo u whenever (t,u)=1.

TUESDAY, FEBRUARY 13, 1990 8:40 A.M.

49

Maximum Independent Sets with Minimum Intersection

Dana L. Grinstead*, John M. Cockerham and Assoc.

In the continuing theory of multiset/ single property problems, we consider the problem of finding two maximum independent sets of a graph G which have minimum intersection.

Define M(G) to be the minimum possible cardinality of the intersection of two maximum independent sets of G. It is shown that determinimg if M(G) = 0 (that is, determining if G has two disjoint maximum independent sets) is NP-Hard for an arbitrary graph G. Then linear algorithms for determining M(T) for a tree T and for finding two maximum independent sets of T whose intersection has cardinality M(T) are given.

50.

EDGE DOMINATION AND GRAPH STRUCTURE

Georges*, Halsey, Sanaulla and Whittlesey, Trinity College

An edge dominating set D of a graph G is a subset of the edge set of G such that every edge not in D is adjacent to at least one edge in D. The edge domination number of G, denoted $d_e(G)$, is the cardinality of a minimum edge dominating set of G. Forbidden subgraph characterizations of graphs with one dominating edge have been established for arbitrary graphs and bipartite graphs specifically. Formulas for the edge domination number of various classes of graphs have been established. Among these are: the products of complete graphs; the t-point suspensions of paths and cycles; the complete n-partite graphs; full m-ary trees; the prisms P(n, 1); helms; crowns; and gears.

A set of edges P is said to be perfect if for every pair of distinct edges e and f in P, N(e) and N(f) are disjoint. Necessary and sufficient conditions are given for the existence of a perfect edge dominating set in k-regular graphs of order n. We characterize all those generalized Petersen graphs P(n,k) which have perfect edge dominating sets.

51

Finding Maximum Subgraphs in Hypercubes

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Hypercube networks have emerged as popular networks. Several practical algorithms have been implemented on hypercubes demonstrating its efficiency and flexibility. Usually, the size of the hypercube is fixed whereas the size of the problem may vary. This leads to an important problem of simulating an algorithm designed for a large size hypercube H_n on a small hypercube H_m. This can be accomplished by embedding H_n into H_m while optimizing cost measures such as dilation and congestion.

We consider the following related problem: Given V a set of k vertices, pack V into a hypercube H so that the induced subgraph has maximum size. In this paper, we show efficient ways of packing V into H. Our efficient packing schemes also lead to efficient embeddings of H_n into H_m.

52

An efficient parallel algorithm for bit-reversal

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A simple recurrence is used to derive an $O(n)$ algorithm for the bit reversal permutation. This algorithm can be performed in $O(\log\log n)$ parallel steps using $\lceil\sqrt{n}\rceil$ processors. An implementation on the Alliant FX/8 performs up to twice as fast as the previously best known algorithm.

TUESDAY, FEBRUARY 13, 1990 9:00 A.M.

53

EXTENDING INDEPENDENT SETS TO MAXIMUM INDEPENDENT SETS

Nathaniel Dean and Jennifer Zito*
Bellcore

We consider graphs in which every independent set of size k is contained in a maximum independent set. For k less than the size of a maximum independent set, call a graph k **MIS-extendible** if every independent set of k vertices is contained in a maximum independent set. Graphs which are 1 MIS-extendible (called B-graphs) have been studied for a number of years, but have only been characterized for some special cases. In general, k MIS-extendible does not imply $(k-1)$ MIS-extendible or vice-versa (e.g. for $k=2$, stars or even cycles). We prove that 2 MIS-extendible graphs which are not 1 MIS-extendible are the join of a complete graph and a 1 MIS-extendible graph. We characterize trees which are k MIS-extendible and 1 MIS-extendible. We give some results for bipartite graphs and describe a number of open problems.

54

Private Domination: Theory and Algorithms

Sandra M. Hedetniemi, Stephen T. Hedetniemi, David P. Jacobs
Clemson University

Let S be a set of vertices in a graph G=(V,E). A vertex u in V has an **(open) private S-neighbor** if u is adjacent to a vertex w in V-S, w≠u, and no vertex in S-u is adjacent to w. In 1979 Cockayne and Bollobas observed that every graph without isolated vertices has at least one minimum dominating set D such that every vertex in D has a private D-neighbor. We call such a set a **private dominating set**. In this paper we introduce the **(upper) private domination number**, i.e. the maximum cardinality of a private dominating set in a graph. We derive some of its graph theoretical and algorithmic properties, including an NP-completeness proof for arbitrary graphs and a linear algorithm for computing the private domination number of an arbitrary tree.

55

A Class of Critical Squareless Subgraphs of Hypercubes

Puhua Guan, Dept.of Math. University of Puerto Rico

A recursive method of constructing a class of critical squareless subgraphs S(n) of hypercubes H(n) is presented. The number of edges

$$|E(S(n))| = ((1/2) + (3/2n) - t)|E(H(n)|.$$

where $t = 1/(2(\exp([n/3])n)$ for $3 \nmid n$ and $t = 3/(2(\exp([n/3]+1)n)$, for $3 | n$.

56

Hierarchical expansions of partial k-trees and applications.

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Abstract
The computational complexity of many physical design problems in VLSI systems can be reduced by recourse to hierarchical methods. In this paper we develop the theoretical framework for a hierarchical process that results in computationally tractable graph models. Our approach is based on the successive expansion of partial k-trees. We identify the conditions under which the vertices of partial k-trees can be expanded while limiting the growth in k by a small constant factor. It is shown that for an important special case of planar partial k-trees, it is possible under fairly general conditions to limit the growth of k to $2k$ at every expansion step. Based on the above, a bounded decomposition scheme with applications in building block style VLSI design, is identified and described.

Keywords:- Algorithms, Decomposable graphs, Complexity and VLSI.

57

A Theorem on Matroid Homomorphism

Jon Henry Sanders

We call f: M→M' a matroid homomorphism, or more briefly a homomorphism if f: E(M)→E(M') is an onto map of the ground set E(M) of a matroid M onto the ground set E(M') of a matroid M' which preserves circuits of M, i.e., f(C) ∈ $\mathscr{C}$(M') whenever C ∈ $\mathscr{C}$(M). If f^{-1} preserves circuits as well, we call f a homeomorphism. If M has no coloops and f: M→M' is a homeomorphism it is not hard to show that M is isomorphic to a subdivision of M' and that the elements of $f^{-1}(x)$ are in series for each x ∈ E(M'). We call a 1-1 homomorphism f: M→M' a circuit injection. In this case M' is isomorphic to a refinement of M. In this note we prove that when M is connected and M' is binary and does not consist of a single circuit then any homomorphism f: M→M' can be written as a composite, f = h ∘ g, of two homomorphisms h and g where g: M→M" is a homeomorphism, h: M"→M' is a circuit injection and M is a subdivision of M". This will imply that M must be binary. This theorem generalizes a result of a previous paper [J. Sanders, J. Combin. Theory Ser. B 42 (1987), 146-155].

58

On integer and fractional total domination

G.H.Fricke, Wright State University, D.P.Jacobs and A.Majumdar*, Clemson University

A total dominating set of a graph G=(V,E) is a set S of vertices such that every vertex v in V is adjacent to some vertex in S. The *total domination number* γ_t and the *upper total domination number* Γ_t are, respectively, the minimum and the maximum size of a *minimal* total dominating set. A total dominating function for G is the continuous analogue of the characteristic function of a total dominating set. The *fractional total domination* γ_t^o and the *upper fractional total domination* Γ_t^o are, respectively, the minimum and the maximum sum of a *minimal* total dominating function. This paper addresses complexity issues and discusses algorithmic aspects of determining these parameters.

Keywords: total domination, fractional domination

59

Some New Generalized Measures of Connectivity with Application to Hypercube Networks

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January 25, 1990

Abstract

In the paper [1], a generalized measure of connectivity has been introduced using the concept of *forbidden faulty set*, which was defined as the set of neighbors of any vertex. An extention to the notion of connectivity, where only p neighbors of any node are allowed to fail, is given. Defining the class-p $n-$cube as an $n-$cube in which any vertex can at most have p faulty vertices ($0 < p \leq n$), we establish the following:
Theorem 1: In a class-p $n-$cube, $\kappa^{(p)}(Q_n)$=$p \times 2^{n-p}$, where $\kappa^{(p)}(Q_n)$ denotes the vertex connectivity of a class-p $n-$cube.
Theorem 2: Let $V(Q_n)$ be the set of vertices of a class-p $n-$cube. Assume F is a subset of $V(Q_n)$, with $|F| = p \times 2^{n-p}$. Then the following statements are equivalent:

1. $|F|$ is the minimum number of nodes which disconnect Q_n.
2. $Q_n - F$ consists of exactly two components, one of which is an $(n-p)$-cube.

Index Terms- Diameter, Forbidden sets, Hypercube, Node-disjoint paths, Vertex-connectivity.

[1] A. H. Esfahanian, "Generalized Measures of Fault Tolerance with Application to $N-$cube Networks," *IEEE Transactions on Computers*, Vol. 38, No. 11, pp. 1586-1591, November 1989.

60

Polynomial Algorithms on k-Polygon Graphs

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Given an integer k, $k \geq 3$, we define the class of k-polygon graphs to be the intersection graphs of straight line chords inside a convex k-polygon. Thus, permutation graphs form a proper subset of any such class. Moreover, circle graphs = $\cup_{k=3}^{\infty}$ k-polygon graphs. In this talk, we present a polynomial time exact algorithm for solving the minimum dominating set problem on k-polygon graphs, for any fixed k. In addition, we discuss the recognition problem of such graphs.

Key Words: circle graphs, permutation graphs, dominating sets.

61

The Tutte Polynomial for Trees and Posets

Gary Gordon, Lafayette College

The Tutte polynomial f(t,z) is an invariant for graphs and matroids which has been studied extensively. Recently, the definition of this invariant has been extended to greedoids. We will explore the polynomial for three well-studied classes of greedoids: trees, rooted trees and posets. For trees and rooted trees, the polynomial can be viewed as giving the size and the number of leaves of each subtree or rooted subtree. For posets, there is a corresponding interpretation in terms of antichains. For all three classes, there is an easily understood recursive procedure for computing the polynomial. For rooted trees, the polynomial completely determines the rooted tree. This is false for posets and open for unrooted trees. We give some examples and describe some other results for posets.

Key words: Tutte polynomial, greedoid

62

Perfect Dominating Sets

Marilynn Livingston*, Southern Illinois University, and Quentin F. Stout, University of Michigan

A dominating set S of a graph G is *perfect* if each vertex of G is dominated by exactly one vertex in S. We were motivated to study perfect dominating sets (PDS) by problems involving efficient resource allocation and placement in parallel computers. We study the existence and construction of PDSs in families of graphs arising from the interconnection networks of parallel computers. These include meshes, tori, trees, dags, hypercubes, cube-connected cycles, and cube-connected lines. For meshes, tori, hypercubes, and cube-connected lines we completely characterize which graphs have a PDS, and the structure of all PDSs. For trees and dags we give linear time algorithms that determine if a PDS exists, and generate a PDS when one does. Motivated by problems involving placement of more than one resource, we also consider the construction of multiple disjoint PDSs. Our results include distance d domination for any given d.

Keywords: perfect codes, resource allocation, dominating sets, mesh, torus, hypercube, tree, dag, cube-connected cycles, cube-connected lines.

63

Steiner Trees in Hypercubes

Zevi Miller* and Manley Perkel
Miami University Wright State University

We consider the analogue for the hypercube of the classical Steiner problem in the plane. Given a set V of points in the n-dimensional cube Q(n), we ask for the smallest number points in any subtree T of Q(n) such that $V \subset T$. We obtain solutions when $|V| \leq 5$, upper bounds when all points in V have constant weight, and we show NP-completeness even when all points of V have weight at most 2.

64

COMPUTER CONSTRUCTION OF THE DUAL OF A PLANE GRAPH

David R. McIntyre
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Cleveland State University

We present an efficient implementation of an algorithm which plots the plane dual graph corresponding to a given plane graph. The asymptotic time and space complexities of the algorithm are proved.

65

WEAK ULTRAMETRIC PARTITION SETS

John C. Higgins* and Jack Lamoreaux, Brigham Young University
Key Words: complete graph, ultrametric, ultrametric partition, weight function

Denote by K_n the complete undirected graph without self edges on n-vertices with vertices x,y,z, etc. and edges (x,y). An ultrametric on K_n is any positive real valued weight function, denoted by $\mu(x,y)$, which satisfies: $\mu(x,y) \leq$ maximum of $\mu(x,z)$ and $\mu(y,z)$ for all x,y,z in K_n. An ultrametric partition set for K_n is a finite set of pairs (P_i, r_i), where P_i is a partition of the vertices of K_n, r_i is a non-negative real number and both P_i is a proper refinement of P_j and $r_i < r_j$ hold when $i<j$. A weak ultrametric partition set on K_n allows equality for P_i, P_j and r_i, r_j. Each $\partial(x,y)$, a weight function on K_n, may be used to create a set of weak ultrametric partition sets which describe in a natural way the supremum of all ultrametrics less than ∂ and the set of infimum of ultrametrics greater than ∂. The structure of the partition sets also permits an elementary analysis of the computational complexity of an algorithm to construct the unique supremum and set of infimum.

66

REINFORCEMENT IN GRAPHS

Christine Mynhardt, University of South Africa

The reinforcement number r(G) of a graph $G = (V,E)$ is the minimum number of edges which have to be added to G such that the resulting graph G′ satisfies $\gamma(G') < \gamma(G)$, where $\gamma(G)$ denotes the domination number of G. If $\gamma(G) = 1$, then define $r(G) = 0$. Further, G is said to be reinforcement–critical if $\gamma(G+uv) = \gamma(G) - 1$ for every edge $uv \notin E$. Results on the reinforcement numbers of graphs and reinforcement–critical graphs are discussed. A characterisation of reinforcement–critical graphs with domination number two is presented.

67

ON A LEVERAGE PROBLEM IN THE HYPERCUBE

Peter Hamburger and Raymond E. Pippert,
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The leverage of a set S of elements (vertices and edges) of a graph G, with respect to a graphical parameter P, is the change induced in P by the removal of S. We consider the case in which G is a hypercube and P is the sum of the distances between vertices. The determination of the minimum leverage of any set of k edges leads to the question of the existence of a perfect matching in which no two edges lie on a 4-cycle. We give a constructive proof of the existence of such a matching, and note possible applications to other problems.

68

Edge Ranking of Trees

Jitender S. Deogun+ and Yiping Peng, University of Nebraska-Lincoln

A *k-edge ranking* of an undirected graph is defined to be a labeling of the edges of the graph with integers $1, 2, \ldots, k$, with the property that all paths between two edges with the same label i contain an edge with label $j > i$. Finding the smallest k for which a graph has a *k-edge ranking* is known as the edge ranking problem. An $O(n \log n)$ time approximation algorithm for edge ranking of trees was developed by Iyer, Ratliff and Vijayan. In their paper, Iyer et al. mentioned that whether the problem of finding edge ranking of a tree is *NP-hard* is still an open question. In this paper, we resolve this question by developing a polynomial algorithm for finding edge ranking of a tree. The problem of edge ranking of a tree is solved by transfoming the problem into an equivalent node ranking problem. The main result in the paper is an $O(max\{n^2, nr^3\})$ time algorithm for edge ranking of trees, where n is the number of vertices of the tree and r ($< n$) is the edge rank number of the given tree.

TUESDAY, FEBRUARY 13, 1990 11:50 A.M.

69

A Construction for a Delta System Avoiding Set.

Dan Ashlock, Caltech Math Department

A construction is given for an exponentially growing familly of sets that fail to contain a delta system. The familly is self similar and includes the known maximum cardinality examples of sets that avoid a delta system.

70

The Domination Number for the Product of Graphs

R.J. Faudree, R.H. Schelp*, W.E. Shreve

Memphis State University and North Dakota State University

In 1965 V.G. Vizing conjectured that $\gamma(G \times H) \geq \gamma(G)\gamma(H)$ for arbitrary graphs G and H, where γ denotes the domination number. We show this inequality holds when at least one of G or H belongs to a large family of graphs which includes (as a proper subset) all trees. The conjecture was also proved when G or H is a tree by Jacobson and Kinch in 1986.

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"The Domination Number for the Product of Graphs"
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We will not schedule your presentation for Friday.

71

Packings into Hypercubes

Quentin F. Stout, University of Michigan

This work is concerned with the number of disjoint copies of a cubical graph which can be embedded in a hypercube. Such packings are of use in various communication and resource distribution problems in hypercube computers. For any cubical graph G and dimension d, let $p_e(G,d)$ [$p_v(G,d)$] denote the maximum fraction of edges [vertices] of the d-dimensional hypercube Q_d covered by edge-disjoint [vertex-disjoint] copies of G. For a cubical graph G, if $p_e(G,d') < 1$ for some dimension d', then $\lim_{d\to\infty} p_e(G,d) > p_e(G,d')$, while if $p_e(G,d') = 1$ then $\lim_{d\to\infty} p_e(G,d) = 1$. For any tree T of t edges, edge-disjoint copies of T completely cover Q_t, and if $t = i2^k$, where i is odd, then edge-disjoint copies of T cover Q_{ci} for all integral c sufficiently large. Notice that counting arguments show that the only dimensions T could cover are of the form ci. It is also shown that $p_v(G,d)$ weakly monotonically approaches 1 for any cubical graph G.
Keywords: hypercubes, packings, coverings, embeddings, trees.

72

SINGLE NEURON STUDIES

A.K.Dewdney, Computer Science, University of Western Ontario

The recent resurgence of interest in neural nets has tended to obscure the fact that, far from being able to design effective computational networks, we don't even know what single neutrons do! After briefly describing experiments that illustrate the dismal failure of Hopfield networks to solve various combinatorial problems, I explore several problems connected with the logical operation of a single formal neuron.
Let a neuron be a set of integers (T, w1, w2,..., wn). The neuron defines a logical function f(x1, x2,...,xn) to have the value 1 iff the weighted sum of inputs xi equals or exceeds the threshold T. The definition is not very helpful when it comes to describing the function f in logical terms. Define a logical function to be neural if there is a neuron that will compute it. In the belief that any worthwhile investigation of the computational properties of neural nets depends on the logical properties of single neurons, I explore various conditions that might aid in the characterization of neural functions. Another problem of potential importance in such an inquiry is that of determining the smallest threshold that will serve to realize a given n-place neural function. What is the largest such threshold over all n-place neural functions?

73

Learning Algorithms for Constructing Set Systems

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January 24, 1990

Abstract

A very simple algorithm that can be used to construct set systems is described. The algorithm has been used successfully to construct set systems that do not have Property B, to construct set systems not containing delta systems, on Ramsey problems, and to construct small regular graphs of specified girth.

74

Cutpoints in Domination Critical Graphs.

Authors: David P. Sumner, University of South Carolina, Ewa Wojcicka*, College of Charleston.

A graph G is k-domination critical if it has domination number k,and for any edge e not belonging to G, G+e has domination number k-1. David Sumner conjectured that every 3-domination critical graph on more than 6 vertices was Hamiltonian. This conjecture was recently resolved by Ewa Wojcicka. In that proof heavy use was made of the fact that 3-domination critical graphs are 'almost 2-connected' in the sense that removing all the endpoints of such a graph results in a 2-connected graph. This property does not hold for k-critical graphs with k>3, thus motivating a need for information about critical graphs having cutpoints. In this paper we characterize, for arbitrary k, those k-domination-critical graphs that contain cutpoints. Other related results and questions will be presented.

75

Butterflies are Free; Butterfly Networks Aren't

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University of Cincinnati

Abstract. Why do more parallel computer architectures employ the hypercube than the butterfly network? Perhaps the butterfly lacks some essential connectivity, such as a hamiltonian circuit, which the Gray codes provide for hypercubes. We answer the second question by showing that butterflies are never (hamiltonian) free.

76

Provably hard graphs for the crossing number problem

Daniel Bienstock, Columbia University

We show that for every arrangement A of lines (or pseudolines) in the real plane, there exists a graph G(A), containing a subset S(A) of edges, such that in every drawing of G(A) with minimum number of crossings, the set S(A) realizes A. Using some recent results of Goodman, Pollack and Sturmfels, this implies that there exist graphs, such that in every rectilinear drawing with fewest crossings the coordinates of the vertices require more than polynomially many bits. As another application of this construction, we consider the problem of drawing a graph so that each edge is a polygonal line with at most (say) t breakpoints. How large must t be in order to achieve the (unrestricted) crossing number? We show that for graphs with crossing number k or less, the smallest possible value t(k) of t satisfies

$$c\,k^{1/2}/\log k \le t(k) \le c'\,k^{1/2},$$

for constants c, c'.

77

Towards a Bound for the Compression of the LZW Algorithm

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The LZW algorithm is a well known efficient adaptive compression algorithm. It is based on constructing a dictionary containing character strings from the text. Recently we attempted to design variations of the original algorithm which achieve higher compression ratios. In this paper we attempt to find a bound for the LZW approach. That is, using a dictionary which satisfies the prefix property of the LZW algorithm and gives maximum compression.

In order to build such an optimum dictionary, an algorithm based on the dynamic programming technique is given. A polynomial implementation of the algorithm is presented. Numerous experiments were performed using different variations to obtain high compression ratios. The compression ratios obtained are substantially higher than the ones obtained by the LZW algorithm and its variations. This indicates that significantly higher compression can be obtained by improvements to the LZW approach. This observation will direct our future efforts.

78

k-Weight Domination and Fractional Domination of $P_m \times P_n$

E. O. Hare
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A [0,1]-valued function g defined on the vertex set of a graph is a dominating function if $g(N[v]) \geq 1$ for every vertex v. If, instead of [0,1], g has values in {0, 1, ..., k} for some positive integer k, then it is a <u>k-weight dominating function</u> provided $g(N[v]) \geq k$ for every vertex v. The number $\min_g \Sigma_v g(v)$ is the k-weight dominaiton number of graph G. An algorithm for computing this number for $P_m \times P_n$ (for fixed k) is presented. The fractional domination number of G is $\min_g \Sigma_v g(v)$. A construction for an exact computation for $P_2 \times P_n$ is given, as well as other examples for more general $P_m \times P_n$.

79

On a Graph Transformation Where Nodes Are Replaced by Complete Subgraphs

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Texas Instruments
and
East Tennessee State University

Teresa W. Haynes
East Tennessee State University

A graph transformation is given that replaces each node v of a graph with a complete subgraph of size d_v where d_v is the degree of the replaced node. Many invariants of the transformed graph can be calculated from the invariants of the original graph. We show that a transformed graph is recognizable in polynomial time and present results on relationships between the invariants of the original graph and the invariants of the transformed graph.

80

MINIMUM DISTANCE OF BCH CODES OF LENGTH $2^m + 1$

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The main result in this investigation is the estimation of the minimum distance of non primitive BCH codes of length $n = 2^m + 1$. We prove that the generating set of such codes, $A = \{\beta^i\}$, where β is a primitive n-th root of unity and the β^i s are roots of the generating polynomial $g(X) = m_1(X)m_2(X)...m_{2k-1}(X)$, for some values of k, contains three reasonably long subsets of roots with consecutive exponents. This permits the use of Hartman-Tzeng (HT) Bound to get substancial improvement over estimations obtained by use of the BCH bound.

Key words

Codes, BCH, minimum distance, primitive roots

81

Incorporating Updates into the LZW Compression Algorithm

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KEYWORDS: TEXT COMPRESSION, LZW ALGORITHM, DYNAMIC DICTIONARY, DICTIONARY UPDATES.

The LZW algorithm is a well known efficient adaptive compression algorithm. It is based on constructing a dictionary containing character strings from the text. This algorithm is especially appropriate for communication since both the encoder and decoder construct the same dictionary while processing the text. Thus no preprocessing of the text or communication of the dictionary is necessary. We enhance the algorithm by allowing dictionary updating including deletion and addition of entries. To control the updates we add a frequency field for each entry of the dictionary. We consider several variations. Our experiments show a significant increase in the compression ratios obtained. The improvement is highest for texts where the LZW algorithm does not perform so well and for small dictionaries.

82

Total Interval Number of a Tree

Arundhati Raychaudhuri, College of Staten Island (CUNY).

The total interval number of a graph, G, is the smallest positive integer, k, such that every vertex, x, of G can be assigned a union of intervals S(x), where $(x,y) \in E(G)$ iff $S(x) \cap S(y) \neq \phi$, and US(x) is a union of at most k intervals. This paper presents a polynomial algorithm for finding the total interval number of a tree T. The approach is based on determining the minimum number of edge disjoint caterpillars which cover E(T).

Keywords Interval graph, line graph, clique graph, total interval number, caterpillars.

83

Hamiltonian Cycles which extend Transposition Matchings in Cayley Graphs of S_n

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2

Let B be a basis of transpositions for S_n and let $C(S_n, B)$ be the Cayley graph of S_n with respect to B. It is known that $C(S_n, B)$ is Hamiltonian. Every transposition b in B induces a perfect matching M_b in $C(S_n, B)$. We show here that for any b in B, there is a Hamiltonian cycle in $C(S_n, B)$ which includes every edge of M_b. That is, it is possible to generate all permutations of $1, 2, \ldots, n$ by transpositions in B so that every other transposition is b.

84

Weakly Resolvable Designs and Unconditionally Secure Authentication Codes

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A block design is said to be **α-resolvable** if the blocks can be partitioned into sets (resolutions) such that each element occurs α times in each set, or simply **resolvable** if α = 1. An α-resolvable design, α > 1, is said to be separable if one or more of the resolutions can be further partitioned into subsets in each of which each element occurs α' times, α' < α; and nonseparable otherwise. A design is affine if every block has the same number of elements in common with each block in the other resolutions. We introduce a new class of designs which we call **weakly resolvable** in which the blocks can be partitioned into sets such that each element occurs α_i times in set i: α_i not the same for all i, otherwise the design would be α-resolvable. The reason for the interest in weakly resolvable designs is that every nonseparable and affine weakly resolvable design corresponds to a perfect and unconditionally secure authentication code in a natural way.

Key words: Resolvable Designs
Authentication Codes
Weakly Resolvable Designs

85

A Model of Information Processing for a Knowledge Base System

J J Longstaff*, N R Armfield, K Burrow, A N Dacre, N Jennings, I Walker, Leeds Polytechnic, England.

The graph representations of knowledge and knowledge processing in a general purpose Knowledge Base System are presented; these representations are directly implemented by data structures in the system. Firstly, an introduction to the software architecture is given. Then a graph representation of an Object Oriented type system is described, which includes "abstract" nodes for the knowledge types maintained by the system. A KB_type abstract node is associated with operations for creating lower level abstract and application type nodes, and for Object query. A DB_type abstract node includes operations for creating DB_application types, and for general database update operations with automatic constraints maintenance; constraints are defined at the DB_application type level. Other special types can be created, e.g., rule_derived_type, image_type; and software/hardware for processing them included in the system. A predicate calculus based query language is introduced, and query processing is described in terms of intermediate graph representations of queries which must be compatible with the Object type hierarchy. The mapping of query structure graphs onto the Active Graph database model (ACM TODS vol 14 no 1) is described. Finally, the representation of user expertise in the form of graphs which are isomorphic with Object type and query structure graphs is presented. These graphs are used to drive adaptive help, and the values associated with their nodes are updated after each interaction. Certain concepts described in the paper are the subject of commercial exploitation.

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Changing and unchanging of the node covering number of a graph

Linda M. Lawson and Teresa W. Haynes
East Tennessee State University

A set of nodes that cover all the edges of a graph G is called a node cover for G. The minimum cardinality among the sets of node coverings is called the node covering number of G, denoted by α_0. We present results on the changing and unchanging of α_0 when a graph is modified by deleting a node, deleting an edge, or adding an edge. These results are important in applications of graph theory where a property of a system's graphical model changes or remains intact when it is modified either by a component failure or the addition of a link.

87

Partitioning Planar Graphs into Independent Sets and Forests

S.L.Hakimi, University of California, Davis
E.F.Schmeichel*, San Jose State University
J.Weinstein, PRC Scientific Support Systems

We first consider the problem of vertex partitioning a planar graph into an independent set and a forest. We prove this problem is NP-complete, even for planar graphs with maximum degree at most 4. By contrast, the problem is polynomial for general graphs with maximum degree at most 3. We then consider the problem of partitioning a planar graph into two independent sets and a forest. Wegner gave the first example--a maximal planar graph on 49 vertices--showing that such a partition is not always possible. We prove that this problem also is NP-complete, even for planar graphs in which each face has 3 or 4 sides. The complexity remains open for maximal planar graphs.

88

Coding Theory and Hyperovals

William Cherowitzo, University of Colorado at Denver

It is well known that in the dual code of the binary code of a projective plane of even order n, the code words of weight n + 2 correspond to hyperovals in the plane. We investigate this connection seeking a classification of hyperovals in projective planes of even order, with special attention paid to the Desarguesian case.

Keywords: Hyperovals, Cyclic Codes, Sets of Even Type, Matroids

TUESDAY, FEBRUARY 13, 1990 4:20 P.M.

89

Proofs for Non-Identities Using Characteristic Functions

Irvin R. Hentzel

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David Pokrass Jacobs *

Dept. of Computer Science, Clemson University, Clemson, SC 29634-1906, USA

This paper deals with the problem of establishing a proof that a nonassociative polynomial is not an identity in a variety of algebras. The traditional method used is an ordinary counterexample. Here a new method is given called a characteristic function. With such a function, person A can convince person B that the polynomial fails, much in the same way that a counterexample supplies a convincing proof. We give evidence to support that in many cases, the characteristic function is easier to construct for person A and easier to verify by person B. We present an iterative algorithm for finding such a characteristic function. Finally we describe how the algorithm was used to discover a new result in the theory of nonassociative algebras. We also show how the method yields shortened proofs of several known results.

Key Words. alternative algebra, characteristic function, counterexample, nonassociative algebra, polynomial identity.

90

Bounded Di:crete Representations of Interval Orders

Garth Isaak, RUTCOR, Rutgers University

A discrete representation of an interval order is an interval representation for which each interval has integral endpoints. A representation is bounded if each interval is constrained with upper and lower bounds on its length. Given an interval order and length bounds, we give a polynomial procedure which determines whether or not it has a bounded discrete representation. This answers a question posed by Ken Bogart. The method uses Farkas' Lemma to reduce the problem to finding a shortest path or detecting a negative cycle in a corresponding digraph. Furthermore, we use this digraph to determine lists of forbidden suborders in cases with constant lower bounds of 0 or 1.

91

COVERING COMPLETE GRAPHS BY CLIQUES

Peter Horak*, University of Nebraska and Slovak Technical University, Czechoslovakia
Norbert Sauer, University of Calgary, Canada

A (k,n)-covering S of the complete graph K_n is a family of k cliques in K_n chosen so that every edge of K_n lies in at least one clique. Let c(S) be the size of the lasgest cliques in S. Define f(k,n)=min c(S), where the minimum is taken over all (k,n)-coverings S. In other words, every (k,n)-covering must have a clique of size at least f(k,n).
To determine f(k,n) is an extremaly difficult problem as it includes in itself the question of the existence of Balanced Incomplete Block Designs with $\lambda=1$.
The values of f(k,n) for $k\leq 7$ and all $n\geq n_0$ as well as several asymptotic properties of f will be presented.

Keywords: covering, clique, BIBD.

92

The Maximum Order Complexity Profile of Cryptosystem PGM

Spyros S. Magliveras*
Nasir D. Memon
Kok. C. Tam
University of Nebraska-Lincoln

Abstract

For a given sequence $\sigma = \{x_1, x_2, \ldots, x_n\}$ where the elements x_i belong to an alphabet $\mathcal{A}$, the *maximum order complexity* (MOC) of σ is defined to be the length of the shortest feedback shift register (not necessarily linear) which generates the sequence. The *maximum order complexity profile* (MOCP) of σ is defined to be the sequence $\mu = \{y_1, \ldots, y_n\}$ where $y_k = \text{MOC}\{x_1, \ldots, x_k\}$. The maximum order complexity profile provides a strong randomness criterion, particularly applicable to deciding the degree of unpredictability of sequences generated from cryptographic systems. In this paper, we study the MOCP of sequences based on cryptosystem PGM.

TUESDAY, FEBRUARY 13, 1990 4:40 P.M.

93

An update on conjectures of Graffiti.

Siemion Fajtlowicz, University of Houston.

I will discuss a few partially solved and a few new conjectures of Graffiti.

Some of them are about graphs derived from certain number-theoretical relations and they often result in conjectures related to Prime Number Theorem, or in general about the distribution of primes.

Other conjectures involve Randic index, relation between the average distance and the minimum degree of a graph, and the rank of adjacency matrix over the two-element field.

94

An Application of Signed Graphs to International Relations

Nancy S Mauldin*, College of Charleston
Edwin Coulter and William Hare, Clemson University

Making foreign policy decisions involves evaluating complex relationships among nations and determining the costs and benefits of each decision alternative. In mathematics, graph theory provides an excellent tool to model international relations. Using signed balanced graphs, the model developed here evaluates the relationships among six countries for the period 1977 to 1982 in terms of their tendency toward alignment or non-alignment. The complete graph K_6 is used to model the relationships among the US, Russia, China, Pakistan, India and Japan. The relationship between two countries is denoted by a positive or negative signed edge between the two vertices representing the countries. A graph is ***balanced*** if there are no cycles having an odd number of negative edges. The goal was to create a model which can be used by diplomats to organize and prioritize alternatives for directing a limited number of resources in order to achieve political stability.

95

Vertex-switching reconstruction of triangle-free graphs

M. N. Ellingham and Gordon F. Royle*, Vanderbilt University

In 1985 R. P. Stanley proved that all graphs of order n are reconstructible from their deck of vertex-switchings, provided that $n \equiv 0 \pmod 4$. This result is not true in the case $n = 4$, but computer results show that it is true for $n = 8$, leaving the situation $n \geq 12$, $n \equiv 0 \pmod 4$ to be resolved. We show that the number of triangles in a graph of order $n \neq 4$ is vertex-switching reconstructible, a result which can in fact be generalised to any subgraph of order less than $n/2$. Using this and some structural arguments we prove that all triangle-free graphs of order $n \neq 4$ are vertex-switching reconstructible.

96

"BINARY CODES ARISING FROM GRAPHS"

by
H.Tapia R.

Let G be a finte connected graph with vertices $X_1,\dots,X_p$ and identify an arc having vertices X_i,X_j with the monomial X_iX_j of the polynomial ring $R = \mathbb{Z}_2[X_1,\dots,X_p]$. Let I be the ideal of R generated by the monomials X_iX_j, $i \neq j$ such that the vertices X_i,X_j determine an arc of G. let M be the maximal ideal of R generated by all the indeterminates $X_1,\dots,X_p$, and denote by L(G) the finite dimensional $\mathbb{Z}_2$-linear space I/MI, which has as a natural basis the cosets of the arcs of G. Let $T: L(G) \longrightarrow L(G)$ be the $\mathbb{Z}_2$-linear function defined by $T(\bar{a}) = \sum \bar{b}$ where the sumation is taken over all cosets $\bar{b}$ such that the arc b is incident to the arc a, and let C be the kernel of this linear function. The purpose of this talk is to describe the length, dimension and minimal distance of C in terms of the graph G. Some results connecting matchings and codes are proved along the same lines.

97

MORE ABOUT EUCLID'S PROOF OF THE INFINITUDE OF PRIMES

Alberto Cáceres and Liduvina Cruz*

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In his note "A Remark on Euclid's Proof of the Infinitude of Primes", *The American Mathematical Monthly*, Vol 96, No.4 (1989) 339-341, John B. Cosgrave defines the numbers $N_i^{(n)}+1 = p_1 \cdot p_2 \cdots \widehat{p}_i \cdots p_n+1$ where $p_1, \ldots, p_n$ are the first n primes and the circumflex means omission. Using Dirichlet theorem on the distribution of primes, we prove that for the case $p_2 = 3$ there are infinitely many numbers $N_2^{(n)}+1$ which are divisible by p_2. For higher primes we give sufficient conditions for the existence of infinitely many numbers $N_i^{(n)}+1$ which are divisible by p_i. The feasibility of our conditions is based on the even distribution of primes among the non zero residue classes modulo p.

98

Opsut's Conjecture and Critical Graphs

Chi Wang

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keywords: competition graph, competition number, Opsut's conjecture

A graph $G = (V(G), E(G))$ is said to be a competition graph of an acyclic digraph $D = (V(D), A(D))$ if $V(G) = V(D)$ and there is an edge in G between vertices $x, y \in V(G)$ if and only if there is some $v \in V(D)$ such that $v \neq x, y,\ xv, yv \in A(D)$. The competition number $k(G)$ of a graph G is the minimum number of isolated vertices needed to add to G to obtain a competition graph of some acyclic digraph.

Opsut conjectured in 1982 that if $\theta(N(v)) \leq 2$ for all $v \in V(G)$, then the competition number $k(G)$ of G is at most 2, with equality if and only if $\theta(N(v)) = 2$ for all $v \in V(G)$. (Here, $\theta(H)$ is the smallest number of cliques covering the vertices of H.) Though Opsut (1982) proved that the conjecture is true for line graphs and recently Kim and Roberts (1989) proved a variant of it, the original conjecture is still open. In this paper, a class of critical graphs is defined. It is proved that Opsut's conjecture is true if and only if it is true for critical graphs. The results of Opsut (1982) and Kim and Roberts (1989) are generalized. All K_4-free critical graphs are characterized. Some structural properties of critical graphs are discussed and further problems are listed.

99

Reconstructing Graphs From Their Homomorphic Images

Dan Pritikin, Miami University, Oxford OH 45056

An elementary homomorphic image (EHI) of a simple graph is the simple graph obtained by identifying a single pair of nonadjacent vertices v and w, i.e. replacing v and w by a vertex whose neighborhood is the union of the neighborhoods of v and w. If given isomorphic copies of the EHIs of a graph, it is conjectured that (with minor exception) these uniquely determine the original graph. It is proved that Husimi trees (graphs all of whose blocks are complete) are indeed reconstructible from their EHIs, and discussed as to how this is a natural step in the right direction towards the conjecture. Related reconstruction problems are considered.

Key Words: Graph reconstruction, homomorphism

100

X-VMS-To: IN%"hoffmanf@servax.bitnet"

Connections Between Exponential Sums, Algebraic Curves and Sequence Design

Oscar Moreno and Vijay Kumar
University of PR; University of Southern California

Sequence design for multiple-access communication systems involves the identification of a family of (+1,-1) sequences such that the maximum magnitude of the inner product between one sequence and a cyclically shifted version of the second (not necessarily distinct from the first) be kept as low as possible. Of course, if the two sequences are the same, we exclude the trivial case of zero shift. In this talk we point out how number-theoretic bounds and results on algebraic curves can prove useful in the design/analysis of such sequence families.

Some of the best known families of sequences have been derived from a pair of maximum-length, linear-feedback shift-register sequences (m-sequences). The inner products in this case are in general, exponential sums of the Weil type. Such sums are related directly to the number of rational points on certain plane curves. Interestingly, while curves having a large number of rational points are of importance in constructing error-correcting codes, in the case of sequence design, one is interested in curves whose number of rational points deviates as little as possible from that lying on the projective line.

The inner product between a m-sequence and its reciprocal gives rise to a Kloosterman sum. By relating the value of the Kloosterman sum to the number of rational points on certain elliptic curves, Lachaud and Wolfmann identified the possible values of a Kloosterman sum. We show how this result proves two longstanding conjectures regardi the inner product between an m-sequence and its reciprocal.

101

On a property of certain pythagorean triples

Julio R. Bastida, Dept. of Math., FAU, Boca Raton, FL 33431

A pythagorean triple that can be obtained from a primitive one by a factor representable by the quadratic form $x^2 + xy + y^2$ is shown to generate solutions of certain diophantine equations of order 4 and 8.

102

On the SIMULATION of MESHES by K-ARY HYPERCUBES

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Dept. of Computer Science, University of Texas at Dallas, Richardson, TX 73083.

In this paper we present simulations of mesh connected networks by k-ary hypercube machines. We show how to embed an $[m \times p]$ grid G into the smallest hypercube that has at least as many points as G, called the *optimum-hypercube* for G. In order to minimize simulation time, we derive embeddings that assign the nodes of G to nodes of its optimum hypercube that keep grid-neighbors as close as possible in the hypercube. The maximum distance between images of grid-neighbors is called the *dilation* of the embedding. Without loss of generality, we may assume that $\lceil \frac{m}{k^{\lfloor \log_k(m) \rfloor}} \rceil \le \lceil \frac{p}{k^{\lfloor \log_k(p) \rfloor}} \rceil$. (In the case where this latter inequality is not satisfied, one would simply interchange the m for the p.) We show that there is a dilation $r+2$ embedding of 2-dimensional grids into their optimum k-ary hypercubes, where

$$r = \begin{cases} \frac{m}{k^{\lfloor \log_k(m) \rfloor}} & \text{if } m \equiv 0 \ (mod\ k^{\lfloor \log_k m \rfloor}) \\ \lceil \frac{m}{2k^{\lfloor \log_k(m) \rfloor}} \rceil & \text{otherwise} \end{cases}$$

Key words: hypercube, meshes, embeddings, graph, dilation.

103

On the Minimum Cost of Sum Graphs

Gerd H. Fricke*, Wright State University
Frank Harary, New Mexico State University

Let $G = (V,E)$ be a graph with vertices $V = \{v_1, v_2, \dots, v_n\}$. Then G is a sum graph if there exists a set of distinct, positive integers $L = \{a_1, \dots, a_n\}$, where a_i is assigned to v_i such that $v_i v_j \in E$ if and only if $a_i + a_j \in L$.

If G is a sum graph then the sum cost $sc(G)$ is the minimum sum $\sum_{i=1}^{n} a_i$, where the minimum is taken over all valid assignments L. We show the following: Let G be the complete bipartite graph K_{2n} minus one edge together with an isolated node. If $n \geq 4$ then $sc(G) \leq n^2 + 4n + 9$ and the minimum cost cannot be attained by using 1 as one of the node values.

104

Face-vertex incidence matrix of a cubic planar graph

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In this paper we classify the cubic planar graphs with respect to the rank of their face-vertex incidence matrix. We also give several applications of this matrix.

References

[1] H. Oral, Constructing self-dual codes using graphs, J. Combin. Theoryh B, (to appear).

105

A new approximation algorithm for the Steiner tree problem

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The Steiner tree problem in networks is the problem of finding the shortest tree connecting a given set of nodes in a connected undirected distance network. Karp proved that the problem is NP-Complete. For this problem we present an approximation algorithm which runs in polynomial time and produces a near optimal solution. We also investigate its performance in comparison with well established techniques.

Keywords: Steiner tree, Networks, Approximation Algorithm

106

Rooted maps on surfaces.

E. Rodney Canfield, University of Georgia

Let S be a connected compact 2-manifold without boundary. A map is a graph G embedded in S in such a way that each maximal connected component of S-G is simply connected; that is, each face is topologically a disk. A map is rooted if an edge, a direction along the edge, and a side of the edge are distinguished. In this paper, a recursion given elsewhere by Bender and Canfield is used to obtain some new closed formulas for various generating functions. Tables of the first few coefficients of these power series are computed. Both orientable and non orientable surfaces are considered.

107

RESOLVABLE MENDELSOHN DESIGNS WITH BLOCK SIZE FOUR

F. E. Bennett*, Mount Saint Vincent University
Zhang Xuebin, Nanjing Architecture and Civil Engineering Institute

Let v and k be positive integers. A (v,k,1)-Mendelsohn design (briefly (v,k,1)-MD) is a pair (X, B) where X is a v-set (of __points__) and B is a collection of cyclically ordered k-subsets of X (called __blocks__) such that every ordered pair of points of X are consecutive in exactly one block of B. A necessary condition for the existence of a (v,k,1)-MD is $v(v-1) \equiv 0 \pmod k$. If the blocks of a (v,k,1)-MD can be partitioned into parallel classes each containing v/k blocks where $v \equiv 0 \pmod k$ or (v-1)/k blocks where $v \equiv 1 \pmod k$, then the design is called __resolvable__ and denoted briefly by (v,k,1)-RMD. It is known that a (v,3,1)-RMD exists if and only if $v \equiv 0$ or $1 \pmod 3$ and $v \neq 6$. In this paper, it is shown that the necessary condition for the existence of a (v,4,1)-RMD, namely $v \equiv 0$ or $1 \pmod 4$, is also sufficient, except for $v = 4$ and possibly excepting $v = 12$. These constructions are equivalent to a resolvable decomposition of the complete symmetric directed graph K_v^* on v vertices into 4-circuits.

108

Load Balancing Graphs II

Hesham H. Ali*, Computer Science University of Nebraska at Omaha
Alfred Boals, Naveed A. Sherwani, CS, Western Michigan University

We introduced the concept of load balancing graphs as a modeling tool for load balancing of networks in case of node failures. Load balancing graphs model networks that allow a job to be shifted to other processors without re-executing the entire job leading to minimal interruption to the network and increase in overall efficiency.
In this paper we generalize the concept of load balancing graphs. We define a graph to be (m,n)-Generalized Load Balancing(GLB) if the adjacency set of each vertex v is dominated by a set of m vertices called cover set v or Cv. Where Cv = (Adj(v) U Adj(Adj(v)) - v) and n refers to its diameter.
We show that a GLB(m,n) graph can be constructed for any m>1, n >= 1. It is also shown that arbitrary large graphs can be constructed for a given m, n. We investigate connectivity, eccentricity and regularity properties of these graphs. We also compare different existent networks with GLB(m,n) graphs.

WEDNESDAY, FEBRUARY 14, 1990 9:00 A.M.

109

NEW BANDWIDTH REDUCTION ALGORITHMS

C. GowriSankaran,* Dawson College, Montreal,
J. Opatrny, Concordia University, Montreal.

The most successful bandwidth reduction algorithms for graphs are algorithms based on generation of a level structure of small width for a given graph. We present here two bandwidth reduction algorithms which combine the level structure of a pseudo-diameter of a graph with a minimization of the widths of level structures of the connected components of the complement of the pseudo-diameter. Such a treatment of subgraphs, which is absent in existing algorithms, results in level structures of reduced widths for many classes of graphs. The first one of our algorithms uses a recursive approach whereas the second one uses a separator set to compute a level structure of a subgraph.

Key words: graph bandwidth, bandwidth reduction algorithms, level structure.

110

Almost-Planar Graphs

Bradley S. Gubser, Louisiana State University

Kuratowski's theorem, perhaps the most well-known theorem of graph theory, states that K_5 and $K_{3,3}$ are the only non-planar graphs G for which both $G\backslash e$ and G/e are planar for all edges e of G. An *almost-planar* graph is a non-planar graph G for which $G\backslash e$ or G/e is planar for all edges e of G. In this talk, we characterize almost-planar graphs.

111

A PROBLEM ON COMPLETE DIFFERENCE SETS

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Israel

If D^* is the set of nonzero elements of the complete plane difference set $\{0, d_1, d_2, \ldots, d_k\}$ then at least $k^2 - 2$ members of Z_{k^2+k+1} can be written as the sum of exactly three, not necessarily different members of D^*. It is conjectured that 0 is always representable in this way and also that for sufficiently large k, perhaps $k > 8$, every element g of Z_{k^2+k+1} is representable. We could not decide whether $4D^* = Z_{k^2+k+1}$ is true but it turned out that $5D^* = Z_{k^2+k+1}$ provided $k > 2$.

112

COMBINATORIAL PROBLEMS IN DISTRIBUTED LOAD BALANCING

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We define load balancing for multiprocessor systems where load balancing decisions are based strictly on local load information (i.e., the loads of the immediate neighbors). Continuing work presented two years ago at this conference, we derive combinatorial results about the behavior of this type of load balancing. In particular, we are interested in problems relating to the initial load α: We derive the minimum α for a given system that guarantees that every node has nonzero load once the system is balanced and we determine the effect that the size of α has on the number of load balancing steps required for obtaining a balanced system. The last problem depends heavily on the system topology; while in general, the number of steps grows as α grows, there exist certain graphs where balancing can be done in time independent of the size of α.

Key words: Distributed computing
Load balancing
Balanced graphs

113

On the Complexity of Recognizing Tough Graphs

Douglas Bauer*, Stevens Institute of Technology, Hoboken, NJ 07030
Aurora Morgana, University of Rome, Rome, Italy
Edward Schmeichel, San Jose State University, San Jose, Ca 95192

We consider the relationship between the minimum degree δ of a graph and the toughness of a graph. Let $t \geq 1$ be a rational number. We first show that if $\delta(G) \geq tn/t+1$, then G is t-tough and the degree bound is best possible. On the other hand, we show it is NP-hard to determine if G is t-tough, even for the class of graphs with $\delta(G) \geq ((t/t+2)-\epsilon)n$.
Key Words: toughness, minimum degree

114

DOUBLE CYCLE COVERS AND THE PETERSEN GRAPH, II

Paul A. Catlin, Wayne State University, Detroit MI 48202

Any graph G with no cut edge and with at most 13 edge cuts of size 3 either has a certain type of double cycle cover ($G \in \mathcal{S}_3$, as defined below), or it is contractible to the Petersen graph. This improves earlier results of Jaeger and of Catlin. Define $\mathcal{S}_3$ to be the family of graphs G for which there is a partition $E(G) = E_1 \cup E_2 \cup E_3$ such that the set of odd-degree vertices of G equals the set of odd-degree vertices of $G[E_i]$, $1 \leq i \leq 3$. Note that $G \in \mathcal{S}_3$ if and only if G has a double cycle cover in which the cycles can be partitioned into 3 edge-disjoint sets that each induce subgraphs, namely $G - E_i$ ($1 \leq i \leq 3$), whose components are all eulerian. (The Petersen graph is not in $\mathcal{S}_3$.) The family $\mathcal{S}_3$ is also the family of graphs having a 4-flow.

Keywords: double cycle cover, 4-flow, eulerian, Petersen graph

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ABSTRACT

57 6-SETS ON 46 POINTS WITH NO PAIR OF POINTS REPEATED

JOHN WESLEY BROWN UNIVERSITY OF ILLINOIS

Marshall Hall Jr. attempted the construction of a BIBD(46,69,9,6,1) from a FPP of order 8. This resulted in 39 6-point lines, 24 5-point lines and 6 4-point lines on 42 of the 46 points with each occurring 9 times.

We have filled in the missing 4 points consistently in 18 of the 30 partial lines, and have shown that this is the best that can be done. Further, we have shown that the 39 6-point lines cannot be completed to a full design by any rearranging of the points in the partial lines.

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Message-Id: <9002022244.AA18421@csvax.caltech.edu>
To: hoffmanf@servax.bitnet
Subject: URGENT: abstract for conference

A Construction for Fault-tolerant Communication Multi-trees

David Schweizer (*) and Dan Ashlock, Caltech

We exhibit a construction for an infinite family of fault-tolerant communication networks. The construction can be viewed as a superposition of trees. The construction has been experimentally verified to perform better than similar networks with random interconnections. Many combinatorial problems arise naturally from the construction, including orthogonal Hamilton cycle decompositions of the complete graph on p vertices, p prime.

keywords: fault-tolerant, communication, graph theory

117

CYCLE IDEALS OF DISTANCE-REGULAR GRAPHS

A. D. Meyerowitz / Florida Atlantic University / Boca Raton, FL 33431

The finite geometries underlying certain distance regular graphs have been used to elucidate eigenvectors and eigenspaces in the incidence algebra. In work with Paul Terwilliger, the opposite approach is being taken; starting from a distance regular graph, eigenvectors are sought which will reveal the point-sets of underlying geometric structures. The plan is to recognize these vectors as zeros of an ideal associated with the cycles in the graph. The goal of this talk is to introduce this program with a well chosen example or two.

keywords: algebraic graph theory, distance-regular graphs,association schemes, finite geometries

118

On separation of graphs of low genus

Ljudmil Aleksandrov[1] and Hristo Djidjev[1,2]

[1]*Center of Informatics and Comp. Techn., Bulgarian Academy of Sci.*
[2]*School of Computer Science, Carleton University, Ottawa*

Abstract. Let G be an n vertex connected graph of genus g and T be a spanning tree of G. Any simple cycle of G that contains exactly one edge not in T is called a *fundamental* cycle of G (regarding T). It is well known that there exist $2g$ fundamental cycles of G whose removal leaves behind a planar graph. Using this fact and the Lipton-Tarjan lemma for separation of planar graphs with one fundamental cycle it follows that there exist $2g+1$ fundamental cycles of G that divide G into components of no more than $(2/3)n$ vertices each. The main result of this paper shows that only $g+1$ cycles suffice to divide G in the above manner. The tool used to obtain this result is a special sparse weighted graph associated with the genus g embedding of G. As a corollary we prove that a set C of no more than $\sqrt{6g+6}\sqrt{n}$ vertices of G exists that divide G into components of no more than $(2/3)n$ vertices each. This is an improvement over the best previous $\sqrt{12g+6}\sqrt{n}$ bound on the size of C. Similar results are obtained for graphs embedded on nonorientable surfaces.

119

ON THE EXISTENCE OF H DESIGNS

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Let m and r be positive integers. Let $\mathcal{T}$ be a collection $\{T_1, T_2, \ldots, T_m\}$ of disjoint r-element sets, and let S denote their union. By a *transverse* of $\mathcal{T}$ we mean a subset of S that meets each T_i in at most one point. An $H(m,r,4,3)$ design on $\mathcal{T}$ is a collection of 4-element transverses of $\mathcal{T}$, called blocks, such that each 3-element transverse is contained in exactly one of them. We show that for $m > 3$, $m \neq 5$, an $H(m,r,4,3)$ design exists if and only if certain obvious divisibility conditions are satisfied, and that an $H(m,6,4,3)$ design exists for every positive integer $m \neq 3$. This last result is needed for the study of minimal coverings of triples by quadruples.

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M. R. EMAMY-K.
P. GUAN
I. J. DEJTER
UNIVERSITY OF PUERTO RICO
MATHEMATICS DEPARTMENT
RIO PIEDRAS CAMPUS

ON THE FAULT TOLERANCE IN A 5-CUBE

Here, a new proof for verification of the minimum number $\alpha(5)$ of faulty edges in a 5-cube such that no square is fault free, is given. This proof is more geometrical and leads to some algorithmic method for the computation of upper bounds for $\alpha(n)$ at large values of n.

WEDNESDAY, FEBRUARY 14, 1990 11:30 A.M.

121

CARDD (Computer-Aided Representative-graph Determiner and Drawer

Michael Wayne Powell, Teresa Haynes, Linda M. Lawson,

East Tennessee State University

CARDD is an expert system and graph drawer that will determine and display a graph representative of a family of graphs defined by a set of invariants. CARDD uses forward chaining inference to construct a graph satisfying all user specified invariants (whenever such a graph exists) and creates a display that can be interactively modified. We present a preliminary report on the on-going development of CARDD with the current version supporting eight invariants: number of nodes, number of edges, maximum clique, node independence number, chromatic number, maximum degree, minimum degree, and domination number.

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The Jordan Curve Theorem

Harold Bell, University of Cincinnati and University of Miami

An elementary proof of the Jordan Curve Theorem is presented.

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Generalizations of the Equivalence between Complete Sets of Orthogonal Latin Squares and Affine Planes

Charles Laywine*
Brock University

Gary Mullen
The Pennsylvania State University

Using affine resolvable designs and complete sets of mutually orthogonal frequency squares and hypercubes, we provide several generalizations of Bose's equivalence between affine planes of order n and complete sets of mutually orthogonal latin squares of order n. We also characterize those complete sets of orthogonal frequency squares and hypercubes which are equivalent to affine geometries.

Key words. Affine resolvable BIB designs, mutually orthogonal latin and frequency squares and hypercubes, affine geometries.

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Reliability of Broadcasting in Communication Networks

Kenneth A. Berman* and Jerome L. Paul, University of Cincinnati

Let G be a *communication network* with vertex set V of size n and edge set E of size m, and let t be a "time function" for E, i.e. t is a mapping from E to the positive integers such that $t(e_1) \neq t(e_2)$ if e_1 and e_2 are adjacent edges of G. For $s, v \in V$, a *t-relay* path from s to v is a path $P = u_0e_1u_1e_2 \ldots u_{p-1}e_pu_p$, $u_0 = s$, $u_p = v$ such that $t(e_1) < t(e_2) < \ldots < t(e_p)$. The *arrival time* $\alpha(P)$ of P is defined by $\alpha(P) = t(e_p)$ (the arrival time of the trivial t-relay path consisting of the single vertex s is defined to be 0). Note that information known to s will arrive at v at time $\alpha(P)$ if it is successively relayed from u_{i-1} to u_i at time $t(e_i)$ during communication e_i, $i = 1,2,\ldots,p$. The *t-broadcast multiplicity of v from source s*, $\beta_{sv}(t)$, is the number of distinct arrival times over all the t-relay paths from s to v. Let $\sigma_s(t) = \sum_{v \in V} \beta_{sv}(t)$. It is obvious that $\sigma_s(t) \leq 2m$. In this paper we show that $\max_t \{ \sigma_s(t) \} = 2m - n + 2$, and present an $O(m)$ algorithm for finding a time function t which acheives this maximum. We also show that the problem of finding a time function t such that $\beta_{sv}(t) \geq b_v$, $v \in V$, for a given set of positive integers b_{sv}, $v \in V$, is NP-complete.

Keywords: communication network, reliability, NP-complete

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A Fast Generalized Approximate String Matching Algorithm

Lawrence V. Saxton* and Nalin Wijesinghe, University of Regina

The problem of looking up a string in a directory or dictionary when the exact spelling of the string is not known is called the *approximate string matching problem*. It is useful in finding similar tokens in information retrieval applications and in correcting spelling mistakes and other errors in text processing applications. For the normal *exact string matching problem*, a binary search technique is known to solve the problem in O(log n) time, for a directory of n elements. For the approximate string matching problem, a dynamic programming method has been developed, which unfortunately has worst case complexity linear in the product of the length of the search string and the sum of the length of all the strings in the directory. This makes the algorithm too time consuming for the very large directories associated with its projected applications. We describe a two phase algorithm, called the *approximate binary search*, to solve the problem. The first phase is a modified binary search which restricts the size of the directory to be searched. The second phase is adapted from the dynamic programming method and is applied only to the restricted directory. The algorithm has been used successfully in the development of an online spelling corrector.

Keywords: approximate search, dynamic programming, error correction, similar strings, spelling correction, string matching

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THE COMPLEXITY OF CLUSTERING IN PLANAR GRAPHS

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ABSTRACT

The problem of finding the maximum clique in a graph is one of the most fundamental graph problems and has many applications. In a planar graph, although the absence of any large cliques makes the clique problem trivial, there may still be a need for finding a dense subgraph. The idea of a cluster is useful. An h-cluster in a graph is a set of h vertices which maximizes the number of edges in the graph induced by these vertices. We show that the connected h-cluster problem is NP-complete on planar graphs.

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Some Balanced Incomplete Block Design Constructions

Malcolm Greig, University of British Columbia

Key words and phrases: BIBD
AMS 1980 subject classifications: Primary 05B05

The object of this paper is the direct construction of (v,k,1) difference families for BIBD's with v a primepower. Over 600 designs with k > 6 and v < 32768 are given, with a few small examples of (v,k,2) families. Some computational aspects are discussed.

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The Effect of Improving Links on Network Reliability

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One of the fundamental considerations in the design of communication networks is reliability. In this paper, we investigate the problem of identifying the links of a network with the property that their improvement will lead to maximum improvement of network reliability, given that only a fixed number of links can be improved.
The network reliability criteria considered here is the expected number of connected pairs of nodes in the network.

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ON MINIMUM BREADTH FIRST ODD-CYCLE COVER

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Abstract

For a spanning tree of a graph the set of non-tree edges each of which gives an odd basis cycles is an odd-cycle cover of the graph. The odd cycle cover has been earlier used to obtain maximum cut of a graph. A breadth-first search of a graph gives a spanning tree and a corresponding odd-cycle. One can have distinct spanning trees and odd-cycle cover corresponding to each vertex as the root. Among these odd-cycle covers there is one with minimum size. We present results giving conditions that makes a breadth first search give such an odd cycle cover. Interestingly, for some graphs, the odd-cycle cover of minimum size corresponding to a breadth first search may be the minimum odd-cycle cover of the graph.

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More on Distance One Realizable Graphs

R. Prairie, C. Purdy*, and G. Purdy
University of Cincinnati

Abstract. Two years ago the last two authors reported on a project aimed at determining conditions under which a graph on n vertices could be "realized" in the plane (with edges allowed to cross) with all edges having length one. We have completed the classification of all graphs on nine or fewer vertices in terms of distance one realizability, and we are presently finishing the classification of graphs on 10 vertices. Besides producing some interesting geometric examples, this project has been a challenging exercise in

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*Minimum Iterated Closure – Dual Closure Sequences in Matroids

by Safwan Akkari, Indiana-Purdue at Fort Wayne, Indiana
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(*visiting* Indiana-Purdue at Fort Wayne, Indiana).

Let M be a matroid on set S with rank function ρ and let ρ^* be the rank function of the dual matorid M^*. For set $A \subseteq S$, define ΣA as the smallest set containing A which is closed in both M and M^*, or equivalently as the largest set obtainable from A by applying alternatingly the closure σ and dual closure σ^*. Let $\alpha(M)$ be the size of a smallest subset A of S such that $\Sigma A = S$. Let $\ell(M)$ be the smallest number of closures and dual closures necessary to expand into S some set A of size $\alpha(M)$. Then we show that $\ell(M) \leq 2[\min(\rho,\rho^*) - \alpha(M)] + 1$. Examples are given showing this bound is achieved for matroids M of any specified rank ρ.

Key words: matroid, closure, dual closure, electrical circuit

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Abstract

Bhaskar Rao Ternary designs and Applications

D. G. Sarvate, College of Charleston

Generalized Bhaskar Rao n-ary Designs with elements from abelian groups are defined. This paper studies a special case of Generalized Bhaskar Rao n-ary Designs called Bhaskar Rao Ternary Designs. A Bhaskar Rao Ternary Design, X, is a v x b matrix of 0's, ±1's and ±2's such that the inner product of any two rows is 0 and the matrix obtained by replacing each entry of X by its absolute value is the incidence matrix of a Balanced Ternary Design. Applications of these designs to the construction of Partially Balanced Ternary Designs and Balanced Ternary Designs are presented. Some construction methods and necessary conditions for the existence of Bhaskar Rao Ternary designs are given.

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DOMINATION ON COCOMPARABILITY GRAPHS

Dieter Kratsch, Friedrich-Schiller-Universitat, Jena, GDR
*Lorna Stewart, University of Alberta, Edmonton, Alberta, Canada

Two well-known graph classes which admit many polynomial time algorithms for NP-complete graph problems are the interval and the permutation graphs. But if one generalizes both in a natural way to chordal and comparability graphs, respectively, things change and many problems remain NP-complete. In particular, this is the case for the well-known domination problem and several of its variants.
Both classes of graphs have the additional property of being contained in the cocomparability graphs, i.e., the complements of comparability graphs.
We present polynomial time algorithms for the dominating set problem and several of its variants on cocomparability graphs.

Keywords: domination, cocomparability graphs, graph algorithms

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Self-dual embeddings of Cayley graphs for certain nonabelian groups

Nora Hartsfield
Western Washington University

Orientable and nonorientable self-dual embeddings of Cayley graphs for the alternating, symmetric, and metacyclic groups are constructed.

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"Embedding Butterfly Networks into Butterfly Networks"

Dr. Said Bettayeb and David A. Hoelzeman*, Louisiana State U

In this paper, embeddings of any size butterfly network with and without wrap-around connections into another butterfly network of any size with and without wrap-around connections are presented. The embeddings given show that any size butterfly network B(m) without wrap-around connections can be embedded into any size butterfly network B(n) with and without wrap-around connections with optimal dilation one. The remaining embeddings show that any size butterfly network Bw(m) with wrap-around connections can be embedded into any size butterfly B(n) with and without wrap-around connections with dilation at most three. The load factor for all of these embeddings is shown to be one when m<n, two when m=n and the guest graph does not have wrap-around connections and the host graph does have wrap-around connection, and O(2**(m-n)) when m>n.

Key terms: Butterfly Network, embedding, dilation, load factor.

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BLOCKING SETS IN FINITE PROJECTIVE SPACES AND UNEVEN BINARY CODES

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A. Beutelspacher defined a *t-blocking set* of PG(m,q), $m \geq t+1$, to be a subset B of PG(m,q) such that any (m-t)-flat meets B and no t-flat is contained in B. Beutelspacher has established a bound for the cardinality $|B|$ of a t-blocking set B that is tight when q is a square. For $q = 2$ and $t = 1$ his bound gives $5 \leq |B| \leq (8 - 2\sqrt{2})\cdot 2^{m-3}$. I am able to prove: THEOREM 1. *There exists a 1-blocking set* B *in* PG(m,2), $m \geq 3$, *of cardinality* n *if and only if* $5 \leq n \leq 5\cdot 2^{m-3}$. This is deduced from the following theorem on linear codes: THEOREM 2. (W. E. Clark, L. A. Dunning and D. G. Rogers) *If* $1 \leq r \leq 3$, *then there exists no uneven* [n,n-r,4] *binary code. If* $4 \leq r$, *then there exists an uneven* [n,n-r,4] *binary code iff* $r + 1 \leq n \leq 5\cdot 2^{r-4}$. (A binary code is *uneven* if it contains a codeword of odd weight.)

Keywords:
blocking set, finite projective space, binary linear code, odd codeword

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A Linear-Time Algorithm for Constructing Minimum Height B-Trees

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B-trees is a useful data structure for external file organization. When the file indexes are represented by a B-tree, the number of external storage (such as disk) accesses required for operations as search, insertion and deletion is equal to the height of the B-tree. Thus, minimum height B-trees are desirable. We present an $O(n)$-time algorithm for constructing minimum height B-trees, given that all keys are in sorted order. For the case that keys are unsorted, our algorithm can be used to construct minimum height B-trees in $O(n \log n)$ time. Our algorithms are optimal in time.

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On a Problem of Robertson and Seymour

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As a forerunner to proving that every 2-connected graph has a 2-representative embedding in some surface, Robertson et al suggest proving that every 2-connected planar graph has a 2-representative non-planar embedding. This is done here for cubic planar graphs.

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PRODUCT NETWORKS: A FAMILY OF GRAPHS FROM GROUP MODEL

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Akers and Krishnamurthy developed a formal group-theoretic model, called the ***Cayley graph model***, for designing and analyzing processor/communication interconnection networks. Using this model they developed two classes of interconnection networks, called the ***star graph*** and the ***pancake graph***. Analysis has shown that star graphs are markedly superior to the widely used n-cube. In this paper using the concept of Ceyley graph model, we propose a new family of interconnection networks. We study their such properties as degree, diameter, connectivity, and fault-tolerance. Both the n-cube and the star graph are members of this new family of networks. For a given size, the n-cube has the largest diameter and the star graph has the smallest diameter; others have diameters between the n-cube and the star graph.

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Lottery Schemes and Covers

by
M. Morley & G. H. J. van Rees*

At present, there are many lottery schemes for sale purporting to offer the gambler a guaranteed win on his "lucky" numbers for a small outlay. In mathematical terms, they are selling a (v,k,t)-cover with a close to minimal number of blocks. A (v,k,t)-cover is a set of blocks(sets) in which the varieties(elements) are taken from a v-set,every lambda set occurs in some block and all blocks contain k varieties. C(v,k,t) is the # of blocks in a minimal (v,k,t)-cover. We prove that C(12,6,4) = 42 and C(11,5,3) = 21. This is done with the help of a construction that gives C(4t-3,2t-1,t-1) <= (2t-2)!/((t-1)!(t-1)!)+1, t >= 3. Also a new recurrsive procedure is developed that gives a quite good upper bound and helps prove that C(12,8,4) = 12 and C(13,9,5)=19.

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From: kuplinsk@math.rutgers.edu (Julio Kuplinsky)

FINDING HARD-TO-COLOR GRAPHS BY COMPUTER

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Julio Kuplinsky, Dimacs and Dept. of Mathematics, Rutgers

A usual measure of the performance of graph coloring heuristics is performance guarantee (Johnson, Wigderson). This is an asymptotic quantity, which basically measures the ratio of the number of colors used by the heuristic to the chromatic number in the worst case (WHEN n IS LARGE). This gives no information on the performance of the heuristic on small graphs. We try to understand the behavior of several well-known heuristics by 1) finding small (in some cases smallest) graphs on which the algorithm does not provide the chromatic number and 2) experimenting on the computer with randomly generated graphs from which vertices and edges are removed one at a time until a "minimal" graph of this nature is obtained. In this discussion a distinction should be made between graphs where the heuristic always performs badly from those where this behavior depends on choices made during the execution of the algorithm.

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Straight Ahead Cycles in Drawings of Eulerian Graphs

Heiko Harborth, Techn. Univ. Braunschweig, West Germany

Drawings D(G) are mappings of the vertices of a graph G into different points (also called vertices) of the plane, and mappings of the edges of G into Jordan curves such that two curves (also called edges) have at most one point in common, either a crossing or an endpoint (vertex).
Paths in drawings are considered which pass through all vertices straight ahead, that means, there are equal numbers of remaining edges to left and to right of the path. Then every drawing of an Eulerian graph is partitioned into disjunct straight ahead cycles. It is conjectured that for every Eulerian graph a drawing exists which determines a straight ahead Eulerian cycle. Proofs for special classes of graphs are given. Moreover, drawings with many small straight ahead cycles are discussed.

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A colourful algorithm for the maximum clique problem

Wendy Myrvold, University of Victoria

Using the observation that if a graph G can be properly vertex-coloured using $k-1$ colours then G has no clique of order k, we develop a simple new algorithm for the maximum clique problem. Experimental evidence is given to show that the performance of this algorithm compares very favourably to algorithms suggested by previous authors.

WEDNESDAY, FEBRUARY 14, 1990 4:40 P.M.

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From: <JAMISON@HARTFORD>

Optimal Algorithms for P4-Sparse Graphs

Beverly Jamison*, University of Hartford and
Stephan Olariu, Old Dominion University

A graph G is p4-Sparse if no set of five vertices in G induces more than one chordless path of length three. P4-sparse graphs are a natural generalization of both cographs and P4-reducible graphs, finding applications in scheduling, clustering, computational semantics, and memory organization. We propose optimal algorithms that, given a P4-sparse graph as input, solve the following problems: coloring, maximum clique, largest stable set, clique cover, domination, strong domination, number of maximum size cliques, number of transitive orientations, number of maximum stable sets, and center, among others.

Keywords: Algorithms, Graph Coloring, Perfect Graphs.

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Spanning eulerian subgraphs

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A graph H is collapsible if for every even subset $R \subseteq V(H)$, H has a spanning connected subgraph with R as the set of the odd-degree vertices. The reduction of a graph G can be obtained from G by contracting all maximal collapsible subgraphs of G. It has been noted that knowing the reduction of small graphs is useful in finding spanning eulerian subgraphs and hamiltonian line graphs. In this note, we study the reduction of small graphs. As an application, we show that if G is a simple graph with order n and $\kappa'(G) \geq 3$, and if n is large and if for every edge $uv \in E(G)$, $d(u) + d(v) \geq \frac{n}{6} - 2$, then either G has a spanning eulerian subgraph or G can be contracted to the Petersen graph. For 3-edge-connected graphs, this is stronger than a conjecture of Benhocine, Clark, Köhler and Veldman.

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On Minimal Paths through a Shuffle-exchange Network

Roy Fuller, University of Arkansas

There has been recent theoretical interest in finding Cayley graphs with small average minimal path lengths, allegedly because such graphs might be utilized in computer network design. However, the graphs actually used in practical networks do not have to be associated with any mathematical structure. Shuffle-exchange (SE) networks are practical networks that are not based on a (single) mathematical structure.
Despite their long history as computer networks, no minimal path analysis seems to have been published. We present an algorithm for finding all minimal paths between two nodes of a SE network. Average minimal path lengths are calculated for SE networks of size up to 2 to the 20th.

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The SetPlayer System for Implicit Set Manipulation

Dave Berque and Mark Goldberg,* Rensselaer

The power set, POW(A), of a given a set A is the set of all subsets of A. Power sets and their derivatives are a standard object of research in Combinatorics. Unfortunately, no conventional software system allows efficient manipulation of power sets. In this talk, we present an interactive command-driven software system called SetPlayer, which in addition to supporting a variety of standard set operations, is capable of expressing, storing, and manipulating formulas representing power sets.

We describe several computataional and combinatorial problems related to the design of any system of this type, and give an illustration of the current version of SetPlayer. Our goal is to show, that the system can be used as a tool during the empirical stage of research in Combinatorics.

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A LINEAR ALGORITHM FOR FINDING THE PERIOD OF A STRING OF CHARACTERS

P.S. Nair, Creighton University, Omaha

J. Opatrny*, Concordia University, Montreal.

For a given string of characters $u=a_1a_2\cdots a_n$ we define the period of u as the shortest prefix $v=a_1a_2\cdots a_j$ of u such that $u=v\ v\ \cdots\ v\ w$ for some prefix w of v. The problem of finding the period of a given string occurs in the area of pattern recognition. We present a linear algorithm for finding the period, and show that this algorithm also can be used to find a detailed structure of the period of the string.

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VERTEX-TRANSITIVE GRAPHS ON A SURFACE

Laszlo Babai, Univ. of Chicago and Eotvos Univ., Budapest

THEOREM. Given an arbitrary surface S, we prove that all but a finite number of connected vertex transitive graphs embeddable on S admit a symmetrical embedding (automorphisms extend to the surface) on a surface with nonnegative Euler characteristic (sphere, projective plane, torus, or the Klein bottle).

In particular, apart from the cycles and four families of stripe-like graphs, all but a finite number of these graphs are factors of semiregular plane tilings.

The size of the exceptions is bounded by a constant times the Euler characteristic of S.

This theorem generalizes previous results by V. Proulx and T. W. Tucker. The proofs use different, geometric methods. A limit construction leads to an infinite planar graph, and cases are distinguished according to the connectivity and the number of ends of this limit graph.

The result was independently obtained by C. Thomassen.

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On Total Covers of Graphs

Yousef Alavi*, Liu Juiqiang and Songlin Tian, Western Michigan University
Jianfang Wang, Academia Sinica, Beijing, Zhongfu Zhang, Lanzhou Institute, China

A total cover of a connected graph G is a subset of $V(G) \cup E(G)$ which covers all elements of $V(G) \cup E(G)$. The total covering number $\alpha_2(G)$ of a graph G is the minimum cardinality of a total cover in G. Alavi et. al, in a paper on the subject, had posed the conjecture that for a connected graph G of order P, $\alpha_2(G) \leq \lceil \frac{p}{2} \rceil$. Here we give some properties of connected graphs which have a total covering number $\lceil \frac{p}{2} \rceil$ and prove the above conjecture.

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Efficient Computer Algorithms for Construction of Triple Systems

Maurice Eggen, Trinity Univ., and Roger Eggen*, Univ. of North Florida

Recent published articles indicate a renewed interest in triple systems and their applications in computer science. The role of combinatorial designs in computer science is not limited to graph theory. A t-(l,m,n) design (V,B) is an n-set V together with a collection of m-subsets B of V with the property that each l-subset occurs in an m-subset exactly t-times. When l=2, m=3, and t=1, the systems are called Steiner triple systems. It is well known that Steiner triple systems exist if and only if $n \equiv 1,3 \pmod 6$. When l=2 and m=3 the systems are simply called triple systems (for any t). In this paper the authors present, for those interested in developing new applications, efficient computer algorithms for construction of any triple system, and provide a complete discussion of when such systems exist. The constructions are based on latin squares. The paper presents special methods for the case t=1, t=2, t=3, etc., and provides algorithms suitable for implementation in any computer language. as well as actual implementations in the C programming language. Algorithm analysis as well as special considerations for parallel processing hardware are discussed.

Keywords: Algorithms, design, combinatorial design, parallel algorithms

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On Characterizing Independence and Overlap Graphs in Testing VLSI Circuits

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In testing VLSI circuits, the problem of identifying the maximum set of independent faults is known to be NP-complete. In this paper, we characterize the independent and overlapped faults and study the corresponding graphs. Using the characterization of the independence and overlap graphs, we develop a heuristic for solving the problem of finding maximum independent faults. We also specify the special cases in which the algorithm finds the optimal solution.

Key Words : VLSI testing, independence graphs, overlap graphs, automatic test generation.

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The Number of Non-zero Entries in Recursively Defined Tables

Neal Brand*, Saial Das, and Thomas Jacob, University of North Texas

Let p be a prime number. The structure of Pascal's triangle modulo p is well known. From this structure it is easy to compute the number of non-zero entries in Pascal's triangle modulo p up through row n. We show this number has the same order as the number of non-zero entries through row n for the table of Stirling numbers modulo p of either kind and for variations of Pascal's triangle modulo p.

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Chromatic uniqueness of certain bipartite graphs

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Abstract: It is proved that if G is obtained from a complete bipartite graph $K_{m,m}$ or $K_{m,m+1}$ by deleting d disjoint edges, $0 \leq d \leq m$ and $m \geq 2$, then G is chromatically unique; the cases $d=0$ and $d=1$ were already known. The case $d=2$ gives a partial answer to Problem 12 stated by Koh and Teo, "The Search for Chromatically Unique Graphs" (to appear). of determining the χ-uniqueness of graphs obtained from the complete $K_{m,n}$ graph removing two of its edges; in fact, we prove that a graph obtained from $K_{m,m}$ or $K_{m,m+1}$ by deleting any two edges is chromatically unique. We also prove that for each $k \geq 2$, the graphs obtained from $K_{m,m+k}$ removing any two of its edges is χ-unique if m is large enough (depending on k).

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A Parallel Algorithm for Maximum Matching in Cographs

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The problem of finding Maximum Matching is: "find a subset of edges of maximum cardinality such that no pair of edges in the subset have a vertex in common". The problem has known randomized parallel NC algorithm for general graphs, however, a deterministic parallel algorithm for the general graphs is still unknown. In this paper we present a parallel NC algorithm which finds out Maximum Cardinality Matching when the input is restricted to a special class of graphs, namely Cographs.

Complement Reducible Graphs, or Cographs in short, are the graphs that can be formed starting from a vertex under the closure of operations of union and complement. They are the underlying graphs of transitive series parallel digraphs and are also the comparability graphs of posets known as multitrees. Cographs have a canonical representation called cotree, which forms the basis of the proposed algorithm.

The algorithm is designed to run on a Concurrent Read Concurrent Write Parallel RAM (CRCW P-RAM) model of computation. The algorithm generalizes the parallel tree techniques and requires $O(\log^2 n)$ time and uses $O(n^2)$ processors.

Keywords: Cographs, Maximum Matching, NC-algorithm

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Some graph problems and the realizability of metrics by graphs

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The problem of the graph realization of metrics has been much studied. We give two examples to show that special realizations of the simplest structure, the tree-realizations, can be used for characterizing the solutions to some graph problems.

Consider a metric (S,d) and a weighted connected graph $G=(V,E,w)$ with positive edge weights (lengths) and $S \subseteq V$. Let $dist(G;i,j)$ denote the length (weight sum) of the shortest path in G from vertex i to j according to w. The quantity $dist(G;i,j)$ is called the distance between vertices i and j induced by G. Graph G is a realization of the metric (S,d) if $dist(G;i,j)=d(i,j)$ for all i and $j \in S$. We show that the minimum-cost Hamilton circuit can be easily found for a metric (S,d) if it is induced by a tree (on a possibly larger set of vertices). This tree is often referred as the distance tree of the metric. The total length of the optimal circuit is twice the total length of the tree. Finally we prove that all Hamilton circuits of S have the same length with respect to the metric distance d if and only if the metric is induced by a star.

The second problem deals with the shortest path trees of a given connected graph with positive edge weights. The structure of these spanning trees usually depends on the choice of the vertex from which the shortest distances must be found to all other vertices in the graph. We discuss an interesting case: what happens if there is a common shortest path tree for all vertices. We give a characterization of this case in the terms of the tree-realizability of the induced metric associated with the original graph. It implies that the minimum spanning tree gives the unique common spanning tree.

Keywords: Hamilton circuits, shortest path trees, graph realization of metrics, tree-realizable metrics

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Sets With No Integral Solutions to $x + y = kz$

Fan Chung, Bellcore and John Goldwasser*, West Virginia University

A set of positive integers having no solutions to $x + y = z$ is called a sum-free set. We generalize this notion. For k a fixed positive integer, what is the largest subset of the set of all positive integers less than or equal to n which has no solutions to $x + y = kz$? For $k=3$ and $n \geq 23$ we show that the odd integers less than or equal to n is the unique largest such subset (verifying a conjecture of Erdos). For $k \geq 4$ the nature of the problem changes considerably and optimal solutions have long strings of consecutive integers.

Key word: Sum-free set

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Set T-Colorings of Graphs

Barry A. Tesman, Dickinson College

Given a graph G, a finite set of nonnegative integers T, and a positive constant k, assign to each vertex x of $V(G)$ a set $S(x)$, of size k, of distinct positive integers (called colors). The assignment is restricted by:

$$\{x,y\} \in E(G) \Rightarrow |x_i - y_j| \notin T, \text{ for all } x_i \in S(x),\ y_j \in S(y).$$

Such an assignment is called a T k-coloring of G. The criteria for efficient T-colorings are similarly defined for T k-colorings. Recent results and open problems are presented concerning these criteria.

Key words: graph coloring, T-coloring, T k-coloring

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Numerical Solution of Optimal control Problems with Bounded State Constraints

John Gregory, Cantian Lin, Southern Illinois University-Carbondale

We present new, efficient and accurate numerical methods to solve a class of optimal control problems.

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d-regular graphs with every edge on t triangles, for certain values of $2d - t$

Peter D. Johnson Jr., Auburn University

For a simple graph $G, u, v \in V(G)$, and $e = uv \in E(G)$, let $N(u)$ denote the set of vertices adjacent to u; let $t(e) = |N(u) \cap N(v)|$, the number of triangles e sits on, in G, and $J(e) = |N(u) \cup N(v)|$; let $m = |E(G)|$ and $n = |V(G)|$; let $t = \frac{1}{m} \sum_{e \in E(G)} t(e)$ and $J = \max_{e \in E(G)} J(e)$.

The following generalization of a famous theorem of Mandel and others is fairly well known: $m \leq \frac{n(t+J)}{4}$, with equality if and only if G is regular and $t(e)$ is a constant function of e. [The famous theorem is: $t = 0 \Rightarrow m \leq \frac{n^2}{4}$ with equality if and only $G = K_{\frac{n}{2},\frac{n}{2}}$.] It is known that when $J = n$, equality holds if and only if G is complete r-partite, for some r, with partite sets of equal size. Here we find the extremal graphs when $J = n - 1$, and when $J = n - 2$.

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On the discrepancy of 3 permutations

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Keywords: discrepancy

A well known conjecture of J. Beck states that the hypergraph consisting of the intervals of three permutations of a set has discrepancy at most K for some absolute constant K. (We regard a permutation as an ordering of the elements of the underlying set.) The discrepancy of the hypergraph $\mathcal{H}$ on the set X is $\mathrm{disc}(\mathcal{H}) = \min_{\chi:X\to\{-1,1\}} \max_{H\in\mathcal{H}} |\sum_{x\in H} \chi(x)|$, i.e. split the underlying set in two pieces so that the sets in the family $\mathcal{H}$ are split as evenly as possible. (For a reference on this problem as well as on discrepancy in general we refer the reader to Spencer, J.H., *Ten lectures on the probabilistic method.* SIAM, Philadelphia, 1987.) The justification for the conjecture is that for two permutations the discrepancy is 2.

In this paper we show an upper bound of $O(\log|X|)$, not only for three, but for any constant number of permutations. The proof also gives an efficient algorithm for coloring the underlying set.

166

Constraints on the chromatic sequence for trees and graphs.

Ewa Kubicka, Emory University

This paper deals with the chromatic sequence associated with a specific graph G, that is the sequence in k of the minimum sums of colors taken over all proper colorings of the graph G using exactly k colors. It is shown that for trees this sequence is constrained, in fact it is inverted unimodal, while for arbitrary graphs it is unconstrained. This means that for any permutation of numbers 2 through n, a graph G can be found whose chromatic sequence after sorting into nondecreasing order realizes the given permutation.

167

COMPUTER EVALUATION OF SINGULAR AND HYPERSINGULAR INTEGRALS

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Abstract

There are numerous monographs on numerical integration [1], [2]. They discuss very thoroughly various methods of integration of non-singular functions, but deal very superficially with the problem of integration of singular ones. Though some theoretical results have been published [3], we are unaware of any standard subroutine available for the singular integrals evaluation. This lack of the standard software resulted in some cases in ignoring the singularities during computations [4] which definitely undermined the accuracy of the numerical results. The development of such a subroutine is the purpose of this paper.

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An algorithm and a standard subroutine are developed for the evaluation of singular integrals over arbitrary two-dimensional domains. The integrand is a product of a Green's function with another function having a week singularity at the boundary of the domain. Formulae are derived for an accurate estimation of the integral in the neighbourhood of the singularities. The integral over the rest of the domain is evaluated by a library subroutine. The software developed is applied to a study of some contact problems of the theory of elasticity for non-classical domains.

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168

BICLIQUE COVERS and BOOLEAN FACTORIZATIONS of TOURNAMENT CODES

Greg Bain and J. Richard Lundgren*
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John S. Maybee
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There are several interesting practical problems involving codes that use the ternary alphabet $\{0,1,*\}$. A code C of length k over the alphabet $\{0,1,*\}$ is called a tournament code if, for any two distinct codewords $\underline{a}, \underline{b}$, exactly one of the following two conditions is true: (i) there exists j such that $(a_j, b_j) = (0,1)$; (ii) there exists j such that $(a_j, b_j) = (1,0)$. If (i) holds we say $\underline{a} \to \underline{b}$; this defines the tournament. The maximal value of $|C|$ over all tournament codes of length k is called $t(k)$. The actual values of $t(k)$ are unknown, but upper and lower bounds have been found.

We have shown that this problem is the dual of finding the minimum boolean rank of an $n \times n$ tournament matrix or finding the minimum size of a biclique cover of a tournament. We have investigated the structure of tournament matrices, and by breaking certain tournament matrices into specially constructed blocks, we have found useful boolean factorizations of the tournament matrices. In particular, we show that if an $n \times n$ tournament matrix $T = AB$ where A is $n \times k$, and B is $k \times n$, then we can construct a code C of length k satisfying $|C| = n$. Here all the arithmetic is boolean. The methods used provide an easy means of generating several infinite series of codes satisfying $\lim_{n\to\infty} n/k = \infty$ and strongly suggest that we may have already found the best possible factorizations.

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Divided differences and combinatorial identities.

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We show that some basic properties of divided differences can be used to obtain combinatorial identities. This is done applying general propositions, like the chain rule, to particular functions and taking suitable values for the arguments of the divided differences. In this way we get numerous identities, for example

$$\sum_{k=1}^{p} (-1)^k \begin{bmatrix} 2p \\ p+k \end{bmatrix} k^n = 0 \quad , \quad p \ge 2,\ n \text{ even} \ ,\ 2 \le n \le 2p,$$

(problem 1330, Math. Mag. 62, No. 4, 1989), and

$$\sum_{j=0}^{n} z_j^m \prod_{\substack{k=0 \\ k\neq j}} \left[1 - \frac{z_k}{z_j} \right]^{-1} = h_m(z_0, z_1, \ldots, z_n) \ ,$$

where h_m is the complete symmetric polynomial of order m. This is a generalization of the identity used by Good in his proof of Dyson's conjecture.

This work is an extension of our paper *Interpolation and combinatorial functions, Studies in Applied Math.* 79: 69-92, 1988.

170

Total Colourings of Dense Graphs

Hugh Hind, University of Southern Mississippi

Given a graph G, a total colouring of G is a mapping from $V(G) \cup E(G)$ into some set of colours such that no two adjacent elements and no two incident elements of $V(G) \cup E(G)$ are assigned the same colour.

Recent results of Flandrin and Li, giving new sufficient conditions for the existence of a Hamiltonian cycle, will be mentioned. These results will then be used to show that at most $\Delta(G) + 2$ colours are needed to totally colour a graph G with $\Delta(G) = c|V(G)|$ for c sufficiently large. This answers a question posed by Hilton.

KEYWORDS: total colouring, Hamiltonian cycles

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A fast NC recognition algorithm for P4-reducible graphs

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Abstract

The P4-reducible graphs are a natural generalization of the well-known class of cographs, with applications to scheduling and clustering. More precisely, the P4-reducible graphs are exactly the graphs none of whose vertices belong to more than one chordless path with three edges. A remarkable property of P4-reducible graphs is their unique tree representation up to isomorphism. In this talk we present a parallel algorithm to recognize P4-reducible graphs and to construct their corresponding tree representation. Our algorithm runs in O(log n) time using $O(\frac{n^2+mn}{\log n})$ *processors in the EREW-PRAM model.*

172

Sum Distance in Digraphs

Songlin Tian, Western Michigan University

For vertices u and v of a digraph D, the sum distance sd(u, v) between u and v is the sum of the lengths of a shortest u – v path and v – u path in D. If D is strong, then this distance is a metric. The s–eccentricity se(v) of a vertex v in D is the maximum value of sd(v, w) over all vertices w in D. The s–eccentricity set of D is the set $\{se(v) \mid v \in V(D)\}$. The sets of positive integers which are s–eccentricity sets of oriented graphs are characterized. Other results and questions concerning sum distance will also be presented.

173 A comparison theorem for multinomial coefficients.

W. Wei[1], J. Shapiro[2], D.F. Hsu[3], V. Frants[1]

Abstract.

It is proved that if two nonnegative integral vectors $(m_1, m_2, \ldots, m_k)$ and $(n_1, n_2, \ldots, n_k)$ satisfy $(m_1, m_2, \ldots, m_k) \prec (n_1, n_2, \ldots, n_k)$, then $\binom{n}{n_1, n_2, \ldots, n_k} \le \binom{n}{m_1, m_2, \ldots, m_k}$ and the inequality becomes equality if and only if $(m_1, m_2, \ldots, m_k) = (n_1, n_2, \ldots, n_k)$. As a direct consequence of this, we can easily obtain the maximum value

$$\max_{n_1+n_2+\ldots+n_k=n} \binom{n}{n_1, n_2, \ldots, n_k} = \frac{n!}{(\lfloor \frac{n}{k} \rfloor !)^{k-r} \left((\lfloor \frac{n}{k} \rfloor + 1)! \right)^r}$$

where $n = k\lfloor \frac{n}{k} \rfloor + r$, $0 \le r < k$. This value gives an explicit upperbound for the cardinality of an s-system defined by Sterner.

1) Sichuan University, Chengdu, China
2) City University (Baruch College), New York
3) Fordham University, New York

174 **Forbidden Trees and On-Line Graph Coloring**

by H. A. Kierstead, S. G. Penrice*, and W.T. Trotter
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We survey recent results related to the following conjecture, which was proposed independently by Gyarfas and Sumner: For every tree T there exists a function $f_T(\omega)$ such that for every graph G with clique size ω which does not induce T, $\chi(G) \le f_T(\omega)$, where $\chi(G)$ denotes the chromatic number of G.

In particular, we study on-line coloring algorithms in relation to the conjecture. An on-line vertex coloring algorithm is one which takes each vertex of a graph from a list and assigns it a color without any information about subsequent vertices on the list; the algorithm may not change a vertex's color once it has been assigned. We say that a family of graphs F can be effectively colored if there exists an on-line algorithm which colors every graph G in F using a number of colors which depends only on $\omega(G)$. Our main results show that for certain trees T, the family of graphs which do not induce T can be effectively colored.

Key Words: Tree, clique size, chromatic number, on-line coloring algorithm.

175 Maximizing a Submodular Function by Integer Programming

Heesang Lee and George L. Nemhauser*

The problem of maximizing a submodular function generalizes many well-known combinatorial optimization problems including max-cut and uncapacitated facility location. We formulate the problem of maximizing a submodular function as an integer program and derive classes of valid inequalities for the convex hull of feasible solutions.

176 Graphs Whose Vertex Independence Number is Unaffected by Single Edge Addition or Deletion

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Corner Brook, Newfoundland

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Halifax, Nova Scotia

Doug Rall*, Furman University,
Greenville, S C

For any graph G and any edge e in G the vertex independence number of the graph $G - e$ is at least as large as that of G. Let E^+ (E^-) represent the class of graphs whose vertex independence number remains the same upon the addition (deletion) of any edge. We give a constructive characterization of E^+ and E^- trees.

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An Analytic Approach to a Combinatorics Problem

John W. Hilgers

Michigan Technological University

Consider the split multinomial distribution
$\sum_{P_n^+}\binom{N}{k}a^k - \sum_{P_n^-}\binom{N}{k}a^k = \Delta$ where a,k,u are indexed sets, $\{a_i\}$ are probabilities, u is a given M-vector, $P_n^{\pm}$ are the index sets, $\{k_1,\ldots,k_M: k_i \geq 0$ integers, $\sum k_i = N,\ \pm k\cdot u > 0\}$.

These differences arise in the study of high sum wins games. An integral representation for Δ affords insights into its behavior, particularly as $N \to \infty$, that are not otherwise readily available.

178

COLORING MIXED GRAPHS

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Pierre Hansen, Rutcor and School of Business, Rutgers University.
Dominique deWerra, Ecole Polytechnique Federale de Lausanne.

A mixed graph has some arcs and some edges. We discuss the problem of coloring a mixed graph. The color of the head of an arc should be less than the color of the tail, while the color of the ends of an edge should be different. This concept can serve as a model for scheduling jobs some of which have precedence constraints while others have simultaneity constraints. We characterize those mixed graphs that admit a coloring and give bounds on its chromatic number in terms of the chromatic number of the underlying graph. We discuss an O(m^2) algorithm for coloring mixed trees. Finally, we give computational results obtained with a branch-and-bound algorithm on arbitrary mixed graphs.

Key words: vertex coloring, mixed graph.

179

AUGMENTATION OF STATE GRAPHS AND EASILY TESTABLE REALIZATIONS OF SEQUENTIAL MACHINES*

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Recently Das *et al.* proposed an efficient procedure of augmentation of the state graph of an arbitrary sequential machine for testable realization by using extra input symbols and output terminals to the original machine. This modification gives a *reduced* upper bound on the lengths of the checking experiments, and also permits coverage of faults that may cause an increase in the number of machine states. The procedure was an extension of an earlier method discussed by Fujiwara *et al.* In this paper an alternative method of augmentation is developed for testable realizations of sequential machines. The suggested modification not only provides an improved bound on the lengths of the checking experiments, but at the same time retains the advantages of the earlier technique in respect of faults coverage.

*This research was supported in part by the Natural Sciences and Engineering Research Council (NSERC) of Canada under Grant A 4750.

Key Words

State graph, sequential machines, testable realizations, checking experiment design.

180

Some Generalizations of the Line Graphs Based on Edge-rotations

Gary Chartrand, Héctor Hevia*, and Elzbieta B. Jarrett, Western Michigan University

For a graph G of size $q \geq 1$ and an integer n with $1 \leq n \leq q$, the n–subgraph distance graph $L_n(G)$ of G is that graph whose vertices correspond to the edge–induced subgraphs of size n in G and where two vertices of $L_n(G)$ are adjacent if and only if one of the corresponding subgraphs can be obtained from the other by the rotation of exactly one edge. Since $L_1(G)$ is the line graph of G, the graphs $L_n(G)$ are also called generalized line graphs. Other generalizations of line graphs can be formulated based on edge–rotations. Some of these are presented and related properties are discussed.

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SOME CONSTRUCTIONS OF SELF-ORTHOGONAL HAMILTONIAN PATH DECOMPOSITIONS OF $2K_n$

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Let $2K_n$ denote the complete multigraph in which there are two edges joining every pair of distinct vertices. Heinrich and Nonay (1985) and later Horton and Nonay (to appear) have asked when $2K_n$ can be partitioned into Hamiltonian paths so that any two paths have exactly one edge in common. Such a decomposition is called a self-orthogonal Hamiltonian decomposition of $2K_n$ and the existance of these objects allows the solution of certain related problems. The authors mentioned above have described a method of construction that depends upon finding 2-sequencings (i.e. terraces of groups) with additional properties. New constructions of 2-sequencings of the required type are given.

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Some Results in Finite Graph Ramsey Theory

David S. Gunderson* and Norbert Sauer, University of Calgary and Voltěch Rödl, Emory University, Atlanta.

For given finite graphs G and H, when can we assert the existence of, or lack of, a Ramsey graph F with $F \longrightarrow (G)_r^H$? If there exists an ordering $\leq$ of G so that all H-subgraphs of $(G, \leq)$ are order-isomorphic, then as an easy consequence of the Ramsey theorem for ordered hypergraphs [e.g. Nešetřil, Rödl, 77,83], such an F exists. It is natural to conjecture, and in fact is claimed by some, that this is not only a sufficient condition for the existence of such an F, but necessary. We show the situation to be more complicated and present a counterexample. We also characterize those triples G,H and r for which a Ramsey F exists.

KEYWORDS: Ramsey, graph, ordered hypergraph, chromatic number.

183

COVERING GRAPHS

Hal Fredricksen Naval Postgraduate School

A cover C of a graph G is an independent subset of the vertices of G with the property that the closest distance of any point in G - C to the cover is at most 1. For the binary n-cube with Hamming distance, covers are related to single error detecting/correcting codes. We show covers of other graphs. In particular, we discuss covers of the de Bruijn graph and bounds for minimum and maximum cardinality of covers. Algorithms and contructions to achieve the best known covers are given.

184

Planarity of Double-Rotation Graphs

Gary Chartrand, Héctor Hevia, and Elzbieta B. Jarrett*, Western Michigan University

For distinct subgraphs F and F' of the same size in a graph G, we say that F can be rotated into F' if there exists a pairing of the edges of F with F' such that every pair consists of two adjacent edges. The double-rotation graph $R_2(G)$ of a connected graph G is defined as that graph whose vertices correspond to the connected subgraphs of G having size 2, and where two vertices of $R_2(G)$ are adjacent if and only if one of the corresponding subgraphs can be rotated into the other. We describe the graphs G for which $R_2(G)$ is planar.

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THE COMPLEMENT OF TWO LINES IN A FINITE PROJECTIVE PLANE

Lynn Margaret Batten University of Manitoba

Let S be a finite linear space which has parameters the complement of two lines in a projective plane of order n. We discuss conditions under which S re-embeds in a projective plane of order n.

Key words: projective plane

186

Disjoint Maximal Independent Sets

David M. Arasmith, Emory University

Consider the property P: For a graph, G, and a function, $f: V(G) \rightarrow \{red, blue\}$, either there is a pair $u, v \in V(G)$ such that $(u,v) \in E(G)$ and $f(u) = f(v) = blue$ or there is a maximal independent set, $M \subset V(G)$, such that $f(u) = red \ \forall u \in M$. The central question: For what graphs does P hold? When restricted to certain classes of graphs the question can be answered. It can be shown that a tree that does not possess a *treble-core* has the property P. Along with this characterization of all *good* trees there exists a polynomial algorithm for determining simultaneously whether a tree contains a pair of adjacent blue vertices, a red MIS, or neither. In contrast, determining if a general bipartite graph is good can be shown to be *NP-complete*.

The property P has roots in a property satisfied by certain collections of partially ordered sets. Namely, for any red/blue coloring of a poset, S, there is either a blue maximal chain or a red maximal antichain. Property P is analogous to this property for posets of height 2.

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Degree sums along edges

Hong-Jian Lai, West Virginia University

Let G be a simple graph with n vertices. We proved that if for any edge $xy \in E(G)$, $deg_G(x) + deg_G(y) \geq (n+2)/2$, then for any pair of vertices u, v in G, there is a (u,v)-trail that is dominating in G.

188

EXTREMAL PROBLEMS AND THE DIAMETER OF DELETED GRAPHS

P. K. Card, Hendrix College
R.J. Faudree*, Memphis State University
C. M. Wong, Grinnell College

For positive integers d and m let G_n be a graph of order n and let v_i, $1 \leq i \leq m$, be an arbitrary collection of m elements of $V(G_n)$. Let $D_{d,m}$ be the property that the graph $G_n - \{v_1, v_2, ..., v_m\}$ has a path of length at most d between each pair of vertices. The problem of determining the smallest number of edges in a graph G_n with property $D_{d,m}$ and the structure of such graphs will be considered. For some specific d and m precise results will be obtained and bounds will be given in other cases. Also the extremal problem for the corresponding edge deletion property $D'_{d,m}$ and the structure of such extremal graphs will also be considered.

189

Symmetric Moore Geometries

Frederick J. Fuglister, John Carroll University

Definition. *A generalized Moore geometry of diameter d with parameters a, b, c is a finite incidence structure in which*

(1) each point lies on $a+1$ lines;
(2) each line contains $b+1$ points;
(3) the diameter is d;
(4) there are no circuits of length less than $2d$; and
(5) every two points at distance d are joined by exactly $c+1$ paths of length d.

It is known [Fuglister, 1987] that the diameter of a non-trivial generalized Moore geometry can be at most 13. Various authors have obtained smaller bounds in the special cases $c = 0$, $c = a$, $c = b$, and $b = 1$.

Define a generalized Moore geometry to be *symmetric* if $a = b$. We show that the diameter of a non-trivial symmetric Moore geometry can be at most 3. Some remarks on possible parameters for symmetric Moore geometries of diameters 2 and 3 are made.

190

Recursive Edge Colorings of Recursive Graphs

by G. Benson, W. Gasarch, T. Grant*, U. of MD. at College Park

Abstract

A recursive (computable) graph is a (possibly infinite) graph whose vertex and edge sets are computable. We study edge 2-colorings and recursive edge 2-colorings of such graphs. We have shown that there exists a recursive graph that can be edge 2-colored in a triangle-free manner (i.e., no triangle is monochromatic) but no recursive edge 2-coloring will be triangle-free. The problem of determining whether or not a recursive graph has a triangle-free coloring is Σ_3 complete. Our results also apply for highly recursive graphs.

191

A Generalization of Dirac's Theorem for K(1,3)-Free Graphs

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R. J. Gould, Emory University
M. S. Jacobson, University of Louisville
L. M. Lesniak, Drew University
*T. E. Lindquester, Rhodes College

It is known that if a 2-connected graph G of sufficiently large order n satisfies the property that the union of the neighborhoods of each pair of vertices has order at least $\frac{n}{2}$, then G is hamiltonian. In this paper, we obtain a similar generalization of Dirac's Theorem for $K(1,3)$-free graphs. In particular, we show that if G is a 2-connected $K(1,3)$-free graph of order n such that the cardinality of the union of the neighborhoods of each pair of vertices is at least $\frac{(n+1)}{3}$, then G is hamiltonian. We also investigate several other related properties in $K(1,3)$-free graphs such as traceability, hamiltonian-connectedness, and pancyclicity.

192

Bounds on the "Growth Factor" of a Graph

David C. Fisher, University of Colorado at Denver

Consider the following problem: *How many j letter words, T_j, can be made from n letters if certain pairs commute?* For example, if we use the letters a, b and c, and if $ab = ba$ (but $ac \neq ca$ and $bc \neq cb$), then there is 3 words with 1 letter (a, b, c), 8 words with 2 letters (aa, ab, ac, bb, bc, ca, cb, cc), 21 words with 3 letters, 55 words with four letters, etc.

Let G be the graph with a node for each letter and edges connecting commuting letters. For $k \geq 1$, let c_k be the number of complete k node subgraphs in G. Let the *dependence polynomial* of G to be

$$f(z) \equiv 1 - c_1 z^1 + c_2 z^2 - c_3 z^3 + \cdots.$$

Let $r(G)$ be the reciprocal of the smallest real root of the dependence polynomial. Then it can be shown that $\lim_{j\to\infty} T_j^{1/j} = r(G)$. Since it describes the asymptotic growth rate of T_j, $r(G)$ is called the *growth factor* of G. (The dependence polynomial has also been used to derive new bounds on the number of triangles in a graph, and to find new properties of other graph polynomials).

Bounds are given for the growth factor of a graph with n nodes and e edges. The lower bounds are sharp, since they are exact for a complete w-partite graph on $k, k, \ldots, l$ nodes where $k \geq l$. The upper bounds are not as satisfactory.

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193 Local group actions on generalized quadrangles

by S. E. Payne & L. A. Rogers*
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Let F be a flock of a quadratic cone in PG(3, q) and S(F) the associated generalized quadrangle of order (q^2, q), q odd. If F′ is derived from F by a method of L. Bader, G. Lunardon and J. A. Thas, then S(F) is isomorphic to S(F′). The semilinear collineations of PG(3,q) preserving F correspond to a certain group of collineations of S(F). All collineations of the generalized quadrangle S associated with W.M. Kantor s likeable planes $(q = 5^e)$ are determined and used to show (without recourse to the classification of finite doubly transitive groups) that the derived flocks are new (even though the associated generalized quadrangles are not).

Keywords: generalized quadrangles, q-clan, flock.

194 **An Improved Algorithm for the Chromatic Polynomial**

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Abstract: It is well known that computing the chromatic polynomial of a graph G is an NP-complete problem. In this paper the algorithms based on Whitney's Identity found in literature are rewritten recursively, and improved by detecting "hanged cycles", ie. cycles with trees attached to them. We have also employed heuristic techniques. In some cases it is possible to achieve an improvement of 35%.

195 **An inequality for degree sequences**

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We give an upper bound to the power sum of the degree sequence of a simple graph. It implies an integral inequality of unusual character.

196 **Bounds on the Largest Root of the Matching Polynomial**

David C. Fisher and Jennifer Ryan*, University of Colorado at Denver

Let G be a graph with n nodes. For $j = 1, 2, 3, \ldots$, let m_j be the number of matchings on j edges in G. The *matching polynomial* of G is:

$$\mu_G(z) \equiv z^n - m_1 z^{n-2} + m_2 z^{n-4} - m_3 z^{n-6} + \cdots.$$

For $j = 1, 2, 3, \ldots$, let c_j be the number of complete subgraphs on j nodes in G. The *dependence polynomial* of G is:

$$f_G(x) = x^n - c_1 x^{n-1} + c_2 x^{n-2} - c_3 x^{n-3} + \cdots.$$

After a change of variables, the matching polynomial of G is the dependence polynomial of the complement of the line graph of G. We use properties of the dependence polynomial to prove new bounds on the largest root, $t(G)$, of the matching polynomial of G. In particular, we show a monotonicity property holds, namely: $t(H) \le t(G)$, where H is any subgraph of G. This gives rise to many new bounds on $t(G)$ which we compare to known bounds. We give analogous results for weighted graphs.

197 The Generalized Quadrangle with (s,t) = (3,5)

Stanley E. Payne

For q any power of 2 there are generalized quadrangles with parameters (q-1,q+1) and some with parameters (q+1, q-1) which are closely related, sometimes being point-line duals of each other. This connection makes it possible to say a great deal about their spreads and ovoids. For q = 4, giving the unique GQ of the title, it is possible to determine all spreads and ovoids, the (large!) complete collineation group, and a variety of related results.

Key Words: Generalized quadrangle, spread, ovoid, tight set.

198 **Chromatic polynomials of graphs in terms of chromatic polynomials of trees**

Aziz A. Adam, University of the Witwatersrand

Izak Broere*, Rand Afrikaans University

Key words: Graph, colouring, chromatic polynomial.

We study the chromatic polynomial of a graph expressed as a linear combination of the chromatic polynomials of trees. The first few coefficients of these expressions are described, some known results find new formulations and some new ways of calculating chromatic polynomials in this form are developed.

199

A Generalization of Ore's Theorem for Hamiltonian-Connected Graphs

N. Dean, Bellcore, R. J. Gould*, Emory Univ., T. E. Lindquester, Rhodes College

For sets of vertices, we consider a form of generalized degree based on neighborhood unions. Using a neighborhood intersection condition and bounding the generalized degree sum of two independent sets (whose sizes are determined by the intersection condition) of vertices from below, we obtain a direct generalization of Ore's Theorem for hamiltonian-connected graphs. We also consider other highly hamiltonian properties for graphs.

200 **The irredundant Ramsey number s(3,7)**

Johannes H. Hattingh, Rand Afrikaans University

Key words: Ramsey numbers, irredundance

The irredundant Ramsey number s(m,n) is the smallest p such that in every two-coloring of the edges of K_p using the colors red (R) and blue (B) either the blue graph contains an m-element irredundant set or the red graph contains an n-element irredundant set. We develop techniques to obtain upper bounds for irredundant Ramsey numbers of the form s(3,n) and prove that s(3,7) = 18.

201 **A New Species of Local Motion.**

Ray Killgrove and Carl Gordon*, California State University, Los Angeles

Consider the following successor operation on the ordered 5-arcs of a projective plane: If A_0, A_1, A_2, A_3, A_4 is an ordered 5-arc, then its successor is the ordered 5-arc $A'_0, A'_1, A'_2, A'_3, A'_4$, where (for $n = 0, \ldots, 4$) $A'_n = A_{n+1}A_{n+2} \cap A_{n-1}A_{n-2}$ (addition and subtraction modulo 5). In any plane, this is a bijection on the orderd 5-arcs and partitions them into cycles and doubly infinite sequences. In $\mathbf{E}^2$, the successor of a regular pentagon π is a regular pentagon with vertices outside the convex hull of π. The regular pentagons in $\mathbf{E}^2$ lie in doubly infinite sequences. Each of these sequences are *central* in the sense that there is a point C such that for each regular pentagon in the sequence and $n = 0, 1, 2, 3, 4$, A'_n, A_n, and C are colinear. These sequences are also *affinable* in the sense that there is a line k (the ideal line of $\mathbf{E}^2$) such that, for each regular pentagon in the sequence and $n = 0, 1, 2, 3, 4$, $A_{n-2}A_{n+2} \cap A_{n-1}A_{n+1} \in k$ (addition and subtraction modulo 5). A number of questions are considered concerning this species of local motion. For example, in a low order Desarguesian plane there are cycles whose lengths are close to the order of the plane. Do such cycles complete to the plane? Do there exist central and affinable sequences in finite planes? Do cycle lengths serve as distinguishing invariants for these planes?

202 From: nate@thunder.bellcore.com (Nathaniel Dean)

On Bicritical Graphs

Nate Dean, Bellcore, 445 South Street, Morristown, NJ 07960

A graph G is bicritical if the deletion of any pair of vertices of G results in a graph with a perfect matching. Although bicritical graphs, together with certain bipartite graphs, are the fundamental building blocks for graphs having a perfect matching, their structure is still not understood. We derive some results about bicritical graphs with respect to several other graph invariants; for example, every 4-connected, projective-planar (or planar) graph is bicritical.

203 **A Neighborhood Condition Which Implies The Existence Of a Complete Multiparte Subgraph**

Debra Gibi, Tennessee Technological University

Given a graph of G of order n, we define NC_k to be equal to the min $|UN(u)|$ where the minimum is taken over all collections of k independent vertices. Noga Alon, Ralph Faudree and Zoltan Furedi have shown that if G satisfies the neighborhood condition $NC_k > \frac{d-2}{d-1}n$, then for sufficiently large n, G contains a K_d, the complete graph on d vertices. Furthermore, this was shown to be best possible.

We shall show that if G is a graph of order n that satisfies the neighborhood condition $NC_k > \frac{d-2}{d-1}n + cn^{1-1/r}$ for some real number c, then for sufficiently large n, G must contain a $K(r, m_1, \cdots m_{d-1})$, $r \leq m_i$ for each i. Furthermore, if $r = 1, 2$ or 3, this is best possible except for the exact value of c.

204

The Number of Edges in Minimum (K_3,K_p-e,n) Graphs

JING ZOU* and STANISŁAW P. RADZISZOWSKI

Rochester Institute of Technology

The first author is a visiting scholar from Shanghai University of Technology, Shanghai, China

A (G,H,n)-good graph F is defined as a graph F on n vertices not containing G nor $\overline{F}$ containing H. $e(G,H,n)$ is the minimum number of edges in any (G,H,n)-good graph, and any graph achieving it is called a minimum (G,H,n) graph. These concepts are important in the study of Ramsey numbers $R(G,H)$. In this paper we investigate (K_3,K_p-e,n)-good graphs, where K_p-e is a complete graph on p vertices without one edge. In the first part we derive an explicit formula for $e(K_3,K_{k+1}-e,n)$ for $n \leq 3(k-1)$ and obtain inequality $e(K_3,K_{k+1}-e,n) \geq 5n-10(k-1)$ for all positive integers n and $k \geq 7$. In the second part of the paper we derive an explicit formula for $e(K_3,K_{k+1}-e,n)$ for $3(k-1) < n \leq 13(k-1)/4 - sign(k-1 \bmod 4)$ and obtain inequality $e(K_3,K_{k+1}-e,n) \geq 6n-13(k-1)$ for all positive integers n and $k \geq 7$. We notice that $e(K_3,K_{k+1}-e,n) = e(K_3,K_k,n)$ for $n \leq 13(k-1)/4 - sign(k-1 \bmod 4)$ except 7 pairs of nontrivial values n and k, which are listed.

207

On Generalized Perfect Graphs: α-Perfection and Acyclic χ-Perfection

Edward R. Scheinerman and Ann N. Trenk*, The Johns Hopkins University

Given a hereditary family of graphs $\mathcal{P}$ one defines the $\mathcal{P}$-*chromatic number* of a graph G to be the minimum size of a partition $V(G) = V_1 \cup \cdots \cup V_k$ such that each V_i induces in G a member of $\mathcal{P}$. Imitating the standard definition(s) of perfect graph, we define generalized $\chi_{\mathcal{P}}$- and $\alpha_{\mathcal{P}}$- perfect graphs. We show that these concepts can be *different* and that a graph can be perfect (either sense) while its complement fails to be. Our main results are a characterization of $\chi_{\mathcal{P}}$-perfect graphs for 'most' families $\mathcal{P}$ of acyclic graphs and a characterization of $\alpha_{\mathcal{P}}$-perfect graphs for 'most' families $\mathcal{P}$.

t-MATCHING COVERS AND DEGREES

Yu, Qinglin
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Simon Fraser University, Burnaby, BC, Canada, V5A 1S6.

Abstract: A t-matching in a graph G is a set of t independent. The graph G is t-matchings covered if each edge of G belongs to a t-matching. Given positive integer n, Δ and δ we evaluate $f(n,\Delta,\delta)$, the largest integer t so that every graph with n vertices, maximum degree Δ and minimum degree δ, is t-matching covered.

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209 **Hamiltonian cycles and paths of Lyndon words in the n-cube**

Joe Yucas, Southern Illinois University at Carbondale

Hamiltonian cycles and paths are constructed for subsets of Lyndon words in the n-cube. These are then amalgamated to form larger Hamiltonian cycles and paths of Lyndon words in the n-cube.

210 Almost Resolvable Path Decompositions of Complete Graphs

Yu, Min-li
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Simon Fraser University,
Burnaby, B. C. Canada

Abstract: An almost P_k-factor of G is a P_k-factor of G-{v} for some vertex v. We prove that necessary and sufficient conditions for the existence of an almost P_k-factorization of λK_n are $n \equiv 1$ (mod k) and $\lambda nk/2 \equiv o$ (mod k-1).

211 Graphs representing obstruction relations among geometric objects.

Stephan Foldes, GERAD, HEC - Ecole Polytechnique - McGill University

Objects are defined as compact subsets of the plane R^2. For each object, motion rules restrict the possible future positions and shapes of that object. A graph arrow is defined from object A to object B if some motion of A is obstructed (intersected) by B. Under certain conditions this graph is acyclic. This is related to a concept of sequential separability studied in computational geometry. Particular disassembly sequences of the objects correspond to linear extensions of the partial order defined by the obstruction graph.

Keywords: separability, disassembly, obstruction graphs, partial orders.

213

ON DECOMPOSITION OF A BOOLEAN FUNCTION INTO TWO UNATE FUNCTIONS

Jie Wu
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(Key words: boolean function, unate function, Hasse diagram, testable circuit)

In this paper, a necessary and sufficient condition is derived for a boolean function F(x) to be decomposed into two unate functions, $F_1(x)$ and $F_2(x)$, with one being positive and the other being negative. In other words, $F(x) = F_{1(x)} \otimes F_2(x)$, where $\otimes$ represents a specific connective (or relation). It is proved that F(x) can be decomposed if and only if its Hasse diagram does not contain subchains of given structures. This result can be used for structural-level test generation where F(x) is implemented by a logic circuit. Since the processes of sensitization and line justification are easily implementable for unate functions, by using the above result certain type of functions can be decomposed into two unate functions, each of which can be implemented by a circuit that is easily testable.

214

GAME-THEORETIC MEASURES OF CONNECTIVITY

Joseph E. McCanna, University of New Mexico

Measures of a graph's connectedness can be obtained from connectivity-games played on a graph G. These measures can be finer than the usual definition and can reduce to it in special cases. Critical use is made of the concept of the refractory time of a move, which tell how long an opponent must wait before he can un-do the result of a previous move. Various generalizations are also discussed.

215

RATIONAL QUADRANGLES

Arnfried Kemnitz, Technische Universität Braunschweig

A quadrangle is said to be rational if its vertices have mutually rational distances. The set of points with rational distances to the vertices of any given rational triangle is everywhere dense in the plane (Almering 1963). Points with this property are called convenient points of the triangle.

We give a parametric solution for the explicit determination of such convenient points. This approach leads to another, more simple proof of the mentioned result.

216

An Efficient Algorithm for Finding a Longest Dominance Sequence

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Let $S = \{p_i = (x_1(p_i), x_2(p_i), ..., x_d(p_i)) \mid 1 \le i \le N\}$ be a set of points in d-dimensional space E^d. For two points $p, q \in S$, we say that p is *dominated* by q (denoted by $p << q$) if $x_i(p) \le x_i(q)$ for every $i = 1, 2, ..., d$, with strict inequality holds for some i. A *dominance sequence* $\delta = (p_{i_1}, p_{i_2}, ..., p_{i_k})$ of S is a sequence of points $p_{i_1}, p_{i_2}, ..., p_{i_k}$ of S such that $p_{i_1} << p_{i_2} << ... << p_{i_k}$, and the number k of points in the sequence is the *length* of the sequence. A *longest dominance sequence of S* is a dominance sequence of S with the maximum length. In this paper, we present an efficient algorithm for finding a longest dominance sequence of S of N points in E^d. Our algorithm requires $O(N(logN)^{d-1})$ time and $O(dN)$ space. The time complexity of our algorithm can be reduced to $O(Nlogk(logN)^{d-2}+NlogN)$, where k is the number of points in the longest dominance sequence.

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217

On the Existence of Certain Cubic, Triangle-Free Graphs

William A. Waller, University of Houston-Downtown

W. Staton has shown that triangle-free graphs with maximum degree three have independence ratio at least 5/14. There are known to be exactly two cubic, triangle-free graphs of order 14 with independence ratio equal 5/14. It has been an open question if there exists a connected, cubic, triangle-free graph of order 28 with independence ratio equal 5/14. In this paper we present the results of some computer investigations of this problem which verify no such graph exists.

Keywords: independence ratio, cubic graph, triangle-free graph

218

A Characterization of a Class of Generalized Line Graphs

Gary Chartrand, Western Michigan University
Héctor Hevia, Western Michigan University and Universidad Católica de Valparaiso
Elzbieta B. Jarrett, Western Michigan University
Donald W. VanderJagt*, Grand Valley State University

For a graph G of size $q \geq 1$ and an integer n with $1 \leq n \leq q$, the n–subgraph distance graph $L_n(G)$ of G is that graph whose vertices correspond to the edge–induced subgraphs of size n in G and where two vertices of $L_n(G)$ are adjacent if and only if the edge rotation subgraph distance between the corresponding subgraphs is 1. The graph $L_1(G)$ is then the line graph of G. The connected graphs G have been determined for which $L_2(G)$ is planar; those results are extended for $L_n(G)$.

Keywords: line graph, n-subgraph distance graph

219

A HUNDRED CARRIAGES LONG

W D Wallis, Southern Illinois University, Carbondale

The train of a one-factorization is a directed graph which is useful in distinguishing non-isomorphic factorizations. A train for K_{2n} has maximum indegree 2n-1, and one-factorizations which attain this maximum are of some interest. In this talk we describe some constructions for such factorizations.

220

Inclusive Connectivity: A Local Parameter

J. W. Boland* and R. D. Ringeisen, Clemson University

We introduce i-connectivity parameters which are local measures of how "close" a specific graph element is to being a separating element of the graph. This is a direct generalization of vertex cohesion as introduced by Lipman and Ringeisen. Relationships to standard connectivity parameters, interrelationships between i-connectivities and illustrative examples will be presented.

KEYWORDS: Connectivity, Cohesion

221

Cliques and Niche Graphs

Stephen E. Bowser* and Charles A. Cable, Allegheny College, Meadville, PA

In this paper we provide various conditions on the maximal cliques of a graph that insure that it is a niche graph. In particular, we show that any graph whose clique graph is a path of length at least two must be a niche graph. In addition we demonstate that certain low degree trees are niche graphs.

key words: clique, niche graph, tree

222

Computational Complexity of a Generalized Movement Problem

Jitender S. Deogun and Lilu Pan, University of Nebraska
Man C. Kong*, University of Kansas

In a generalized movement model a set of objects are to be scheduled to move from their sources to their destinations in a transport network so as to meet their respective deadlines. The most significant features of this model is the requirement that an object to be scheduled can only move continuously for a bounded amount of time, and then it must break journey at some prespecified nodes with capacity constraints. This requirement of forced interruptions makes our model more realistic in modeling real world transportation problems and renders the classical shortest path approach useless. In this paper, we will formally define this generalized movement problem and investigate the computational aspects of this problem under different sets of assumptions. We show that the generalized movement problem is NP-Complete by showing that some restricted versions of the problem remain NP-Complete. Heuristic and optimal algorithms respectively for the generalized model and restricted versions are developed and analyzed.

223

ON THE INTERSECTION RANK OF A GRAPH

James A. Wiseman

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Rochester, NY 14623

Abstract

This paper studies the dimension of the intersection between the cycle and coboundary groups of a simple graph. Basic equations are derived, which are then refined for bipartite graphs. These are then employed in studying graphs arising from t-designs and circulant matrices.

224

Lower Bounds for Rectilinear Steiner Trees in Bounded Space

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Washington, DC 20057

ABSTRACT

We prove that, in all dimensions $d \geq 2$, the total weight of a worst-case minimal rectilinear Steiner tree in the unit d-cube is at least $n^{\frac{d-1}{d}}$. This is an improvement over the lower bound of $n^{\frac{d-1}{d}}/2$, due to Gilbert and Pollak (1968). Using an upper bound from Snyder (1989), this yields tight bounds on the worst-case weight of a minimal rectilinear Steiner tree in low dimensions.

Our proof uses a generalized version of Hanan's Theorem (1966) that enumerates all possible locations of Steiner points in dimension d. The proof is also the first in which a non-optimal sphere packing attains worst-case asymptotic behavior.

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225

Peripheral and Eccentric Vertices in Products of Graphs

K.B.Reid, Louisiana State University and California State University, San Marcos
Weizhen Gu[*], Louisiana State University

Let G be a simple connnected graph. The *eccentricity* $e(u)$ of a vertex u in G is given by $e(u) = max\{d(u,v) : v \in V(G)\}$. A vertex v is called a *peripheral vertex* of G if $e(v) = dia(G)$. A vertex v is called an *eccentric vertex* of G if $d(v,c) = e(c)$ for some center vertex c of G. Let $P(G)$ and $EC(G)$ denote the sets of peripheral vertices and eccentric vertices of G, respectively. In this paper we discuss these two sets in four products of graphs (cartesian product, the symmetric difference, the disjunction, and the lexicographic product), and characterize pair of graphs so that, in their product, the set of peripheral vertices is the same as the set of eccentric vertices.

226

Sequences, Schedules and Initialization of Unsynchronized Networks

R. E. Newman-Wolfe, University of Florida
S. L. Davis, Kennesaw State College

We investigate properties of sequences that have applications in dynamic, softwire laser networks. If there is no global clock and there is no established communication path between nodes, a set of schedules is needed to tell each node where to direct its transceivers at a given time.

This scheduling problem may be abstracted to a problem involving sequences of k-tuples. Lack of global synchronization may be interpreted as shifting the sequences representing the schedules of the nodes relative to one another. If the n-th k-tuple of sequence i contains j and the n-th k-tuple of sequence j contains i, then these two sequences cross-reference each other at time n, which may be interpreted as the two nodes establishing a link at that time. The property of interest is whether two schedules are guaranteed to cross-reference each other in corresponding positions of their shifted schedules, regardless of the shifts.

227

A Geometry of Linear Dependences Associated to Algebraic Coding Theory

Charles T. Ryan Seton Hall University, South Orange, New Jersey 07079

We investigate a geometry of linear dependences $\sum_{i=1}^{m} F(p_i)$ associated to the Grassmann Varieties G(2,n) where F denotes the classical Plücker embedding. It is a classical result that linear dependences of order three correspond to linear pencils of the Grassmann Variety G(2,n) however an elementary method of arriving at this result will be presented. Linear dependences of order four will be shown to correspond to the non-common points of a pair of intersection linear pencils of G(2,n). Finally taking the images of G(2,4) under the Plücker embedding as the columns of a parity check matrix it will be shown that the total number of linear dependences of order five correspond to a weight enumerator coefficient, Moreover using the geometry of G(2,4) it will be shown that each such dependency corresponds to one of two geometric configurations.

228

End-faithful spanning trees

P.D.Seymour, Bellcore and Robin Thomas*, Georgia Tech

Halin proved that every countable graph has an end-faithful spanning tree and conjectured this result for graphs of all cardinalities. We disprove his conjecture by constructing an infinitely-connected graph in which every spanning tree has a 2-way infinite path, and discuss how restricted (in terms of excluded minors) such a counterexample can be.

229

EVEN TREES

Kurt Barefoot*, New Mexico Tech;
Joseph McCanna, Laszlo Szekely, UNM

We introduce the notion of an even tree and address problems concerning their ennumeration and their existence as spanning trees of given graphs.

230

The Sixteenfold Way through Finite Algebras

Robert W. Quackenbush, University of Manitoba

On page 33 of Enumerative Combinatorics (Volume I), Richard P. Stanley gives a 4x3 table whose rows are the number of functions, injections and surjections (respectively) from an m-set to an n-set subject to the 4 possibilities of the elements of the domain being distinguished or not and the elements of the codomain being distinguished or not. Add a fourth column corresponding to bijections. This is the Sixteenfold Way. Now choose your favorite class of finite algebras; if you are reasonably normal, it will contain countably many isomorphism classes. Let $\{A_n \mid n \geq 1\}$ be a transversal of the isomorphism classes. Form the 4x4 matrix whose (i, j)-entry is the number of orbits of the action of group G_i on the set S_j. Here $S_1 = Hom(A_m, A_n)$; $S_2 = Mono(A_m, A_n)$; $S_3 = Epi(A_m, A_n)$, and $S_4 = Iso(A_m, A_n)$, while G_1 is trivial; $G_2 = Aut(A_m)$; $G_3 = Aut(A_n)$, and $G_4 = Aut(A_m) x Aut(A_n)$. This is the Sixteenfold Way through your class of algebras. I will present matrices for some of my favorite classes of finite algebras.

231

Cycles in Permutation Graphs

Bradley W. Jackson, San Jose State University

A technique for constructing smallest permutation graphs with a given girth is discussed.

232

INDEPENDENCE RATIO OF GRAPH PRODUCTS

P.Hell* (SFU), X.Yu (Vanderbilt), and H.Zhou (SFU).

We observe that the independence ratio of the cartesian product (G power n) has a limit bounded below by the reciprocal of the chromatic number of G and bounded above by the independence ratio of G. We describe several families of graphs in which the limit is either the upper or the lower bound, and we ask whether this is always the case. Other related open problems will be posed.

FRIDAY, FEBRUARY 16, 1990 2:40 P.M.

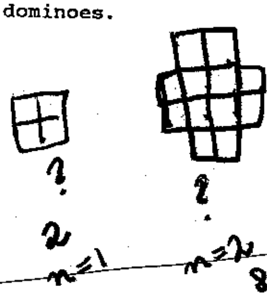
233

An Enumeration Result for Domino Tilings

Greg Kuperberg and Jim Propp (*), University of California at Berkeley.

We describe a one-parameter family of planar regions, called Aztec diamonds, and show that the number of tilings of the order-n Aztec diamond by dominoes is equal to 2 to the power of n(n+1)/2. We know three different proofs of this result (one of them found by Michael Larsen and Noam Elkies); the argument sketched here is the most combinatorial of the three, and involves an operation on tilings that we call ''shuffling''.

Key words: tiling, dominoes.



234

UNIVERSAL CYCLES: WHAT WORKS, WHAT DOESN'T

Glenn H. Hurlbert, Rutgers University

Universal Cycles have their origin in the 1940's with the discovery of de Bruijn Cycles, the most compact way of listing all binary N-tuples. A de Bruijn Cycle is a cyclic listing of 0's and 1's of length 2^N with the property that every binary N-tuple appears exactly once consecutively on the cycle. More recently Chung, Diaconis, and Graham have inaugurated a more general study of compact listings (of the same cyclical type) of other combinatorial structures. These include K-ary N-tuples, permutations of [N]={1,2,...,N}, ordered or unordered partitions of N (or [N]), ordered or unordered subsets of [N] of size K, and K-dimensional subspaces of an N-dimensional vector space over a finite field, among others. Here we concern ourselves with unordered subsets and consider the case K=6 in contrast to the case K=5.

235

OPTIMAL BROADCASTING ON THE STAR GRAPH

Victor E. Mendia and Dilip Sarkar
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Coral Gables, FL 33124

Recently the star graph has been shown as an attractive alternative to the widely used n-cube. Like the n-cube, the star graph possesses rich structure and symmetry as well as fault-tolerant capabilities, but has smaller diameter and degree. However, very few algorithms exist to show its potential as a multiprocessor interconnection network. Many fast and efficient parallel algorithms require broadcasting as a basic step. In this paper, we propose an optimal algorithm for one-to-all broadcasting in the star graph. The algorithm can broadcast a message to N processors in $O(\log N)$ time. The algorithm exploits the rich structure of the star graph and works by recursively partitioning the original star graph into smaller star graphs.

236

On a Class of Kernel-Perfect and Kernel-Perfect-Critical Graphs

Kiran B. Chilukamari, The Ohio State University and Peter Hamburger,*
Indiana University-Purdue University at Fort Wayne

In this paper we present a construction of a class of graphs in which each of the graphs is either Kernel-Perfect or Kernel-Perfect-Critical. These graphs originate from the theory of games (J. Von Neumann and O. Morgenstern [1]). We also find criteria to distinguish Kernel-Perfect graphs from Kernel-Perfect-Critical graphs in this class. We obtain some of the previously known classes of Kernel-Perfect-Critical graphs as special cases of the present construction given here. The construction that we give greatly enlarges the class of Kernel-Perfect-Critical graphs.

Key words: Kernel, Kernel-Perfect, Kernel-Perfect-Critical graphs.

237

A New Family of Distance-Regular Graphs

A.E. Brouwer (Tech. Univ. Eindhoven)
Joe Hemmeter* (Univ. of Delaware)

Let G be a dual polar graph of type $C_d(q)$. Define graph $\hat{G}$ on the same vertex set by having vertices x and y adjacent in $\hat{G}$ if they are of distance 1 or 2 in G. The graph $\hat{G}$ was recently shown, by Ivanov, Muzichuk and Ustimenko, to be distance-regular. In fact, there is a bipartite graph associated with $\hat{G}$. When q is odd, this bipartite graph is new. We will introduce the new graph, and discuss some of its properties.

238

Two-terminal Reliability Bounds based on Edge-Packings by Cutsets

Louis D. Nel *School of Computer Science, Carleton University, Ottawa, Ontario. K1S 5B6*
Heidi J. Strayer *Dept. of Computer Science, University of Waterloo, Waterloo, Ontario. N2L 3G1*
Charles J. Colbourn *Dept. of Comb. and Opt., University of Waterloo, Waterloo, Ontario. N2L 3G1*

Abstract

A simple model of an unreliable network is a probabilistic graph in which each edge has an independent probability of being operational. The two-terminal reliability is the probability that specified source and target nodes are connected by a path of operating edges. Upper bounds on the two-terminal reliability can be obtained from an edge-packing of the graph by source-target cutsets. However, the particular cutsets chosen can greatly affect the bound. In this paper we examine three cutset selection strategies, one of which is based on a transshipment formulation of the k-cut problem. These cutset selection strategies allow heuristics for obtaining good upper bounds analogous to the pathset selection heuristics used for lower bounds. The computational results for some example graphs from the literature provide useful insight for obtaining good edge-packing bounds, and give rise to a new cutset selection problem.

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