RUTH

PROGRAM AND SCHEDULE

Twentieth Southeastern International Conference on

COMBINATORICS GRAPH THEORY COMPUTING

Florida Atlantic University
February 20-24, 1989
Boca Raton, Florida

Co - Sponsored by:
Department of Mathematics
Division of Continuing Education
National Security Agency
U.S. Office of Naval Research

MONDAY, FEBRUARY 20, 1989

REGISTRATION begins at 8:00 AM in the downstairs lobby of the University Center. COFFEE will also be available in the downstairs lobby. GCN (left, or front) and GCS are the two halves of the Gold Coast Room.

	GCN	GCS	109	113
9:00AM	OPENING and WELCOME President POPOVICH Vice President BERRY Deans CARRAHER & HALL			
9:20	BERLEKAMP			
10:20	COFFEE			
10:35	GRAHAM			
11:40	BERLEKAMP			
12:40	LUNCH (On your own--Cafeteria open; many nearby restaurants)			
2:20PM	1 GARGANO	2 FURINO	3 HALSEY	4 KB REID
2:40	5 WIMER	6 McFARLAND	7 LUNDGREN	8 ANACKER
3:00	9 H KIM	10 KW SMITH	11 AKKARI	12 QUACKENBUSH
3:20	13KRISHNAMOORTHY	14 ANDERSON	15 GRABLE	16
3:40	17 GIMBEL	18 BENNETT	19 HOBBS	20 M WU
4:00	21 MATA-MONTERO	22 HAMM	23 TJ REID	24 ELLINGHAM
4:20	25 MUGAVERO	26 STARLING	27 RUSHANAN	28 GRIMALDI
4:40	29 CORNEIL	30	31 HEFNER	32 BOLAND
5:00	33 NEMHAUSER	34 P SMITH	35 DUTTON	36 KUBICKI
5:20	37 AUNG-NAING	38 SAVAGE	39 LAWSON	40 CHARTRAND

6:00 CONFERENCE PARTY at the home of JACK FREEMAN: 741 AZALEA ST, (but park on AURELIA St.) 395-7921

Conference transportation will leave for the motels at 5:45. There will be transportation from the University Center to the party at about 5:50, and from the motels at about 6:15. There will be transportation from the parties back to the motels. As always, we urge car-pooling. It is a pleasant walk to the Freeman home, should you be adventurous.

TUESDAY, FEBRUARY 21, 1989

REGISTRATION HOURS (second floor LOUNGE) 8:15-11:00 AM and 1:30-3:30 PM
GCN (left, or front) and GCS are the two halves of the Gold Coast Room.
FAU Rooms A and C are reached through the second floor Lounge, where COFFEE will be served. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	FAU-A	FAU-C	109
8:30AM	41 JENKYNS	42 HEMMINGER	43 BERMUDEZ	44 PLANTHOLT	
8:50	45 BEASLEY	46 WILSON	47 ELMALLAH	48 WOLDAR	
9:10	49 SHADER	50 CABINISS	51 FULLER	52 LAZEBNIK	
9:30	MILLS				
10:30	COFFEE				
10:45	HALL				
11:50	53 McDOUGAL	54 DEAN	55 BIENSTOCK	56 SHEE	
12:10PM	57 SALI	58 SOTTEAU	59 WALCZYK	60 ULLMAN	
12:30	LUNCH BREAK				
2:00	MILLS				
3:00	COFFEE				
3:20	61 GRAHAM	62 SUTHERLAND	63 BIBELNIECKS	64 ZHENG	65 GRINSTEAD
3:40	66 SIMMONS	67 TAFT	68 FISHER	69 GUAN	70 TW JOHNSON
4:00	71 BATTEN	72 ASHLOCK	73 J RYAN	74CHAIMOVICH	75 KF JONES
4:20	76 CHEROWITZO	77NIEDERHAUSEN	78 TOLMAN	79 CHESTON	80 NOWAKOWSKI
4:40	81 deRESMINI	82 SHAMIR	83 ALAVI	84 TRUNG	85 ROBERTS
5:00	86 EBERT	87 LEWIS	88 HILTON	89 MAKKI	90 McKEE
5:20	91 PAYNE	92 HILGERS	93 GOLDWASSER	94MAGLIVERAS	95 MAYBEE
5:40	96 PLUMMER	97 TROYER	98 SZEKELY	99 WEISS	

6:15 CONFERENCE RECEPTION in the BOARD of REGENTS ROOM on the third floor of the ADMINISTRATION BUILDING

There will be Conference transportation back to the motels at 6:00 PM, returning to the reception about 6:30. There will be transportation from the reception back to the motels.

WEDNESDAY, FEBRUARY 22, 1989

REGISTRATION HOURS (second floor LOUNGE) 8:15-11:00 AM and 1:30-3:30 PM
GCN (left, or front) and GCS are the two halves of the Gold Coast Room.
FAU Rooms A and C are reached through the second floor Lounge, where COFFEE will be served. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	FAU-A	FAU-C	109
8:30AM	101 BURR	102 ARCHDEACON	103 SCHIBELL	104 CT RYAN	105 KUBICKA
8:50	106 VINCE	107 JOHNS	108 SOUKUP	109 BRUALDI	110 BERGSTRAND
9:10	111 CHINN	112 KAINEN	113 FREIMAN	114VELAZQUEZ	115 LALANI
9:30	LOVASZ				
10:30	COFFEE				
10:50	116 REGENER	117 JACOBSON	118 MATHIEU	119 DUNNING	120 STUART
11:10	121 WAGNER	122 STEVENS	123 MAHADEV	124 CUMMINGS	125 EMANY-K
11:30	126 GOLDBERG	127RADZISZOWSKI	128 GB PURDY	129 WE CLARK	130 BOALS
11:50	131FAJTLOWICZ	132BIALOSTOCKI	133TRUSZCZYNSKI	134 EVANS	

12:15PM CONFERENCE PHOTOGRAPH at the OUTDOOR STAGE--We will lead you from the lobby, if you can't find it in your own, but PLEASE PARTICIPATE!!

12:30 LUNCH BREAK

	GCN	GCS	FAU-A	FAU-C	109
2:00	LOVASZ				
3:00	COFFEE				
3:20	136 STUECKLE	137 J WU	138 SCHNEIDER	139LINDQUESTER	140 IWATA
3:40	141 EO HARE	142 MYRVOLD	143 HAMACHER	144 VARMA	145 C LIN
4:00	146 FRICKE	147 COHEN	148 TONG	149 BAREFOOT	150 TOAN
4:20	151 RALL	152 SEN	153 MOON	154 CATLIN	155 CN PURDY
4:40	156 VA RICE	157RAYCHAUDHURI	158 RUBIO	159 GOULD	160 CHOI
5:00	161 ZITO	162 SHIER	163TERWILLIGER	164 PIAZZA	165 KANEVSKI
5:20	166 SMYTH	167 PRITIKIN	168PROSKUROWSKI	169 RAYBURN	170 TIPNIS
5:40					

The CONFERENCE BANQUET will be held at the DEERFIELD BEACH HILTON on HILLSBORO Boulevard at 7:30 PM (seating at 7:15). Conference transportation will be available to the motels at 5:45. There will be transportation to the HILTON from the UNIVERSITY CENTER at 6:15 and from the motels at 6:35. The HILTON's two Cocktail Lounges will be open prior to the BANQUET, for those of you who would like to purchase drinks. Wine will be furnished at the BANQUET. There will be transportation back to the motels after the Banquet.

THURSDAY, FEBRUARY 23, 1989

REGISTRATION HOURS (second floor LOUNGE) 8:15-11:00 AM and 1:30-3:30 PM
GCN (left, or front) and GCS are the two halves of the Gold Coast Room.
FAU Rooms A and C are reached through the second floor Lounge, where COFFEE will be served. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	FAU-A	FAU-C
8:30AM	176 JAMISON	177 MILLER	178 ALDRED	179 GEORGES
8:50	181 HADLOCK	182 PERKEL	183 ZHANG	184 DOMKE
9:10	186 LEISS	187 WILLARD	188 DYMACEK	189 SCHELP
9:30	ERDOS			
10:30	COFFEE			
10:45	SIMS			
11:50	191 TARJAN	192 N WHITE	193 X YU	194 TESMAN
12:10PM	196 SHERWANI	197 WISEMAN	198 BAUER	199 DEJTER
12:30	LUNCH BREAK			
2:00	SIMS			
3:00	COFFEE			
3:20	201 MESNER	202 WALLIS	203 ANSTEE	204 MOSZKOWSKI
3:40	205 YUCAS	206 NOBERT	207 GRIGGS	208 WT TROTTER
4:00	209 CHEE	210 GARRETT	211 FAUDREE	212 ROPP
4:20	213 KRAMER	214 D ZHOU	215 MEYEROWITZ	216 SMALL
4:40	217 KREHER	218 SARRAFZADEH	219 JA HOSKINS	220 ABRHAM
5:00	221RUBIN-THOMAS	222 ISAAK	223 MANVEL	224 SEAH
5:20	225 BRAND	226 SOUMYANATH	227 DESAI	228 S-M LEE
5:40	229 FARLEY	230 WILLE	231 DOOB	232 P-C WANG

The First CONFERENCE LUAU will be held at 6:15 at the outdoor PAVILLION. There will be Conference transportation to the motels at 6:00 PM and back to the LUAU at 6:30. There will be transportation back to the motels after the LUAU.

FRIDAY, FEBRUARY 24, 1989

REGISTRATION HOURS (second floor LOUNGE) 8:15-11:00 AM.
GCN (left, or front) and GCS are the two halves of the Gold Coast Room.
FAU Rooms A and C are reached through the second floor Lounge, where COFFEE will be served. There will be book exhibits in Room 232 from 9:00 to 11:30AM.

	GCN	GCS	FAU-A	FAU-C
8:30AM	233 McKAY*	234 WACHS	235 BOROS	236 DAHLHAUS
8:50		238 McCANNA	239 FISCHER	240 LIPMAN
9:00	237 HIGMAN*			
9:10		242 RABINOWITZ	243 DEO	244 SAZBERG
9:30	LUKS			
10:30	241 RYBA*	(COFFEE AVAILABLE)		
10:50		246 KILLGROVE	247 HARBORTH	248 GOWRISANKARAN
11:00	245 SIMS*			
11:10		250 STERNFELD	251 HIND	252 N ZHOU
11:30	253 STEINER*	254 C GORDON	255 BERMAN	256 G GORDON
11:50		258 JOB	259 DAS	260 HONGXUN
12:00N	257 SCHWENK*			
12:10PM		262 TAPIA-RECILLAS		264 SCHEINERMAN
12:30	261 BASENSPILER*			
1:00	LUNCH BREAK			
2:00	LUKS			
3:00	265 KALLAHER*	266 PEDERSEN	267 SIMION	
3:20		270 ROYLE	271 ALI	
3:30	269 EALY*			
3:40		274 LIANG	275 HARTVIGSEN	

*SPECIAL SESSION ON SOFTWARE FOR DISCRETE MATHEMATICS,
ORGANIZED BY CLIFTON EALY

There will be transportation back to the motels at 4:10.

THANKS FOR COMING!

WE'LL SEE YOU BACK HERE FOR THE TWENTY-FIRST SOUTHEASTERN INTERNATIONAL CONFERENCE ON COMBINATORICS, GRAPH THEORY AND COMPUTING,

< FEBRUARY 12-16, 1990>

By invitation of the organizing committee, the following speakers will present instructional lectures:

Elwyn R. Berlekamp, University of California at Berkeley and Cyclotomics, Inc.
Combinatorial Games
Monday, February 20, 9:20 a.m. and 11:40 p.m.

Ronald L. Graham, AT&T Bell Laboratories and Rutgers University
Universal Cycles for Combinatorial Structures
Monday, February 20, 10:35 a.m.

William H. Mills, Institute for Defense Analyses
(1) *Balanced Incomplete Block Designs with* $\lambda = 1$ Tuesday, February 21, 9:30 a.m.
(2) *Construction of Covering Designs* Tuesday, February 21, 2:00 p.m.

Marshall Hall, Jr., Emory University
Existence and Construction of Designs
Tuesday, February 22, 10:45 a.m.

László Lovász, Eőtvős Loránd University and Princeton University
The Discrepancy of Hypergraphs
Wednesday, February 22, 9:30 a.m. and 2:00 p.m.

Paul Erdős, Hungarian Academy of Sciences
Problems and Results in Combinatorial Geometry
Thursday, February 23, 9:30 a.m.

Charles C. Sims, Rutgers University
A Survey of Group–Theoretic Algorithms
Thursday, February 23, 10:45 a.m. and 2:00 p.m.

Eugene M. Luks, University of Oregon
The Complexity of Computing in Permutation Groups:
I. The Graph Isomorphism Problem, Friday, February 24, 9:30 a.m.
II. Parallel Computation Friday, February 24, 2:00 p.m.

INDEX OF ABSTRACTS

This index contains a list of all authors of abstracts, not just presenters.
The list is complete as of 3:45 PM, Monday, February 13, 1989.

INDEX OF ABSTRACTS

This index contains a list of all authors of abstracts, not just presenters.
The list is complete as of 3:45 PM, Monday, February 13, 1989.

1

ON CONSTRUCTING A SPANNING TREE WITH OPTIMAL SEQUENCING

MICHAEL L. GARGANO, PACE UNIV. CS DEPT. NYC NY 10038

SYLVIA C. FRIEDERICH, PACE UNIV. CS DEPT. NYC NY 10038

Consider an undirected, connected, labeled graph $G=(V,E)$ with card(V)=n and card(E)=m. Consider also the assignment $C: E \rightarrow \{f \mid f: [0, n-1] \rightarrow \mathbb{R}$ and f is monotone nondecreasing$\}$. Each edge is assigned a function $f_e = C(e)$ where $f_e(t)$ $(0 \leq t \leq n-1)$ is a monotone nondecreasing cost function giving the cost subjected to by completing the construction of an edge e at time t. It is assumed that the construction of any edge in G takes exactly one unit of processing time. The ordered edge sequence $(e_1, e_2, \ldots, e_{n-1})$ is defined to be a spanning tree with sequencing if the subgraph induced by the edges $\{e_i\}$ is a subgraph which is a spanning tree of G. The ordered sequence indicates the order in which the edges are to be constructed. The problem is to construct a spanning tree with optimal sequencing, *i.e.*, a spanning tree with sequencing that minimizes the maximum of the incurred costs.

2

DIFFERENCE FAMILY CONSTRUCTIONS USING RINGS

Steven Furino, University of Waterloo

In his "Cyclotomy" paper, Wilson used the multiplicative structure of finite fields to construct difference families in their additive groups. We show that the multiplicative structures of rings with identity can also be used to construct difference families. These ring constructions are used to derive a new, large class of cyclic (v,4,1) 2-designs.

KEY WORDS : difference families, cyclic Steiner 2-designs

3

Ray Separation in Euclidean Space

Mark D. Halsey Worcester Polytechnic Institute, Worcester, MA 01609

Consider a finite set of points X in Euclidean space. X together with affine closure is a combinatorial geometry which we denote as G(X). A partition of X (into two parts) is a hyperplane separation of X if it is induced by a hyperplane in the ambient Euclidean space. A partition of X is a ray separation of X if each line of G(X) is split into two rays by the partition. Assuming that G(X) is binary, we give a necessary and sufficient condition for every ray separation of X to be a hyperplane separation of X.

Key words: mataroid, oriented matroid, hyperplane separation.

4

On Realization Of Majority Preference Digraphs

K.B.Reid and Weizhen Gu, Louisiana State University

A digraph D with vertex set $X = \{x_1, x_2, \ldots, x_k\}$ is $(n,h,k)-$*realizable* if there exists a connected graph G with n vertices, a subset V of h vertices of G (voters) and a subset $C = \{c_1, c_2, \ldots, c_k\}$ of vertices of G (candidates) so that for all distinct i and j in $\{1, 2, \ldots, k\}$, (x_i, x_j) is an arc of D if and only if a majority of the voters in V are closer to c_i than to c_j in G. Johnson and Slater [1988] proved that any oriented connected graph D of order k with arc set $A(D)$ and maximum (undirected) degree Δ is (n,n,k)-realizable, where $n = k^2 + k\Delta - |A(D)|$. We show how to reduce the value of n.

Theorem. *Every oriented connected graph D of order k with arc set $A(D)$ is (n,n,k)-realizable, where* $n = min\{k^2 + 2,\ 2k + 2|A(D)|\}$.

Such a digraph D represents the majorty preferences of voters located at vertices of G for the location of (desirable) candidate facilities on the network given by G.

5

Chordal completion of colored graphs.

T. V. Wimer, Clarion University of Pa.

In 1983, McMorris and Meacham posed the following problem:

> Determine the NP status of the following: Given a properly colored graph, determine if it can be made into a chordal graph by adding edges between vertices of different colors.

In this paper, we show that the problem is polynomial.

6

SYMMETRIC DIFFERENCE SETS IN ABELIAN GROUPS

R. L. McFarland, University of Minnesota, Duluth, MN

A difference set in a group is *symmetric* in case it is invariant under group inversion. In determining the (v, k, λ, n)-parameter values for which there exist symmetric difference sets in Abelian groups it is convenient to consider two cases.

Case I: $v = 4n$.
Then, as noted by P. K. Menon, $(v, k, \lambda, n) = (4m^2, 2m^2-m, m^2-m, m^2)$ for some integer m. Constructions by Menon and R. J. Turyn yield symmetric difference sets with these parameters for all m of the form $\pm 2^r 3^s$, $(r, s \geq 0)$. We show that a necessary condition for existence is that the square-free part of m divides 6. In the range $1 \leq m \leq 100$ there remain six values of m for which existence is undecided.

Case II: $v \neq 4n$.
McFarland has constructed the only known example of a symmetric difference set in an Abelian group with $v \neq 4n$. In some joint work with S. L. Ma it is shown that there are only two (v, k, λ, n)- parameter values in the range $1 \leq n \leq 100,000,000$ for which the existence of such a difference set is undecided.

7

Biclique Coverings of Graphs and Digraphs and Minimum Semiring Ranks of {0,1}-Matrices

Kim Hefner, Naval Postgraduate School, Teri Henson and
J. Richard Lundgren*, University of Colorado at Denver
John Maybee, University of Colorado at Boulder

In this talk we will present results on biclique covering and partition numbers for various classes of bipartite graphs and digraphs. Equivalent results are obtained for boolean and nonnegative integer rank of {0,1}-matrices. In particular, results of Brualdi, Manber and Ross on minimum real rank of k-regular {0,1}-matrices are extended to the semiring ranks. We give partial results on several open questions involving relationships between the different ranks and covering and partition numbers. For example, for what classes of digraphs is the real rank of the adjacency matrix a good bound for the biclique partition number of the digraph?

8

SOME RECONSTRUCTION THEOREMS FOR SEPARABLE GRAPHS WITH BRANCHES LARGE RELATIVE TO MAXIMUM VALENCE AND PRUNED CENTER SIZE

Steven E. Anecker*, Educational Testing Service, Princeton, New Jersey 08541 and Andrew J. Woldar, Department of Mathematical Sciences, Villanova University, Villanova, Pennsylvania 19085

A graph G is said to be branch-reconstructible if it can be obtained from the deck {G-v} where v can be any vertex of V(G)\P(G) where P(G) is the pruned center of G. The results presented are in the context of the following: If G is separable with P(G) a block, then either (i) G is branch-reconstructible or (ii) if B is a branch of G, then so is B-u where u is a non-root noncutpoint of B. In the present work, the authors assume (ii) and obtain asymptotic results on branch-recontructibility using bounds recently obtained on the order of primitive groups not containing the alternating group and arguments from the theory of permutation groups.

MONDAY, FEBRUARY 20, 1989 3:00 p.m.

9

FINDING A MINIMUM DOMINATING SET IN A PERMUTATION GRAPH

C. Rhee and H. Kim*
School of Electrical Engineering and Computer Science
University of Oklahoma
Norman, Oklahoma 73019

In this paper, we present an O(n log n) algorithm for finding a minimum cardinality dominating set in permutation graphs.

10

In Search of a (495, 39, 3) Difference Set

Ken W. Smith, Central Michigan University

Key words: difference set, symmetric design

It is known that an abelian (495, 39, 3) difference set does not exist. But there are two *non-abelian* groups of order 495.

This talk will discuss attempts to find a non-abelian (495, 39, 3) difference set. The attack focuses on the contracted difference set which would be created by the 45 cosets of the normal subgroup of order 11.

11

Almost (2,2)-Matroids

Safwan Akkari, Indiana University-Purdue University at Fort Wayne

We examine the connected matroids M that have the following properties: (1) Every two element deletion of M is disconnected, and (2) M has an element whose deletion is connected, and an element whose deletion is disconnected. We also give upper bounds on the number of elements of such matroids, along with a complete description of the matroids attaining these upper bounds.

12

Extremal Clone Theory: Three Examples

Robert W. Quackenbush, Department of Mathematics, University of Manitoba

A clone C on a set A is a set of functions $f: A^n \to A$ (for n a natural number) such that C contains all projection functions and is closed under composition of functions. C is maximal if the only clone properly containing C is the clone of all functions on A. For A finite, what are its maximal clones? I. G. Rosenberg has shown that there are only finitely many and has explicitly described them. C is a majority clone if it contains a function m(x, y, z) such that m(a, a, b) = m(a, b, a) = m(b, a, a) = a for all a, b in A. If $|A| = n$ and C a majority clone on A, then C is generated by a finite number of functions. What is the maximal functional arity needed in a generating set? H. Lakser has shown that for $n \geq 5$, this number is $n(n-2)$. Let C_n be the set of n-ary functions in C, $c_n = |C_n|$. Let p_n be the number of functions in C_n which depend on all their n variables; the sequences $\langle c_n \rangle$ and $\langle p_n \rangle$ are related by an obvious inclusion-exclusion formula. A sequence of cardinals, $\langle s_n \rangle$, is a p_n-sequence if it equals $\langle p_n \rangle$ for some clone C. If $s_0 > 0$, then $\langle s_n \rangle$ is always a p_n-sequence. If $\langle s_n \rangle$ is a p_n-sequence with $p_0 = 0$ and $p_1 = 1$, then for every f in a clone representing $\langle s_n \rangle$, $f(x, ..., x) = x$. In this case, if $s_n < \infty$ for all n, then $\langle s_n \rangle$ is eventually strictly increasing. The Minimal Extension Problem: Given C with $p_0 = 0$, $p_1 = 1$ and $p_n < \infty$ for all n, is there an N such that if B is any clone with p_n-sequence $\langle q_n \rangle$ such that $p_i = q_i$ for $i \leq N$, then $p_i \leq q_i$ for all n?

13

Two Software Systems for Manipulating Graphs

M. S. Krishnamoorthy, T. H. Spencer, Magda Echeandia, Alyce Faulstich,
George Kyriazis, Eric McCaughrin, Cathy Maroulis, Dave Pape
Department of Computer Science, R.P.I. Troy, NY 12180

In this paper, we discuss two different software systems that were developed at R.P.I. to manipulate Graphs: GraphPack and DrawGraph.

GraphPack is a software system that supports a language LiLa and a User Interface. The system consists of two windows, the textual window and a graphics window. The user can manipulate graphs using the interpreter or a mouse and pull down menus. We have implemented a Kernel (which supports primitive data structures and operations on them), user interface, LiLa compiler and interpreter. Examples of sessions will be provided in the final paper. The system is written in C using Sun Windows.

DrawGraph manipulates undirected graphs and works only with menus and mouse. The user who does not want to program can use this sytem without any difficulty. This system also includes various algorithms for drawing graphs, such as barycentric, symmetric and planar methods. Examples will be provided in the final paper. This system is written in C using X Windows.

14

SOME QUASI-COMPLETE LATIN SQUARES

B. A. Anderson Arizona State University

A Latin square in which each ordered pair (a,b) of distinct symbols occurs once in adjacent row cells and once in adjacent column cells is said to be complete. A Latin square is quasi-complete if each unordered pair {a,b} of distinct symbols occurs twice in adjacent row cells and twice in adjacent column cells. A Tuscan-k square is a row Latin square with the property that for any two symbols a and b of the array and for each m from 1 to k, there is at most one row in which b is the mth symbol to the right of a.

Combining these ideas leads to the notion of k-complete and qk-complete Latin squares. Results of Gordon and Bailey can be extended to give constructions of arrays with these properties if certain types of sequencings and terraces can be found. There is a class of terraces that leads to a number of examples of q2-complete Latin squares. Some members of this class can be lifted to symmetric sequencings that generate Tuscan-2 squares (in fact, nxn 2-complete Latin squares) for infinitely many composite values of n+1. This answers a question of Golomb, Etzion and Taylor.

15

k-Matroid Tree Decompositions of the Complete Hypergraph

David A Grable, Auburn University

In this talk, we discuss a generalization of trees from graphs to hypergraphs called k-matroid trees and the problem of partitioning the edge set of the complete k-uniform hypergraph into k-matroid trees.

16

17

Subgraphs With Minimum Degree Exceeding Half Their Order

J. Gordon Gimbel*, University of Alaska and Edgar M. Palmer, Michigan State University

We present an algorithm which accepts a graph G of order n and searches for a subgraph H of order at least cn where the minimum degree of H exceeds half its order; c is a constant. We show that for c=.21439 the algorithm is successful for almost every graph with edge probability one-half.

18

INCOMPLETE PERFECT MENDELSOHN DESIGNS

F. E. Bennett*, Mount Saint Vincent University

Cheng Maorong, Suzhou University

Let v and n be positive integers. An incomplete perfect Mendelsohn design, denoted by k-IPMD (v,n), is a triple (X,Y,B) where X is a v-set (of points), Y is an n-subset of X, and B is a collection of cyclically ordered k-subsets of X (called blocks) such that every ordered pair (a,b) $\in (X \times X) \setminus (Y \times Y)$ appears t-apart in exactly one block of B and no ordered pair (a,b) $\in Y \times Y$ appears in any block of B for any t, where $1 \leq t \leq k-1$. In this paper some basic necessary conditions for the existence of a k-IPMD(v,n) are easily obtained, namely, $(v-n)(v-(k-1)n-1) \equiv 0 (\mathrm{mod}\ k)$ and $v \geq (k-1)n+1$. It is shown that these basic necessary conditions are also sufficient for the case k = 3, with the one exception of v = 6 and n = 1. Some problems relating to embeddings of perfect Mendelsohn designs and associated quasigroups are mentioned.

19

MINIMUM COVERINGS OF MATROIDS UNDER ITERATED CLOSURE AND DUAL CLOSURE

by Arthur M. Hobbs*, Texas A&M University, College Station, and Andrew Vince, University of Florida, Gainesville.

In an electrical circuit satisfying Ohm's law in every branch and satisfying the two Kirchhoff laws, we would like to minimize the number of measurements of branch currents and/or voltage drops needed to determine all of the currents and voltage drops. But applications of Kirchhoff's laws correspond to closures in the circuit matroid and its dual. Thus we wish to find a minimum subset A of the underlying set S of a matroid M such that a sequence of closures in M and its dual applied to A results in S. We solve this problem for several classes of matroids.

Key words: matroid, closure, dual closure, electrical circuit

20

Constructing Spanning Trees With Many Leaves

Jerrold R. Griggs and Mingshen Wu*, University Of South Carolina

For a connected simple graph G let $L(G)$ be the maximum number m such that G has a spanning tree with m leaves. Linial conjectured that if G has n vertices and minimum degree k, then $L(G) \leq \frac{k-2}{k+1}n + c_k$, where c_k is a constant depending on k. In 1981 Storer proved that Linial's conjecture is true for $k = 3$ with c_3=2. We prove that (1) if $k = 4$, then $L(G) \geq \frac{2}{5}n + \frac{8}{5}$ (which was also obtained by Kleitman and West), and (2) if $k = 5$, then $L(G) \geq \frac{1}{2}n + 2$. Examples show that these bounds are sharp. The proofs give polynomial algorithms for finding such spanning trees.

21

Resiliece of Partial k-tree Networks

Erick Mata-Montero, University of Oregon

We model a network as a graph in which each edge has an associated probability of operation. Edge failures are statistically independent. The resilience of a network is the expected number of pairs of nodes that can communicate. Computing the resilience of a network has been shown to be a #P-complete problem for planar networks and to take $\mathcal{O}(n^2)$ time for n-node partial 2-tree networks. We present an $\mathcal{O}(n)$ time algorithm to compute the resilience of partial 2-tree networks on n nodes, and, for a fixed k, an $\mathcal{O}(n^2)$ time algorithm to compute the resilience of n-node partial k-tree networks given with an embedding in a k-tree. We also indicate how to generalize these algorithms for networks in which both nodes and edges may fail.

Key words: partial k-trees, network reliability, network resilience.

22

Row-Column Directed Block Designs

Dinesh G. Sarvate and Rose C. Hamm*
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There has been much interest in the study of directed designs. A directed balanced incomplete block design is a BIBD with blocks ordered in such a way that each ordered pair occurs an equal number of times in the blocks. We ask whether we can represent a BIBD as a matrix in such a way that each column in the matrix is a directed block and the rows are directed in the sense that each ordered pair occurs an almost equal number of times in the rows. We call such an arrangement a Row-Column Directed Balanced Incomplete Block Design (RCDBIBD). We will present some constructions and examples and show that the necessary conditions for the existence of a BIBD are not sufficient for the existence of a RCDBIBD.

Key Words:
BIBD
Directed dDesigns
Row-column directed designs

23

Fixing elements in minors of binary matroids

Talmage James Reid, The University of Mississippi

A collection of 3-connected binary matroids $\mathcal{F}$ each having four elements is 2-rounded if it satisfies the following condition. Whenever e and f are elements of a 3-connected binary matroid M having a minor in $\mathcal{F}$, then M has a minor which uses e and f and is isomorphic to a member of $\mathcal{F}$. Suppose $\mathcal{F}$ is 2-rounded. Oxley and Reid showed that if $|\mathcal{F}| = 1$, then $\mathcal{F}$ contains the cycle matroid of the wheel graph with six or eight edges. This paper establishes that a similar result holds if $|\mathcal{F}| = 2$. Analagous results for 3-connected simple graphs were obtained by the author.

24

Sum graphs from trees

M. N. Ellingham, Vanderbilt University

A graph G is a *sum graph* if there is a labelling σ of its vertices with distinct positive integers , so that for any two (possibly equal) vertices u and v, uv is an edge of G if and only if $\sigma(u) + \sigma(v) = \sigma(w)$ for some other vertex w. Every sum graph has at least one isolated vertex (the vertex with the largest label). Harary, who introduced the idea of a sum graph at last year's Southeastern Conference, has conjectured that a connected graph with n vertices and m edges can be made into a sum graph by the addition of at most $m - n + 2$ isolated vertices. We verify this conjecture for trees. In other words, we show that any tree becomes a sum graph with the addition of a single isolated vertex.

25

Which Graphs Are Tensor Squares

Michael F. Capobianco and Ann C. Mugavero*, St. John's Univ.

In this paper we investigate graphs which are tensor squares, i.e., graphs H for which the graph equation $G^4 = H$ has a solution. Here G^4 denotes the tensor product (also known as the conjunction) of G with itself $G \wedge G$.
One section presents required definitions and some easy results giving necessary conditions for the graph to be a tensor square. The remainder of the paper presents an algorithm for determining if any given graph, H, is a tensor square, and we conclude with a possible application.

26

A Combinatorial Algorithm in Combinational Logic

Greg Starling, University of Arkansas, Fayetteville

The Quine-McCluskey algorithm is a method for minimizing combinational logic functions. A graph model of such n variable functions is realized on the Boolean n-cube, $(Z_2)^n$. A vertex of the n-cube is a vertex of the graph when the value of the Boolean function is one on that vertex. An edge connects two vertices of the graph when the vertices are adjacent in the n-cube. The components of the graph are then transformed by collapsing each edge into a single vertex with a coordinate missing at the one differing coordinate of the two end vertices. This is done during canonical Gray Code traversals of the n-cube.

Keywords: Quine-McCluskey algorithm; logic function minimization; Gray Code

27

COMBINATORIAL APPLICATIONS OF THE SMITH NORMAL FORM

Joseph J. Rushanan, The MITRE Corp., Bedford, MA

The application of the Smith Normal Form (SNF) over the integers to combinatorics is discussed. By using results that limit the SNF of an integral matrix based on its eigenstructure, the SNF of some graph incidence matrices can be determined easily. Conjectures are given for the SNF for several families of graphs.

Keywords: integral matrices, Smith Normal Form, incidence matrices

28

Graphs from Rings

Ralph P. Grimaldi
Rose-Hulman Institute of Technology

At the Nineteenth Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Professor Frank Harary (currently at New Mexico State University) introduced the following idea called a sum graph. Starting with a nonempty finite subset S of Z^+, one constructs an undirected loop-free graph $G(S)$ where

(1) each element of S is a vertex; and,
(2) for $x, y \in S$ $(x \neq y)$ the edge $\{x, y\}$ is drawn in $G(S)$ if $x + y \in S$.

At that time, properties of such graphs were examined for certain sets S as well as for the case when $(S, +)$ was a finite abelian group.

In this paper we start with a finite ring $(R, +, \cdot)$, and want to use both of the binary operations in R. So here we examine the properties of the undirected loop-free graph $G(R)$ where

(1) each $r \in R$ is a vertex of $G(R)$; and,
(2) for distinct $r, s \in R$, the edge $\{r, s\}$ is drawn in $G(R)$ when $r + s$ is a unit of R.

29

F-G PACKINGS

D.G. Corneil, Department of Computer Science
University of Toronto

An F-G packing of graph H is a collection of copies of G in H such that no two such copies of G share a copy of F. Thus a K_1-K_2 packing of H is a matching. The F-G packing problem is: given graph H and integer k, does H have an F-G packing of size greater than or equal to k. The complexity status of the K_1-G and K_i-K_j $(j > i)$ packing problems has been determined. In this talk we address the K_2-G problem.

31

BICLIQUE COVERINGS AND PARTITIONS OF DIGRAPHS: A SET THEORETIC APPROACH

Kim A. S. Hefner, U.S. Naval Postgraduate School

In this talk, a digraph D is related through its adjacency matrix to a bigraph B. The minimum number of complete directed bipartite subgraphs it takes to cover/partition the arcs of D is equal to the minimum number of complete bipartite subgraphs needed to cover/partition the edges of B. From B, a graph G is obtained. A sequence of subsets which has G as its intersection graph is generated using a minimum of k elements. Constraints are given for constructing the sequence which will enable us to use k–2 of these elements to form a minimum biclique cover/partition of the original digraph D.

32

MOD SUM GRAPHS

J. Boland*, G. Domke, R. Laskar and C. Turner
Clemson University

A graph G is a mod sum graph (MSG) if there exists a positive integer n and a labelling of the vertices of G with elements of { 1, 2, ... , n-1 } such that e = uv is an edge of G if and only if the sum, modulo n, of the labels on vertex u and vertex v is also a label in G. Note that sum graphs as defined by Harary are mod sum graphs but the converse is not true. Among other results we show that paths, cycles and stars are mod sum graphs while complete graphs are not.

MONDAY, FEBRUARY 20, 1989 5:00 p.m.

33

A Mathematical Programming Approach to Edge Coloring

G. Nemhauser* and S. Park, Georgia Inst. of Tech.

We formulate the edge coloring problem on a simple graph as the integer program of covering edges by matchings. For the NP-hard case of 3-regular graphs we show that it is sufficient to solve the linear programming relaxation with the additional constraints that each odd circuit be covered by at least 3 matchings. We give an efficient separation algorithm for recognizing violated odd circuit constraints and a linear programming based constrained weighted matching algorithm for pricing Computational experiments with the overall linear programming system are discussed.

key words: edge coloring, integer programming

34

Lyanzin starters and D-Neofields

Paul Smith* and Wendy Myrvold, University of Victoria

Shortly after Bose, Shrikhande and Parker demolished Euler's conjecture that no grecolatin square of order n existed whenever $n \equiv 2 \bmod 4$, A.I. Lyanzin produced a square of order 10 with a very simple structure. A.D. Keedwell showed that the existence of a Lyamzin square of and order was equivalent to the existence of a special type of neofield which he called a D-neofield. By constructing difference sets we show that D-neofields of order n exist for all $n < 44$ except 2 and 6.

35

Changing and Unchanging Edge Clique Cover Numbers

R. D. Dutton* and R. C. Brigham
Univ. of Central Florida

For certain graphs, the edge clique cover number remains constant with the removal of an arbitrary edge. For certain other graphs it always changes, either positively, negatively or both. A similar phenomenon occurs with the removal of an arbitrary node except that the edge clique cover number never increases. This initial study has led to some mildly interesting results which we present here.

36

Outerplanar graphs and greatest common subgraphs

Grzegorz Kubicki, Western Michigan University

Those connected outerplanar graphs G are determined for which there exist nonisomorphic connected outerplanar graphs of equal size containing G as a unique greatest common subgraph.

MONDAY, FEBRUARY 20, 1989 5:20 p.m.

37

A Parallel Algorithm for the Enumeration of Spanning Trees

Maung Aung-Naing (*) and Eduardo B. Fernandez, Florida Atlantic University

Many algorithms for the enumeration of all spanning trees of an undirected graph have been proposed. One of these algorithms, using fusion of nodes, is discussed and its parallel implementation using Ada tasks is presented. Some general considerations are also discussed.

Keywords: Ada tasking, concurrent programming, parallel algorithms, spanning tree algorithms.

38

Gray Code Sequences of Partitions

Carla Savage, North Carolina State University

Several recent papers have addressed the problem of listing combinatorial objects in such a way that each element on the list differs from its predecessor in some pre-specified way. Such orderings have been called Gray code orderings, after the classical example of listing all n-bit binary numbers so that successive elements differ only in one bit. In this talk we consider Gray code-like orderings of the partitions of an integer and show how to solve a problem posed by Herbert Wilf at the SIAM Discrete Math Conference. We show that for all integers n, there is a way to list the partitions of n in such a way that successive partitions differ only in that one part has increased by 1 and one part has decreased by 1. (A part of size 1 may decrease to 0 or a "part" of size 0 may increase to 1.) Referring to this ordering as Gray code order, the result is actually more general: For all $n \geq k \geq 1$, there is a way to list all partitions of n into integers of size at most k in Gray code order. Unless $(n,k)=(6,4)$, we can arrange for such a list to start at the lexicographically smallest partition and end at the lexicographically largest. As a consequence, we can also list each of the following in Gray code order: (1) all partitions of n whose largest part is k, (2) all partitions of n into k or fewer parts, and (3) all partitions of n into exactly k parts.

39

Changing and Unchanging of the Invariants: Minimum and Maximum Degree, Maximum Clique Size, Node Independence Number and Edge Independence Number

Teresa W. Haynes, Linda M. Lawson*,
East Tennessee State University
Robert C. Brigham, Ronald D. Dutton,
University of Central Florida

We consider the changing and unchanging of an invariant of a graph under three different situations: deleting a node, deleting an edge and adding an edge. It is an ongoing project of the authors to characterize extremal graphs in each of these six categories for selected invariants. This paper presents results on changing and unchanging with respect to the invariants minimum and maximum degree, maximum clique size, node independence number and edge independence number.

40

Least Common Supergraphs of Graphs

Gary Chartrand*, Héctor Hevia, Grzegorz Kubicki, Ortrud R. Oellermann and Farrokh Saba, Western Michigan University and Hung Bin Zou, Intermagnetics General Corporation

Let $\mathcal{G}$ be a set of graphs, all having the same size. A graph G without isolated vertices is said to be a greatest common subgraph of $\mathcal{G}$ if G is a graph of maximum size that is isomorphic to some subgraph in every graph in $\mathcal{G}$, while H is a least common supergraph of $\mathcal{G}$ if H is a graph of minimum size that is isomorphic to some supergraph of every graph in $\mathcal{G}$. Some results concerning least common supergraphs are presented, and some relationships between greatest common subgraphs and least common supergraphs are described.

41

Intersection Graphs on Pythagorean Triples

T. A. Jenkyns, Brock University, St. Catharines, Ont.

Three positive integers A, B and C are said to form a Pythagorean Triple if $A^2 + B^2 = C^2$. Let PT denote the set of all Pythagorean Triples and let G(PT) denote the graph with vertex set PT where two distinct triples are adjacent if and only if they have some integer in common. A number of properties of G(PT) are easily derived and a number of conjectures easily formulated. The main result presented is that G(PT) is connected.

42

Contractible Edge Covers of Size Three in 3-Connected Graphs.

Robert L. Hemminger* and Xingxing Yu, Vanderbilt University

It is shown that if G is 3-connected with $|V(G)| \geq 10$, then, with the exception of one infinite class based on $K_{3,p}$, it takes at least four vertices to cover the set of contractible edges of G.

Keywords: Contractible edges, 3-connected, contractible edge covers.

43

Parallel Computation of Closure on Finite Relations

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ABSTRACT

Let P and V be two disjoint finite sets. Let E be a relation on V. Let I:V→*Powerset*(P) be given. We consider the problem of computing the function M:V→*Powerset*(P) such that for all x∈V,

$$M(x) = \bigcup \{ I(y) \mid y \, E^{*} \, x \}$$

Computing M involves computing the closure of relation E. Traditionally, the graph G=(V,E) is used, and graph algorithms are used to "propagate" the values of M (which are subsets of P) from vertex to vertex in graph G, performing set unions along the way. In this paper we present an algorithm for performing this computation in a completely parallel fashion, as long as P is finite. Each element of P is allocated to one parallel processor, which propagates only that element from vertex to vertex. The result is a speedup factor as high as the number of elements in P, and a much simpler algorithm that requires no set union operations. We present the algorithm, discuss its implementation, and an application to language processing.

44

The Chromatic Index of Regular Multigraphs which are Nearly Full

M. J. Plantholt* and S. K. Tipnis, Illinois State University

Let G be a regular multigraph with degree d and even order n which has at most r parallel edges joining any pair of vertices. We show that if r is even and $d \geq r(6n/7 + 1)$ then G has chromatic index $\chi'(G) = d$. This extends to multigraphs the result by Chetwynd and Hilton that any simple, regular graph with even order n and degree $d \geq 6n/7$ is in Class 1. It is also shown that improvements on the $6/7$ coefficient in the simple graph case will produce corresponding improvements in the multigraph case.

Key Words: Chromatic Index

TUESDAY, FEBRUARY 21, 1989 8:50 a.m.

45

Linear Operators Preserving Properties of Graphs

LeRoy B. Beasley*, Utah State Univ. and Norman J Pullman, Queen's Univ.

We characterize the linear operators on simple graphs on n vertices that preserve: planarity, k-colorability, k-edge-colorability, the chromatic number and the edge-chromatic number.

46

A MORE ELEMENTARY PROOF OF GRUNBAUM'S CONJECTURE

Allan Gray and Steve Wilson*, Northern Arizona University

In 1974, Grunbaum conjectured that a finite regular map of type {p,q} exists for each p and q. In 1983, Vince proved a generalization of Grunbaum's conjecture using Mal'cev's theorem about groups of matrices. In this paper, we prove Grunbaum's conjecture by presenting a construction in permutation groups. As a preliminary to this construction, we prove some lemmas which explore the question of exactly when a group is the group of a rotary map. We explore some of the consequences of the construction, including the question of whether the map constructed is reflexible. And we show a slight refinement which guarantees that the map constructed is reflexible.

47

Reliable Assignments of Processors to Tasks

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In the simple assignment problem, there are n processors, m tasks, and a relation between the processors and tasks; this relation indicates the ability of the processor to perform the task. When the processors fail independently with known probabilities, two performance issues arise. First, with what probability can the operating processors all be kept busy? Second, with what probability can the operating processors perform the same number of tasks that all processors could? In this talk, we formulate these questions on the underlying transversal matroid and outline an application of the reliability factoring theorem to compute exact solutions.

Keywords: reliability models, graphs and matroids, reliability factoring algorithms

48

IRREGULARITY STRENGTHS FOR CERTAIN GRAPHS

Gary Ebert, Joe Hemmeter
Felix Lazebnik, Andrew Woldar*

University of Delaware

A network $G(\omega)$ of strength s is a graph G in which each edge is assigned a positive integer weight, the largest of which is s. The degree of a vertex in $G(\omega)$ is the sum of the weights of its incident edges. We call $G(\omega)$ irregular if distinct vertices have distinct degrees. The irregularity strength $s(G)$ of G is the minimum strength among all irregular networks having G as an underlying graph. In this work, the authors determine $s(G)$ for G a wheel, a $(2\times n)$-grid, and a k-cube.

49

Singular (0,1) Tournament Matrices

Bryan Shader, University of Wisconsin, Madison

A (0,1) tournament matrix of order n is known to have real rank $n-1$ or n. We determine the singular tournament matrices which have score vector $(1,1,2,3,4,5,\ldots,n-4,n-3,n-2,n-2)$. As a consequence, we show that almost all tournament matrices with this score vector are singular.

50

Bi-Embeddings of Graphs

S.L. Cabaniss, San Jose State University

A graph G is said to be (p,q) bi-embeddable if there exist two subgraphs H_1 and H_2 of G with $H_1 \cup H_2 = G$ such that H_1 is embeddable on S_p and H_2 is embeddable on S_q where S_p and S_q are orientable surfaces of genus p and q respectively. A bi-embedding is called perfect if $p = q$. A similar definition can be used for non-orientable surfaces.

This paper presents a review of known results to date as well as some new results for non-orientable surfaces, including the first infinite family of perfect bi-embeddings. In addition, the open question of the *bi-genus* of a graph, especially K_n, will be discussed. The *bi-genus* of a graph G is the minimal genus of a surface upon which G can be bi-embedded.

Key words: embedding, bi-embedding, genus, topological graph theory

51

Computation of Signal Flow Diagrams for Parallel Algorithms

Roy Fuller, University of Arkansas, Fayetteville

In developing algorithms for parallel computers, one of the problems is how the data should be routed between processors. A method now used to determine the *signal flow diagram* involves factorizing a matrix into several sparse matrices, impractical when large numbers of variables are involved. Our method applies to any algorithm which is an iteration of vector transformations **T** (not necessarily linear) of the following type. The input (real or complex) is X(0), X(1), ... ,X(N-1). Let $N = q_r q_{r-1} \ldots q_1 q_0$ be a factorization into (not necessarily prime) factors. Let Q_k ($r \geq k \geq 1$) represent the product $q_{k-1} \ldots q_1 q_0$. Given n satisfying ($N > n \geq 0$), let $(n_r \ldots n_1 n_0)$ denote the indices of n in the base $(q_r, \ldots, q_1, q_0)$, that is, let

$$n = n_r Q_r + n_{r-1} Q_{r-1} + \ldots + n_1 Q_1 + n_0; \quad 0 \leq n_k < q_k.$$

Let $\pi_{k,t}(n) = (n_r \ldots n_{k+1}\, t\, n_{k-1} \ldots n_0)$. Then the output XT(n) depends only on n and input values $X(\pi_{k,0}(n))$, $X(\pi_{k,1}(n))$, ... $X(\pi_{k,q_k-1}(n))$. For such an algorithm, we present a method that reduces to a very simple heuristic for determining the flow diagrams, which are perfect shuffles, and the appropriate revision to the transformation **T**. Thus by a simple computation we can derive the flow diagrams and parallel versions of a class of algorithms that includes fast Fourier transforms for arbitrary N and the bitonic merge sorting algorithm.

Key Words: signal flow diagram, parallel, perfect shuffle.

52 ON THE NUMBER OF IRREGULAR ASSIGNMENTS OF A GRAPH

Gary Ebert, Joe Hemmeter, Felix Lazebnik*, Andrew Woldar
University of Delaware

A network $G(\omega)$ of strength s is a graph G in which each edge is assigned a positive integer weight, the largest of which is s. The degree of a vertex in $G(\omega)$ is the sum of the weights of its incident edges. We call $G(\omega)$ irregular if distinct vertices have distinct degrees. Let $Irr(G, \lambda)$ be the number of irregular networks with underlying graph G and strength at most λ. We prove that

$$| Irr(G, \lambda) - \lambda^q + c_1 \lambda^{q-1} | = O(\lambda^{q-2}) \quad (\lambda \to \infty)$$

where $c_1 = c_1(G)$ and $q = |E(G)|$. Moreover, an explicit expression for c_1 is given.

53

Three Graphs Associated With an nxn $0-1$ Matrix

Kevin McDougal, University of Wisconsin - Madison

We investigate three graphs associated with any nxn $0-1$ matrix. For each type of graph, matrices equivalent by row and column permutations give isomorphic graphs. For matrices with constant row and column sums, we determine properties of these graphs, such as the number of connected components, and estimate the degrees of vertices.

54

Euler Contributions and Matching Extendability in Nonorientable Surfaces

Nathaniel Dean, Bellcore, Morristown, NJ 07960-1910

If a graph G on at least $2k+2$ vertices contains a matching of size k and every such matching is contained in a perfect matching, then G is said to be k-extendable. The theory of Euler contributions is essentially a technique for obtaining local information about a graph embedding and has been used by Plummer to derive an upper bound for the function $f(\gamma)$, the smallest integer k such that no connected graph of orientable genus γ is k-extendable. In this paper we extend the theory to embeddings of graphs in nonorientable surfaces, we define a corresponding function $\bar{f}$ of the nonorientable genus $\tilde{\gamma}$, and we apply the theory to determine an upper bound for $\bar{f}$ in terms of $\tilde{\gamma}$ and to determine f and $\bar{f}$ precisely for some small values of γ and $\tilde{\gamma}$, respectively.

55

Monotonicity in graph searching

D. Bienstock*, P.D. Seymour, Bellcore

Consider a graph as a system of tunnels containing an infinitely fast and cunning fugitive. This fugitive must be apprehended using a minimum number of guards, in a finite sequence of discrete "pebbling" moves: in one move, we may place a guard at a vertex, remove a guard from a vertex, or slide a guard along an edge. To clear an edge $\{u, v\}$ we slide a guard from u to v, while simultaneously having guards posted so that there is no unobstructed path from an unsearched edge to u. If at any point of the game there is an unobstructed path from an unsearched edge to a searched one, the latter becomes instantaneously "recontaminated" and must be searched again. This graph searching game was first proposed by Parsons. LaPaugh proved that there is always a search strategy with minimum number of guards which is monotone, i.e. there is no recontamination. This result was later extended by Kirousis and Papadimitriou to a variant of searching.

In this talk we present a short proof of monotonicity, together with a unified framework for studying the different variants of searching. The proof uses the submodularity of vertex connectivity, and is motivated by, but does not depend on, some ideas from the Graph Minors theory of Robertson and Seymour.

56

A CHARACTERIZATION OF CORDIAL GRAPHS

YONG-SONG HO
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Let $G = (V,E)$ be a graph with vertex-set V and edge-set E. A binary labelling of G is a mapping $f : V \to \{0,1\}$, and for each $v \in V$, $f(v)$ is called the label of v under f and for each $x = uv \in E$ the label of x under f is given by $|f(u) - f(v)|$. We also denote by $v_f(0)$ (resp. $e_f(0)$) and $v_f(1)$ (resp. $e_f(1)$) the number of vertices (resp. edges) with labels 0 and 1 under a binary labelling f of G respectively, and we call f cordial if

$$|v_f(0) - v_f(1)| \le 1 \quad \text{and} \quad |e_f(0) - e_f(1)| \le 1.$$

A graph G is cordial if it admits a cordial labelling. Cordial graphs are first introduced by I. Cahit as a weaker version of both graceful graphs and harmonious graphs. Cahit showed that if G is an Eulerian graph with m edges, where $m \equiv 2 \pmod 4$, then G has no cordial labelling.

In this paper we give a characterization of cordial graphs. A class of non-cordial graphs which are not eulerian but whose sizes are congruent to 2 modulo 4 is provided. A necessary condition for the cordiality of a graph G, which involves only the order and size of G is established. We also obtain a necessary and sufficient condition for the graph $K_n^{(2)}$, consisting of two copies of K_n sharing a common vertex, to be cordial.

57 EXTREMAL THEOREMS WITH INTERSECTION CONDITIONS

Attila Sali
The Ohio State University and Math. Inst. of Hung. Acad. Sci.

The aim of the present research is to find possible generalizations of classical extremal theorems. Let *S(A)* be the set of submatrices of the n x m matrix *A*. Then *S(A)* is a ranked poset with respect to the inclusion, and the rank of a submatrix is the sum of the number of rows and columns minus 1, the rank of the empty matrix is zero. We attack the question: What is the maximum number of submatrices such that any two of them have intersection of rank at least t? We have a solution for t=1,2 using the following theorem of independent interest. Let $m_n(i,j,k) = \max (|F|+|G|)$, where F is i-intersecting, G is j-intersecting families of subsets of an n element set and F and G are cross k-intersecting. Then if $i \leq j \leq k$, $m_n(i,j,k)$ is attained if F is maximal i-intersecting family of subsets containing large subsets, and G consists of all large enough sets that the size of the sets ensures k-intersection with members of F.

58

Realizable Values of the Forwarding Index

M.C. Heydemann, J.C. Meyer, J. Opatrny, D. Sotteau*
LRI, UA 410 CNRS, Universite Paris Sud, Concordia University

For a given connected graph G of order n, a routing R is a set of n(n–1) simple paths, one specified for each ordered pair of vertices in G. The pair (G,R) is called a network. The vertex–forwarding index $x_i(G,R)$ of a network (G,R) is the maximum number of paths of R passing through any vertex of G. The minimum over all possible routings of shortest paths R of a connected graph G is denoted by $x_i m(G)$. We give here an upper bound on $x_i m(G)$ (in terms ofthe number of vertices of G and the parity of $x_i m(G)$), and show that for any positive value k less than or equal to this upper bound, there exists a graph G of order n such that $x_i m(G)=k$.

59

ASPECTS OF IMPLEMENTING PARALLEL BRANCH AND BOUND ALGORITHMS

J.H. Walczyk, Jr. - IBM Corporation, Boca Raton, FL (*)
E.B. Fernandez - Florida Atlantic University, Boca Raton, FL
E. Gudes - Ben Gurion Univ. of the Negev, Beer Sheva, Israel

Artificial Intelligence problems often involve state space trees and branch and bound algorithms that lend themselves quite well to parallel implementations. A number of parallel versions of the basic algorithms have been designed and studied for specific computational models. However, many interesting theoretical and practical questions remain. This paper focuses on derivations of A* and discusses issues concerning local versus global lists, new heuristics, and tie breakers.

Key Words: parallel algorithms, branch and bound, A*

60

The fractional chromatic number of a graph and a construction of Mycielski

James Propp, University of California, Berkeley, Michael Larsen, Princeton University, and Daniel Ullman*, The George Washington University.

The chromatic number $\chi(G)$ of a graph G is the solution to a certain integer program whose dual gives the clique number $\omega(G)$ and whose real relaxation gives the fractional chromatic number $\chi_F(G)$. This implies that $\omega(G) \leq \chi_F(G) \leq \chi(G)$. A natural question is whether $\chi_F(G)$ can be simultaneously far from both $\omega(G)$ and $\chi(G)$. The answer is yes, and a well-known sequence of graphs G_n due to Mycielski provides the example. These graphs have $\omega(G_n) = 2$ and $\chi(G_n) = n$. Here it is proved that $\chi_F(G_n) \sim \sqrt{2n}$ by showing that

$$\chi_F(G_{n+1}) = \chi_F(G_n) + \frac{1}{\chi_F(G_n)}.$$

LIST OF KEY WORDS

fractional chromatic number, fractional clique number, duality theorem.

61

Pseudo Fields and Pseudo Planes

George P. Graham, Indiana State University

L. J. Paige introduced neofields in 1949. In 1966 R. Sandler defined pseudo ternary rings, a generalization of neofields, and their associated pseudo projective planes. A. D. Keedwell continued Paige's ideas in 1967, defining neofields with division property D. In this talk a class of neofields intermediate between neofields and D-neofields is defined and some elementary facts about these systems are presented.

Key words: Neofields, Pseudo ternary rings, Projective planes

62

The Automorphism Groups of the Strong Bruhat Order of the Coxeter Groups B_n and D_n

David C. Sutherland, Middle Tennessee State University

The automorphism group of the strong Bruhat order of the Coxeter group B_n for $n \geq 3$ is isomorphic to $C_2 \times C_2$ where C_2 is the cyclic group of order 2. The automorphism group of the strong Bruhat order of the Coxeter group D_n for $n \geq 5$ and n even is isomorphic to $C_2 \times C_2 \times C_2$. The automorphism group of the strong Bruhat order of the Coxeter group D_n for $n \geq 5$ and n odd is isomorphic to the dihedral group $\mathcal{D}_4$ of order 8. The proofs use the (signed) permutation representations of the Coxeter groups B_n and D_n.

Keywords: Strong Bruhat Order, Permutations, Coxeter Group, Dynkin Diagram, Automorphism, Schensted's Algorithm

63

Neighborhood Subtree Tolerance Graphs

Eric Bibelnieks* and P.M. Dearing, Clemson University

In this paper, we introduce neighborhood subtree tolerance (NeST) graphs which are defined in terms of the tolerance-intersection of neighborhood subtrees of a tree. This class of graphs extends the class of (interval) tolerance graphs which are defined in terms of tolerance-intersection of intervals on the real line. Some relationships between interval tolerance, NeST, and weakly triangulated graphs are examined. The main result shows that NeST graphs are weakly triangulated graphs. In addition, NeST graphs with constant tolerance are shown to be strongly chordal.

Keywords: Neighborhood subtrees, intersection graphs, tolerance graphs, weakly triangulated, and strongly chordal.

64

Area Bound for the Three-Layer Wirings of a Class of Planar Layouts

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and
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University of California - Santa Barbara

A planar VLSI layout is a collection of edge-disjoint subgraphs of the square grid. It is known that the problem of determining whether or not a planar VLSI layout is three-layer wirable is NP-complete. Previous investigations on three-layer wirings of layouts suggested the techniques of stretching the layouts to ensure wirability. In this paper we show that for a layout class, there exist layouts that can not be stretched and three-layer wired in an area less than (7/6 - ε) time of their original layout area, where ε is an arbitrary small positive constant.

65

An Introduction to multiset graph-theoretic problems,

Dana L. Grinstead, University of Alabama in Huntsville

Recently the author and P. J. Slater introduced a "multiset" type of graph-theoretic problem intermediate between finding a single vertex set with a specified property (such as being a minimum dominating set) and finding a partition of the entire vertex set of the graph into such sets (for example, a partition into the maximum possible number of dominating sets). One example of such a multiset problem involves considering minimum dominating sets two at a time and finding a pair which is as disjoint as possible. In this presentation, several other examples of this type of problem will be introduced.

66

Sharply Focused Sets of Lines on a Conic in PG(2,q)

Gustavus J. Simmons, Sandia National Laboratories, Albuquerque, NM

A set of n points, S_n on a conic, Ω, in PG(2,q) defines $\binom{n}{2}$ secants to Ω since no triple of points on Ω can be collinear. The $\binom{n}{2}$ secants determine $\binom{\binom{n}{2}}{2} - 3\binom{n+1}{4} = O(n^4)$ points of intersection in all, $n\binom{n-1}{2}$ which are accounted for by multiple incidences on the n points on Ω. There remain however $3\binom{n}{4}$ points of intersection not on Ω. For a given q and n, the general question is: What is the minimal number of distinct points of intersection off Ω? A set S_n that realizes this minimum is said to be focused. For example, if $n = 6$, $q \geq 7$, the 15 secants can be focused on only 37 points (33 single points of intersection and 4 points on which 3 secants lie). This is the best focus possible. At the other extreme the 15 secants can be defocused to intersect at 45 distinct points. This is also the best (worst?) possible. We will say that S_n is sharply focused on an exterior (to Ω) line ℓ if the $\binom{n}{2}$ secants of Ω defined by S_n intersect ℓ in only n points. We give constructions for several sharply focused sets and pose a number of combinatorial questions concerning focused and sharply focused sets in general.

Key Words: projective geometry, conics, focused sets, incidence properties

67

Invertibility of linearly recursive sequences

Earl J. Taft, Rutgers University

We discuss linearly recursive sequences $\{a_n\}_{n\geq 0}$ over a field k. There are three well-known products $\{a_n\}\{b_n\} = \{c_n\}$ on this space: the rational product $c_n = \sum_{i+j=n} a_i b_j$, the Hurwitz (divided power) product $c_n = \sum_{i+j=n} \binom{n}{i} a_i b_j$ and the Hadamard product $c_n = a_n b_n$. The latter two are the algebra part of a bialgebra (Hopf algebra) structure, dual to the polynomial bialgebra (Hopf algebra) k[x]. We discuss necessary and sufficient conditions for invertibility under the Hurwitz product and under the Hadamard product. For example, at characteristic $k = 0$, $\{a_n\}$ is Hurwitz invertible if and only if it is a non-zero geometric (exponential) sequence $a_n = r^n a_0$. The conditions for Hadamard invertibility involve the nature of the roots of the minimal recurrence relation for $\{a_n\}$. While the results are classical, our methods are strongly Hopf-algebra theoretic.

68

More Conjectures on the Number of Complete Subgraphs in a Graph

David C. Fisher* and Jennifer Ryan, University of Colorado at Denver

Let G be a graph whose largest complete subgraph has w nodes. For $1 \leq j \leq w$, let k_j be the number of complete subgraphs on j nodes (so k_1 is number of nodes, k_2 is number of edges, k_3 is number of triangles, etc.). Let

$$g(z) \equiv z^w - k_1 z^{w-1} + k_2 z^{w-2} - k_3 z^{w-3} + \cdots + (-1)^w k_w.$$

Fisher and Solow showed that the largest (in absolute value) root of $g(z) = 0$ is real and positive. Let r be that root. We will show that:

$$r - k_1 \leq 0, \quad r^2 - k_1 r + k_2 \geq 0, \quad r^3 - k_1 r^2 + k_2 r - k_3 \leq 0, \quad \ldots.$$

We wish to make the following conjectures:

Conjecture 3. For $0 \leq m < w$, the largest (in absolute value) root of $\frac{d^m}{dz^m} g(z) = 0$ is real and positive. (Known to be true for $m = 0$, $m = w-2$, and $m = w-1$.

Conjecture 4. Let r_m be the root from Conjecture 3. Then: $r_0 \geq r_1 \geq r_2 \geq \cdots \geq r_{w-1}$.

If these conjectures are true, a myriad of new relations could be found between the number of complete subgraphs in a graph. For example, it would give an asymptoticly exact lower bound for the number of triangles in a graph in terms of the number of nodes and edges.

69

Improvement of Log Depth Division Circuit

Puhua Guan, Dept. of Math., Univ. of Puerto Rico, Río Piedras, PR 00931

We present a log depth circuit for integer division, which improved the best known algorithm of W. Beame, S. Cook and J. Hoover. We also show that preparing the constants needed for our division algorithm can be done in dog^2 depth.

70

Maximum independent, minimally c-redundant sets in graphs

T. W. Johnson* and P. J. Slater, University of Alabama in Huntsville

We consider a recently introduced class of problems involving the evaluation of "single set, prioritized multiproperty" parameters and discuss in detail the specific multiproperty parameter problem of finding the cardinality of a maximum independent set which minimizes redundance.

71 PROJECTIVE SPACES AS MAXIMAL ARCS

Lynn Margaret Batten, University of Winnipeg

Let S be a projective space of dimension d ≥ 2 and order q ≥ 1 embedded as a maximal arc in a projective plane π of finite order n. Let G be a collineation group of π fixing S and line transitive on S. We show that G fixes no exterior line of S in π.

72

Permutation Polynomials Over the Integers (mod n).

D. Ashlock, California Institute of Technology

In this paper necessary and sufficient conditions are established for a polynomial with coefficients in $\mathbb{Z}_n$ to induce a permutation of $\mathbb{Z}_n$. For each n the number of such permutations is computed and some information about the structure of the group of permutations is derived.

73

Conjectures on the Number of Complete Subgraphs in a Graph

Jennifer Ryan* and David C. Fisher, University of Colorado at Denver

Let k_j be the number of complete subgraphs on j nodes in a graph G whose largest complete subgraph has w nodes. We present two related conjectures giving asymptotically sharp bounds on k_j.

Conjecture 1. $\left(\frac{k_{j+1}}{\binom{w}{j+1}}\right)^{\frac{1}{j+1}} \leq \left(\frac{k_j}{\binom{w}{j}}\right)^{\frac{1}{j}}$. (This is known to be true when $j = 1$ and when $w \leq 2j$).

Conjecture 2. Let ω_j be the largest number satisfying $\left(\frac{k_{j+1}}{\binom{\omega_j}{j+1}}\right)^{\frac{1}{j+1}} \leq \left(\frac{k_j}{\binom{\omega_j}{j}}\right)^{\frac{1}{j}}$. Then $\omega_1 \geq \omega_2 \geq \cdots \geq \omega_w = w$. (The first and last inequalities are known to be true).

74

Solving a Value-Independent Knapsack Problem with the Use of Analytical Methods.

Mark Chaimovich, School of Mathematical Sciences, Tel-Aviv University, Israel.

The value-independent knapsack problem is: maximize $z = \sum_{1\leq i\leq \overline{m}} a_i x_i$, subject to $\sum_{1\leq i\leq \overline{m}} a_i x_i \leq M$, $0 \leq x_i \leq m_i$, $x_i \in \mathbb{Z}$, $i = 1,\ldots,\overline{m}$, where $a_i, m_i \in \mathbb{N}$. By using the methods of additive number theory the range of the linear integer form $\sum_{1\leq i\leq \overline{m}} a_i x_i$ can be described as a collection of arithmetic progressions with a common difference. It allows us to design a new algorithm which works for problems satisfying the density relation $\max_i(a_i \log a_i) = O(\sum_{1\leq j\leq \overline{m}} m_j)$.

While the dynamic programming approach yields an $O(M\sum_{1\leq i\leq \overline{m}} m_i)$ algorithm, the new algorithm is substantially faster and even $O(\sum_{1\leq i\leq \overline{m}} m_i)$ in some cases. It also may be well suited either to solve unbounded (an implicit bound is $m_i = \lfloor \frac{M}{a_i} \rfloor$) and zero-one ($m_i = 1$) problems or to answer the question does the linear equation $\sum_{1\leq i\leq \overline{m}} a_i x_i = M$ has a solution in non-negative integers.

75 Size and independence in triangle-free graphs with maximum degree three

Kathryn Fraughnaugh Jones, University of Colorado at Denver

Let C be the class of triangle-free graphs with maximum degree at most three. A lower bound for the number of edges in a graph of C is derived in terms of the number of vertices and the independence. Several classes of graphs for which this bound is attained are given. As corollaries, we obtain the best possible lower bound for the independence ratio of a graph in C and evaluate some Ramsey-type numbers.

Keywords: graphs, independence, Ramsey numbers

76

Ovali di Roma

William Cherowitzo, University of Colorado at Denver

The remarkable non-Desarguesian projective planes discovered by Figueroa in 1982 can easily be described in purely geometric terms. Using this description, a new family of ovals, called the Ovali di Roma, was constructed in the finite Figueroa planes of odd order. After giving this description of the Figueroa planes, details of this construction and its proof will be given.

Keywords: Ovals, non-Desarguesian projective planes, Figueroa planes

77

Nets and Laurent series

Heinrich Niederhausen, Florida Atlantic University

How general can an index set I be and still allow for an algebra of Laurent series ? Our concept includes the incidence algebra as well as the algebra of formal Laurent series in infinitely many variables. Such Laurent series are the building blocks for generating functions like $1/(1-x\cdot t)$, where $x^n \equiv x_n$ and $t^n \equiv t_n$, $n \in I$, in umbral notation. For $I = \mathbb{N}_0^{\mathbb{N}_1}$, the methods of Umbral Calculus carry $1/(1-x\cdot t)$ to $(1-t/(1+t))^{-1}$ by substitution and evaluation, and determine the coefficients in this power series. They are known as the number of permutations of finite multisets having no two equal consecutive entries.

78

Duality for Graphs of Arbitrary Genus

L. K. Tolman, BYU

By modifying the definitions of "cutset" and "genus" (these modifications include the current meanings), we show that all graphs have duals. (Two graphs are duals if and only if there is an isomorphism between the edge sets such that a set of edges in one is a circuit if and only if the corresponding set of edges in the other is a cutset.) This will include Hasler Whitney's 1932 result as a specific case.

We use the Jordan Curve Theorem and some elementary results from algebraic geometry. The concepts of "α-splitting" and "β-splitting" are introduced, explained and used in the development.

Key words: α-splitting, β-splitting, cutset, dual graphs, genus

79

Receiving in General Graphs

Grant Cheston, University of Saskatchewan, Canada

Keywords: receiving, centrality, algorithm, network communication

The receiving problem is concerned with getting a distinct message from every vertex in a graph to a specified vertex, the receiver, i.e., an all-to-one message accumulation process. The objective is to determine the minimum time to complete all message transfers, the receiving time for the receiver, where we assume that one unit of time is required for a message to be transferred from one vertex to an adjacent vertex. In this version of the problem, we further assume that a vertex can only do one thing at a time: either send a message or receive a message. The receiving center of a graph is the set of vertices with the minimum receiving time. For trees, the receiving center is the same as the centroid, and for biconnected graphs, the receiving center is the whole graph. For other graphs, the receiving center depends upon the structure of the centroid biconnected component. We present several criteria which show that a vertex is in the receiving center.

80

On well-covered graphs with no 4 nor 5-cycles

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R. Nowakowski*, Dalhousie University, Halifax, Canada

A well-covered graph is one in which every maximal independent set of vertices is a maximum.

Let H(4, 5) represent the collection of graphs such that G belongs to H(4, 5) if and only if G has neither a 4-cycle nor a 5-cycle. We obtain a complete characterization of the well-covered graphs in H(4, 5).

TUESDAY, FEBRUARY 21, 1989 4:40 p.m.

81

Some Remarks on the Hall Plane of Order 16

Marialuisa J. de Resmini, Università di Roma "La Sapienza"

The plane in the title, say π, is the derived plane of PG(2,16), the Desarguesian plane of order 16. Therefore, π inherits some properties of PG(2,16) which make it quite different from the other translation planes of order 16. Of course, π is generated by quadrangles and the distribution of such quadrangles is studied. A quadrangle which does not generate π generates a Fano subplane of π. A Fano subplane of π is either maximal or extends to a unique Baer subplane whereas other translation planes of order 16 contain Fano subplanes which complete to more than one Baer subplane, e.g. the Dempwolff plane and the Johnson-Walker plane. Some results on the distribution of Fano and Baer subplanes are shown.
The most interesting fact about π is that it contains at least three non-equivalent types of complete 16-arcs. (The arcs of one of these types were already known.) The classification into inequivalent types is carried out by looking at the distribution of the tangents to the arcs in the dual plane.
Some results on small complete arcs and hyperovals are provided too.

82

Asymptotically Optimal Chromatic Configurations in Random Graphs and Matrices

Eli Shamir
The Hebrew University, Jerusalem
On visit at Bellcore, Morristown, NJ

In a graph G (or hypergraph) of size n let $\alpha(Q,G)$ be the maximum size of a vertex set satisfying the property Q. For example Q_0 is edge-free, Q_K is K-free, Q_ϵ is the property of having $\leq \epsilon \cdot n$ edges. A Q-chromatic configuration in G is a decomposition:

(1) $V = \bigcup_{i=1}^{s} A_i$, A_i disjoint, A_i satisfy Q.

(2) $X(Q,G)$ is the minimum possible s in (1).

A desirable goal is to prove:

(3) $\alpha(Q,G) \cdot X(Q,G) = n(1+o(1))$.

for an asymptotic family of graphs (with size $n \to \infty$), i.e. there exists a covering of almost all of V by Q-sets of nearly maximum size.

Recently, this goal was achieved in the 1-o(1) probability-sense, for Q_0 (the ordinary chromatic number), for the random graph spaces $G_{n,p}$ with p=constant or $p(n) \to 0$ (also for hypergraphs) [Bollobas, Luczak, Matula, Shamir]. We extend it to general Q_K, where K is a connected pattern and also to the property Q_ϵ for random matrices.

The method of proof is a confluence of martingale estimates and second moments estimates.

83

On Total Covers of Graphs

Yousef Alavi*, Liu Juiqiang and Songlin Tian, Western Michigan University
Jianfang Wang, Academia Sinica, Beijing, Zhongfu Zhang, Lanzhou Institute, China

A total cover of a connected graph G is a subset of $V(G) \cup E(G)$ which covers all elements of $V(G) \cup E(G)$. The total covering number $\alpha_2(G)$ of a graph G is the minimum cardinality of a total cover in G. Alavi et. al, in a paper on the subject, had posed the conjecture that for a connected graph G of order P, $\alpha_2(G) \leq \lceil \frac{P}{2} \rceil$. Here we give some properties of connected graphs which have total covering number $\lceil \frac{P}{2} \rceil$ and prove the above conjecture.

84

POSSIBLE AUTOMORPHISM GROUPS OF AN S(3,5,26)

Earl S. Kramer*, Spyros S. Magliveras*, Tran van Trung**
University of Nebraska-Lincoln*, University of Heidelberg**

In this work we consider all possible automorphism groups for a Steiner system $S(3,5,26)$. Initially there is a very large number of possible groups for these Steiner systems but we are able to reduce the possibilities to a feasible number of cases. The techniques used include a combination of group theoretic methods and knowledge of the structures of the Steiner systems $S(2,4,25)$ and their automorphism groups. The structures of all possible groups for an $S(3,5,26)$ are completely determined and the groups are listed in the present paper.

Keywords: Steiner systems, Automorphism groups

85

On Opsut's Conjecture and Other Problems Concerning Competition Numbers, Double Competition Numbers, and Niche Numbers

Suh-ryung Kim, St. John's University, and Fred S. Roberts*, Rutgers University

If G is a graph, its competition number $k(G)$, double competition number $dk(G)$, and niche number $n(G)$ are, respectively, the smallest number of isolated vertices to make G into a competition, double competition, or niche graph. We present a number of recent results and open questions about these numbers. Opsut's conjecture, motivated by the special case of line graphs, says that if the neighborhood of every vertex of graph G can be vertex-covered by at most two cliques, then $k(G) \leq 2$. We prove a variant of this conjecture. We also describe the solution to Harary's problem about the largest $k(G)$ for G of n vertices (which gives a theorem which generalizes that of Turan), a result and open question about $dk(G)$ for G bipartite, and the conjecture that every graph with finite $n(G)$ has $n(G) \leq 2$.

86

UNITALS IN THE DESARGUESIAN PLANE

R.D. Baker and G.L. Ebert*, University of Delaware

E.F. Assmus and J.D. Key have recently undertaken a program which attempts to use algebraic coding techniques to classify translation planes. One aspect of their work deals with the intersection of unitals in a Desarguesian projective plane P of order q^2, where $q=p^r$ for some prime p, and they have asked if any hermitian unital must meet an arbitrary unital in a number of points that is congruent to 1 modulo p. This is known to be true for the intersection of two hermitian unitals, and in fact the possible intersection configurations have been determined by Kestenband. In this paper we construct an infinite family of nonhermition unitals and address the problem of how these unitals intersect other unitals.

87

Circular Avoiding-Sequences with Minimum Sum

Dean S. Clark and James T. Lewis*, The University of Rhode Island

Let x and n be positive integers. We say the finite sequence $a_1,\ldots,a_n$ of positive integers is a <u>circular sequence of length n which avoids x</u> if no set of consecutive terms of the infinite sequence $a_1,\ldots,a_n,a_1,\ldots,a_n,\ldots$ sums to x. If $a_1,\ldots,a_n$ are arranged consecutively in a circle, then no "arc" of terms sums to x, even if "wrap-around" is allowed. For example, the circular sequence 1,1,1,5 avoids x=4,x=12,x=20,... . In a previous paper (to appear in J. Discrete Applied Math.) we showed that the minimum sum $a_1+\ldots+a_n$ for a circular sequence of length n which avoids x is $\geq 2n$ and that this lower bound can be attained if and only if x/gcd(x,n) is odd. In the present paper we study the minimum sum when x/gcd(x,n) is even.

90

Interval Competition Multigraphs of Food Webs

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Interest in interval competition graphs of acyclic digraphs dates from Joel Cohen's observation that competition graphs of naturally occurring food webs tend to be interval graphs.

We show that this tendency fails under the natural upgrading to competition <u>multi</u>graphs. Moreover, even when a competition multigraph is interval, a particular subpath representation (or interval assignment) for it may only closely correspond to certain digraphs among all those having the competition multigraph.

We study this correspondence between digraphs and representations and the modeling issues involved with interval competition multigraphs.

On two conjectures about edge-colouring hypergraphs.

88

A.J.W. Hilton, Reading University, England

Berge, Füredi and Meyniel have independently conjectured that the chromatic index $\chi'(H)$ of a linear hypergraph H satisfies the inequality

$$\chi'(H) \leq 1 + \Delta(H_2) .$$

where $\Delta(H_2)$ denotes the maximum degree of the graph formed from H by replacing each edge of cardinality at least three by a complete graph on the same set of vertices, and by removing all loops. Here we prove a stronger statement in the case when the set of edges of cardinality at least three is independent.

Similarly Berge conjectured that the chromatic index of the hereditary closure $\hat{H}$ of any linear hypergraph H satisfies

$$\chi'(\hat{H}) = \Delta(\hat{H}) .$$

Here we prove this conjecture in the special case when there are no edges of cardinality greater than three, and the set of edges of cardinality three is independent.

89

ON THE STEINER TREE PROBLEM

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A number of important practical combinatorics and combinatorial optimization problems can be formulated and solved by network related algorithms. One such problem with a wide variety of practical applications such as communication networks is the Steiner tree problem. The Steiner tree problem in networks is the problem of finding the shortest tree connecting a given set of nodes in a connected undirected distance network. Since the problem of finding a minimal Steiner tree in networks is NP-Complete there is a strong evidence that no polynomial time algorithm can be found to solve this problem. Hence research has been towards finding good approximation algorithms, including one of the author's recent algorithm. We survey approximation algorithms that are guaranteed to be a fixed percentage away from the minimal one. We also discuss their performances under different circumstances.

91

Tight Pointsets in Finite Generalized Quadrangles II

Stanley E. Payne, University of Colorado at Denver

A set A of points in a generalized quadrangle (GQ) with parameters (s,t) is i-tight provided $|A| = i(1+s)$ and each point of A is collinear with exactly $s+i-1$ other points of A. Tight sets, which correspond to certain eigenvectors of an associated incidence matrix, were introduced by us in *Congressus Numerantium* 60 (1987), 243–260. Several new constructions are given here, and some progress is made in determining 3-tight sets. A new model of $W(4)$ (the unique GQ with 85 points and 85 lines) is helpful in considering the case $(s,t) = (3,5)$.

Key Words: Finite Generalized Quadrangles, eigenvector, tight set.

92

High Sum Wins Contests With Discretely Distributed Random Variables

John W. Hilgers
Michigan Technological University

The variation of probabilistic advantage with N of two competing random variables each sampled N times is examined using a half infinite Fourier type integral. This approach is extended to the case of discretely distributed variables and illustrated with several examples.

93

Subpermanents of (0,1)-Matrices

John L. Goldwasser, West Virginia University

The t-subpermanent of a matrix A, denoted by $p_t(A)$, is the sum of the permanents of all t x t submatrices of A. Let $\Lambda(n,k)$ be the set of all (0,1)-matrices with all row and column sums equal to k. For a given t, for which matrices A in $\Lambda(n,k)$ is $p_t(A)$ a maximum? A minimum? This paper gives a partial answer to these questions and offers some conjectures.

94 THE LINEAR COMPLEXITY PROFILE OF CRYPTOSYSTEM PGM

Spyros S. Magliveras* and Nasir D. Memon, University of Nebraska-Lincoln

A fast and secure cryptographic system, called *PGM*, was invented in the late 1970's by Spyros Magliveras. *PGM* is based on non-solvable permutation groups of moderate size. In particular, *PGM* is based on the prolific existence of certain kinds of bases, called *logarithmic signatures*, for the given group. In this paper we present recent results on the Linear Complexity Profile , of *PGM*-generated pseudorandom sequences. By the *aperiodic linear complexity* (ALC) of a given sequence $\Sigma = \{x_1, x_2, \cdots, x_n\}$; $x_i \in Z_m$, we mean the length of the shortest linear-feedback shift register which generates the sequence followed by some arbitrary sequence. For a given sequence Σ of length n, the function $F_\Sigma : Z_n \to Z_n$ defined by $F_\Sigma[k] =$ ALC $\{x_1, x_2, \cdots, x_k\}$ is called the *linear complexity profile* (LCP) of the sequence. We show that *PGM is* LCP–*strong*, in the sense that the LCP follows closely but irregularly the line $y(k) = k/2$.

Keywords: Cryptosystem, *PGM*, Permutation Groups, LCP, LFSR

95

Generalized Competition Graphs

J.R. Lundgren and J.S. Maybee, University of Colorado
and Craig Rasmussen

We consider the competition graph corresponding to a loopless symmetric digraph D with corresponding graph H. The competition graph G of D is then the 2–step graph $S_2(H)$. We obtain a variety of of special results when H has given properties. In particular, we characterize the conditions on H for which $S_2(H)$ is an interval graph. We also investigate conditions on H such that $S_2(H)$ is chordal. Some of our results are extensions to the loopless case of theorems obtained by Raychaudri and Roberts on symmnetric digraphs with loops. Our theorems have application to the channel assignment problem.

96

CLAW-FREE TRIANGULATIONS OF THE PLANE

M.D. Plummer, Vanderbilt University

A planar graph G is maximal planar if given any embedding of G in the plane, all faces are triangles. (The triangular graph K_3 has a unique planar embedding and so do all larger maximal planar graphs since they must be 3-connected. So *maximal plane* and *maximal planar* are equivalent. A graph with these properties is also said to be a *triangulation*.) Graph G is claw-free if it contains no induced subgraph isomorphic to the complete bipartite graph $K_{1,3}$.

We give a constructive characterization of the family of claw-free maximal planar graphs. In particular, there are precisely eight such graphs having no separating triangle, while if G is maximal planar with a separating triangle, then G is shown to belong to one of three infinite families or to a fourth family consisting of precisely seven graphs.

Keywords: planar, triangulation, claw-free

97

3- 4- and m-dimensional Catalan Numbers

Stephanie F. Troyer: & S.L. Snover

We generalize the Catalan sequence $\frac{(2n)!}{n!(n+1)!}$ by defining a Catalan word on m symbols (a_i) to be a word of length mn containing each symbol n times in such a way that a_i dominates a_{i+j} in every initial segment of the word. We use a correspondence with paths on m dimensional lattices to prove that the number of such words is a generalized Catalan number Cat (m,n) = $\frac{(m-1)!!\,(mn)!}{n!(n+1)!\ldots(n+m-1)!}$.
We also discuss the geometry associated with the three and four dimensional cases Cat (3,n) = $\frac{2\,(3n)!}{n!(n+1)!(n+2)!}$ and Cat (4,n) = $\frac{12\,(4n)!}{n!(n+1)!(n+2)!(n+3)!}$.

98 ON A MATRIX DISCREPANCY PROBLEM

T. D. Porter and L. A. Székely*, University of New Mexico

We solve the following problem posed by R. Entringer:
Let $A=(a_{ij})$ be an $n \times m$ matrix, $S_1 \cup S_2 \cup \ldots \cup S_m = \{1,2,\ldots,n\}$. What is

$$\max\ \min\ \max_{i=1}^{m} \sum_{j \in S_i} a_{ij}$$

where the minimum is taken for all disjoint decompositions of $\{1,2,\ldots,n\}$, and the outer maximum is taken for all $n \times m$ matrices with constant row sum 1.

99

A Good Case For Shellsort

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ABSTRACT

For many sorting algorithms, such as insertion sort and simple-minded quicksort, the worst case occurs for permutations in reverse order.

We prove that the running time for Shellsort using any $O(\log N)$ increment sequence on permutations in reverse order is $O(N \log N)$, which is apparently much better than average. Shellsort appears to be the first non-trivial sorting algorithm with this property.

101

ON INEQUALITIES INVOLVING VERTEX–PARTITION PARAMETERS OF GRAPHS

Stefan A. Burr*, Dept. of Comp. Sci., City College, C.U.N.Y.
Michael S. Jacobson, Dept. of Math., Univ. of Louisville

Let P be a property of graphs. Then define $\chi_P(G)$ to be the smallest integer k such that the vertices of G can be partitioned into k classes in such a way that each class induces a graph with property P.Call P hereditary if it is closed under taking induced subgraphs and disjoint unions, and is neither nor empty nor universal. A theory of inequalities involving different parameters of the type χ_P is developed which includes a number of properties of interest. The main result is the following:
If properties P and Q are hereditary, then

$$\chi_P(G) \le \chi_P(Q)\chi_Q(G)$$

for all graphs G, where $\chi_P(Q)$ is the maximum value of χ_P among all graphs having property Q. Moreover, this inequality is sharp when P satisfies certain conditions. Some applications are given for specific choices of P and Q; some of these involve Ramsey–type questions.

Key words: Vertex–coloring, Ramsey theory, Generalized colorings, Generalized chromatic number.

102

Constructing Polygonal Graphs of Large Girth and Degree

by *Dan Archdeacon**, THE UNIVERSITY OF VERMONT,
and *Manley Perkel*, WRIGHT STATE UNIVERSITY

An (r, m)-polygonal graph is an r-regular graph of girth m together with a set C of m-cycles such that each path of length two lies in a unique element of C. Using a construction which doubles m we build (r, m)-polygonal graphs with r and m arbitrarily large. Quotients of these give other examples of polygonal graphs.

103

PROCESSOR INTERCONNECTION NETWORKS FROM CAYLEY GRAPHS.

Steve Schibell * and Dick Stafford, Department of Defense

An essential component of a computer based upon large - scale parallel processing is its interconnection network. Interconnection networks are modeled by graphs. At the 1986 SIAM international conference on parallel processing, Sheldon Akers and BalaKrishnan Krishnamurthy suggested using Cayley graphs of groups as models for interconnection networks. Since then there has been an explosion of activity directed toward applying group theory to the design of network architectures for supercomputers. In this talk we present our work on the routing problem. This research can be regarded as a first attempt to find general purpose routing algorithms for interconnection networks. In addition we shall present some promising new interconnection networks.

104

Uniformly Packed Codes Associated to Clifford Algebras

Charles T. Ryan Seton Hall University

We will arrive at a more detailed understanding of a well known uniformly packed code by means of an investigation which reconstructs its dual in terms of the geometry of the Quadric Grassmann Variety G*(3,7). The elements of G*(3,7) are isotropic subspaces of maximal diminsion lying on a quadric and may be associated to pure spinors in much the same way that elements of the Grassmann Variety G(k,n) are associated to decomposable elements of an exterior algebra. In particular it will be shown that this code may be taken as the linear span of the rows of an incidence matrix whose rows and columns are in turn indexed by the points of G*(3,7). This geometric interpretation along with a natural action of the orthogonal group O(7) on G*(3,7) will be used to exhibit the minimal weight codewords as affine charts of a quadric grassmanian and will lead to a more explicit description of the codes automorphism group. Essential parameters of a related family of linear block codes will also be discussed.

105

Graphs that require many colors to achieve their chromatic sum

Paul Erdös, Hungarian Academy of Sciences
Ewa Kubicka * and Allen J. Schwenk, Western Michigan University

The chromatic sum of a graph is the minimum total of the colors on the vertices taken over all possible proper colorings using natural numbers. In an earlier article we showed that even for trees we may need arbitrarily many colors, but the smallest tree requiring k colors is of order $O((2+\sqrt{2})^k)$. Our object is to find the smallest possible graphs that require t colors beyond their chromatic number k. We present three constructions depending on the ratio $\frac{t}{k}$. The resulting graphs grow linearly, quadratically, cubicly, and exponentially in k depending on the t chosen. The construction is proven to be best possible for t = 1 and all k.

106

A New Chromatic Number

Andrew Vince, University of Florida

A generalization χ_n, n=2,3,..., of the chromatic number of a graph is introduced such the colors are integers modulo n, and colors on adjacent vertices are required to be as far apart as possible. The ordinary chromatic number χ is one of the χ_n and a new invariant $\chi^* = \inf \chi_n$ can be though of as the "best possible" coloring. For example, the chromatic number of an odd cycle is $\chi(C_{2m+1}) = 3$, but C_{2m+1} can "almost" be 2-colored. In fact $\chi^*(C_{2m+1}) = 2 + 1/m$.

107

Bounds for mar(G) and med(G)

Garry L. Johns, Saginaw Valley State University

For a connected graph G, the distance d(v) of a vertex v is the sum of the distances from v to every other vertex of G. The minimum value for d(v) is called the median of G and is denoted by med(G). Similarly, the maximum value of d(v) is called the margin of G and is denoted by mar(G). In this talk some known bounds are extended and various values for B are discussed where B satisfies mar(G) ≤ B[med(G)].

Key words: distance, median, margin

108

"Organized C" - AN EASY WAY TO PROGRAM ALGORITHMS

Jiri Soukup, Code Farms Inc, Richmond, KOA 2Z0, Canada

"Organized C" is a new method for automatic handling of data structures in C or C++ programs. You don't have to learn a new language, and you still get more than what the object oriented languages can offer. The method is a spinoff from the VLSI CAD. It is based on a graph of pointers, and it is especially suited to complicated graph and combinatorial algorithms. It is easy to learn, strongly typed, protected against typical errors, and it creates a code which is easy to maintain. Thought this is hard to believe, it has practically no run time or memory overhead compared to hand coded C. The method is equally applicable to small programs or big programming systems, and contains number of database-like features (saving/retrieving whole organizations, extensibility, selfID, etc). The program runs on any UNIX or DOS machine. Current experince indicates about 3-4 times faster time for coding and debugging.

109

Subcodes of Hamming Codes

Richard A. Brualdi*, University of Wisconsin and Vera S. Pless, University of Illinois at Chicago

Let C be a binary code of length n. The covering radius of C is the maximum distance of a binary n-tuple to the code. Although Hamming codes are an important and familiar family of codes, there seems to be little known about their subcodes. In this note we consider the covering radii of a chain of codes of consecutive dimensions that begins with a Hamming code and ends with the zero code.

110

Posets with Isomorphic Upper and Lower Bound Graphs

Deborah J. Bergstrand,* Williams College
Kathryn F. Jones, University of Colorado at Denver
William R. Sherman, Williams College

If (P,≤) is a finite partially ordered set (poset), then the <u>upper bound graph</u> of P is the graph G = (V,E), where V = P and xy is an edge iff x ≠ y and there exists a z in P such that x,y ≤ z. The <u>lower bound graph</u> H of P is defined analogously. If P is isomorphic to its dual, then clearly G is isomorphic to H. Here we show that the converse is also true in the case of height-1 posets (those with only maximal and minimal elements).

111

Primal Graphs with Maximum Degree Two

Phyllis Z. Chinn*, Humboldt State University
R. Bruce Richter, Carleton University
Mirosław Truszczyński, University of Kentucky

It has previously been shown that there exists a unique set Π of *primal graphs* such that every graph has an edge-decomposition into non-isomorphic elements of Π and such that the only such decomposition of a graph in Π is the trivial one.

Here we explore primal graphs of maximum degree 2. All the primal graphs which contain C_4 are determined. It is shown that there are an infinite number of unknown primal graphs of maximum degree 2 and a few of these are determined.

KEY WORDS: graph factoring, graph decompositions, primal graphs

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THE BOOK THICKNESS OF A GRAPH, II

Paul C. Kainen, George Washington University

This paper is a sequel to our original work with Frank Bernhart (JCT, B27, 1979). Let bt(G) denote the page number (book thickness) of graph G and let bw(G) be the cyclic bandwidth (length of the longest edge with respect to a cyclic ordering of the vertices, minimized with respect to all such vertex orders). Thm 1 bt(G) $\leq$ 2bw(G)-1. Theorems by Nash-Williams or Scheinerman and West give: Cor f(G) $\leq$ 3bw(G)-1, where f(G) is either arboricity or interval number. Thm 2 bt(G) $\leq$ 1+locr(G), where locr(G) denotes an outerplanar variant of Ringel's local crossing number.

Keywords: book thickness, bandwidth, local crossing number

113

On a System of Two Boolean Linear Equations.

Gregory A. Freiman, School of Mathematical Sciences, Tel-Aviv University, Israel.

The solvability of a system of two boolean linear equations is considered. Although the problem is known to be NP-hard, the methods of analytical number theory allow us to solve its dense version in polynomial time. New approach is applicable to systems which coefficients are bounded relatively to the number of unknowns and which right hands are in the certain wide neighborhood of the middle point of the sum of coefficients (unlike the dynamic programming which is well suited only for small right hands). New method can also be effectively used to solve a special case of 0-1 knapsack problem with two constraints when the objective function is a linear combination of the constraints.

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ON THE NUMBER OF INFORMATION SYMBOLS OF LONG GOPPA CODES

Oscar Moreno, Ivan Velazquez* and Ivelisse Rubio, University of Puerto Rico

In this paper we compute the number of information symbols of long Goppa Codes when the Goppa polynomial does not have repeated roots. We do this using character sums techniques .

115

Graphs and Posets: Some Common Parameters

J. Lalani*, R. Laskar and S.T. Hedetniemi, Clemson University

Given a poset (X,P), a set L of linear extensions is a realizer if $\cap L = P$. The minimum cardinality over all minimal realizers is the dimension of P, whereas the maximum cardinality is the rank of P. A set $\{L_1, L_2, \ldots, L_r\}$ of linear extensions (not necessarily a realizer) is irredundant if for every $i \neq k$, $\cap(L_k - L_i) \neq \phi$. The irredundance number of P, ir(P) and upper irredundance number IR(P), are respectively the minimum and maximum cardinalities over all maximal irredundant sets of linear extensions. It is well known that

$$ir(P) \leq dim(P) \leq rank(P) \leq IR(P) \leq |I|/2,$$

where I denotes the set of all ordered incomparable pairs in (X,P). This paper gives a necessary condition in terms of the width of a poset for which the upper bound of IR(P) is not attained.

116

Primal graphs on up to 14 edges

E. Regener* and S. Stairs
Concordia University, Montreal, Canada

The set Π of *primal* graphs is such that every simple graph has an edge-decomposition into non-isomorphic graphs $\in \Pi$ and the elements of Π are just those G for which this decomposition is trivial. Hitherto the primal graphs were completely known only up to those with 8 edges. We prove that there are no $G \in \Pi$ with 10 or 12 edges, and 22 with 14 edges. The results were suggested by a computer search.

Key words: Primal graphs, graph factors, edge decomposition, depth-first search.

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LOWER BOUNDS FOR LOWER RAMSEY NUMBERS.

Ralph Faudree, Memphis State University, Ronald J. Gould, Emory University, Michael S. Jacobson,* University of Louisville, Linda Lesniak, Drew University.

For any graph G, let i(G) and μ(G) denote the smallest number of vertices in a maximal independent set and maximal clique, respectively. For positive integers m and n, the lower ramsey number, s(m,n) is the largest integer p so that every graph of order p has $i(G) \leq m$ or $\mu(G) \leq n$. In this paper we give several new lower bounds for s(m,n) as well as determine precisely the values s(1,n).

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Partial Order Graphs and Spectral Perturbations

Jeffrey L. Stuart, University of Southern Mississippi

A ZME-matrix is a real matrix all of whose positive integer powers are Z-matrices, and all of whose even positive integer powers are irreducible. It is known that ZME-matrices are diagonalizable with real spectra, and that the set of spectral projectors for a ZME-matrix form a partially ordered set under a simple combinatorial relation on the entries of the projectors. We describe the combinatorial relation, characterize the partial order graphs that can occur, and show that the partial order graph determines all spectral perturbations of a fixed ZME-matrix which are again ZME-matrices.

Keywords: spectral perturbation, Z-matrix, partial order graph

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p–adic Computation of the Rational Normal Form of a Matrix

Marie–Helene Mathieu, Computer Science, Concordia University, Montreal, Canada H3G 1M

We consider the problem of bringing a given matrix into"cyclic form," from which the rational form can be computed easily. Matrices are taken to have p–adic integer entries,and computations are done with rational integer approximations to p–adic integers. We give bounds on the precision necessaryto ensure that the resulting cyclic form is indeed similar tothe original matrix. We also give a criterion for deciding whether the cyclic form is correct. In practice, this criterion suffices to achieve the cyclic form at a much lower level of precision than is implied by the theoretical bounds.

Keywords: rational canonical form, p–adic.

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Undetected Error Probabilities for Message Bits of Certain Cyclic Codes

Chi-Chin Chao and Larry A. Dunning,* Bowling Green State University

Given any fixed linear block code used for error detection, the undetected error rates for the message symbols depend on the encoding map or generator matrix used. An algorithm is given for finding a generator matrix which minimizes the undetected error rates for all message symbols simultaneously for transmission over a binary symmetric channel. The algorithm is basically an application of the greedy algorithm of matroid theory to the code's dual vector space considered as a matroid with certain weight distributions as the weighting function. The algorithm has been applied to some cyclic codes of small dimension. The optimal generator matrices produced are always also optimal with respect to Unequal Error Protection, but the converse of this result is found to be false. A (35,18) cyclic code is found to have separation vector exactly equal to (10,10,10,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4) improving a table of van Gils [IEEE Tr. Info. Th., vol. IT-29, pp.866-876, 1983]. In general, the optimal encodings found are nonsystematic. It is shown that a systematic generator matrix for a cyclic code always has the same undetected error rate for every message symbol regardless of the particular generator matrix chosen and that rate is always at least as large as the rate for any message symbol using an optimal encoding. For every optimal generator matrix of a cyclic code which was computed, the maximum undetected error rate of all message bits was exactly equal to the error rate of any message bit using a systematic encoding. Whether or not this is true in general is an open question.

Index Terms: coding theory; matroids; cyclic codes; unequal error protection; combinatorial search algorithms

WEDNESDAY, FEBRUARY 22, 1989 11:10 a.m.

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DECOMPOSITION OF 3-CONNECTED GRAPHS

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Cunningham and Edmonds have proved that a 2-connected graph G has a unique minimal decomposition into graphs, each of which is either 3-connected, a bond or a polygon. They define the notion of a good split, and first prove that G has a unique minimal decomposition into graphs, none of which have a good split, and second prove that the graphs that do not have a good split are precisely 3-connected graphs, bonds and polygons. This paper provides an analogue of the first result above for 3-connected graphs, and an analogue of the second for minimally 3-connected graphs. Following the basic strategy of Cunningham and Edmonds, an appropriate notion of good split is defined. The first main result is that if G is a 3-connected graph, then G has a unique minimal decomposition into graphs, none of which have a good split. The second main result is that the minimally 3-connected graphs that do not have a good split are precisely cyclically 4-connected graphs, twirls ($K_{3,n}$ for some $n \geq 3$) and wheels. From this it is shown that if G is a minimally 3-connected graph, then G has a unique minimal decomposition into graphs, each of which is either cyclically 4-connected, a twirl or a wheel.

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Ramsey Numbers for Stars Versus Complete Multipartite Graphs

Scott Stevens, James Madison University

Let G and H be simple graphs. We define the <u>Ramsey number</u> r(G, H) to be the smallest integer M for which the following holds: if the edges of a complete graph K_M are colored red and blue, either the red graph contains a copy of G or the blue graph contains a copy of H. We focus on the cases where G is a star K(1,n) and H is a multipartite graph $K(k_1, k_2, \ldots, k_t, m)$, with each $k_i \geq 2$. Burr, Faudree, Rousseau, and Schelp have determined the value of these Ramsey numbers for all m sufficiently large; for fixed $k_1, k_2, \ldots k_t$, n, their proof requires an m at least on the order of n^{2nk+2}, where $k = k_1 + k_2 + \ldots + k_t$.

In this paper, we show that the formula for these Ramsey numbers given by Burr, et al. applies equally well for much smaller m. The new lower bound on the required size of m is on the order of only n^{k+1}. Central to our work is an exact characterization of the critical graphs for stars versus complete bipartite graphs.

<u>ON RECOGNIZING MIXED DIAGONAL CONSECUTIVE ONES GRAPHS</u>

123

MARGARET COZZENS AND N.V.R. MAHADEV*

A 0-1 matrix is said to have consecutive ones property for columns if its rows can be rearranged so that in each column all the ones appear consecutively. A linear time algorithm to recognize such matrices is given by Booth and Lueker. A graph is called a mixed diagonal consecutive ones graph (MDC-graph) if there is an assignment of zeros and ones along the diagonal of its adjacency matrix so that the resulting matrix has consecutive ones property. In other words, a simple graph on n vertives is an MDC-graph if there is an ordering of vertices from 1 to n such that for each vertex x, either its neighborhood N(x) or its closed neighborhood N(x) U {x} appears consecutively in that order. We give efficient algorithms to recognize these graphs and to find a minimum coloring and maximum clique for these graphs.

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Circular Hamming Arrays

L.J. Cummings, University of Waterloo

A Hamming array is an m × n array of symbols from a finite alphabet such that adjacent rows differ in exactly one position. A Hamming array is circular if the first and last rows are considered adjacent. For example, a Gray code is a circular Hamming array. We discuss necessary and sufficient conditions for both an array and its transpose to be circular Hamming arrays. We determine all strings over finite alphabets whose n-windows when taken in order as rows of an array yield a Hamming array.

125

On the Cut-number of the 5-Cube

M. R. Emamy-K.
University of Puerto Rico

A method of approach to the solution of cut-number of the 5-cube ($k(c^5)$) is proposed. This method is based on the existing solutions of the problem for the 4-cube. Basic lemmas of the proofs for $k(c^4) = 4$ are extended to that of 5-cube. Theorems analogous to this result will solve the cut-number problem for the 5-cube.

126 On the Maximum Size of a Graph Partitioning.

Mark Goldberg*, Thomas Spencer, RPI

It is well known that every graph with p vertices and q edges can be vertex partitioned so that the size of the partition (= the number of edges cut) is at least $w_a = (q/2)(1 + o(1))$. In fact, for almost all graphs, the maximum size w_{max} of a partition is asymptotically equal to w_a, if $(q/p) \to \infty$. Our main result is the following lower bound for w_{max}.

Theorem. *For every connected graph with p vertices and q edges, there is a vertex partition of size* $(2q + p - 1)/4$.

We also describe the class of graphs for which the bound is an equality. Thus, this establishes the asymptotics of $\min w_{max}(G)$, where G ranges over the set of sparse graphs ($q = O(p)$).

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The Ramsey Number $R(K_5{-}e, K_5{-}e)$

STANISŁAW P. RADZISZOWSKI*, Rochester Institute of Technology
ALBERTO TUBILLA, Universidad Nacional Autónoma de México

We show that the exact value of the Ramsey number $R(K_5{-}e, K_5{-}e)$ is 22. The lower bound is obtained by a self complementary graph G, which is the unique up to isomorphism graph on 21 vertices such that neither G nor $\bar{G}$ contain a $K_5{-}e$. The uniqueness of the latter and the upper bound are established by using several computer algorithms. The previously best known bounds for this number were 21 and 23, respectively.

Computer Viruses and Collision-free Hash Functions Based on Discrete Log

128 George B. Purdy, University of Cincinnati

A function f(x,y) is collision-free if it is computationally infeasible to find two pairs (x,y) and (x',y') such that f(x,y) = f(x',y'). We construct such a function and prove that its collision-freeness is as secure as the difficulty of obtaining a discrete log modulo p, where p is a large prime (for example, a 100-digit prime). The current estimate for obtaining such a log is roughly 50 million Cray weeks. The function f(x,y) could be used to 'hash' a software file to obtain a checksum which could then be used to detect unauthorized alterations due to, for example, computer viruses.

129

Construction of Byte Error Control Codes Using Partial Steiner Systems.

W. Edwin Clark*, *Department of Mathematics, University of South Florida;* Larry A. Dunning, *Department of Computer Science, Bowling Green State University;* D. G. Rogers, *School of Mathematical, East Anglia University.*

A design theoretic approach to binary linear codes which are single-error-correcting (sec) and double-error-detecting (ded) with the additional capability of detecting any error within a single byte of even width (bed) is developed. A construction of sec-bed-ded codes using ordinary binary linear codes in conjunction with partial Steiner systems is given. The construction produces some codes with higher rates than known previously. The codes constructed may inherit further special properties from the Steiner systems they are derived from. In particular, some "rotational odd weight column" codes are obtained.

KEYWORDS: binary linear code, single-error-correcting, double-error-detecting, byte-error-detecting, partial Steiner system.

130 Vertex Diameter Critical Graphs

Alfred Boals*, Naveed Sherwani, Western Michigan University
Hesham Ali, University of Nebraska-Omaha

A graph is vertex k diameter critical (DC(k)) if the graph has diameter k and the deletion of any vertex increases its diameter. We show that any graph is an induced subgraph of a DC(2) graph. We investigate several classes of graphs with respect to this property.

131 An update on conjectures of Graffiti.

Siemion Fajtlowicz, University of Houston.

I will discuss some solved and a few new conjectures of Graffiti.

Among solved conjectures are " chromatic number ≤ rank" refuted by Alon and Seymour, " residue ≤ independence" proved by Favaron, Maheo and Saclé , a few conjectures related to Turan's Theorem, some of which were proved by Shearer and Tura, and " chromatic number ≤ frequency of mode of degree", now completely solved by Erdős , Staton and myself.

Some of the new conjectures are about graphs in which independence can be expressed in terms of spectral invariants and some other are about graphs derived from certain number-theoretical relations.

132 **ZERO SUM RAMSEY THEOREMS**

Arie Bialostocki* and Paul Dierker

We replace the notion of "coloring by n colors and asking for monochromatic configurations" known in Ramsey theory by "coloring by Z_n, the ring of integers module n, and asking for certain polynomial identities." We review known results in this direction, state some conjectures and prove some particular cases. Some of the results are related to the Erdős Szekeres Theorem of 1935, others generalize the Erdős-Ginzburg-Ziv Theorem of 1961.

CONJECTURE 1 *If the edges of K_n are mapped arbitrarily into the integers, then there exists a spanning tree in K_n the sum of whose edges is divisible by $n-1$.*

CONJECTURE 2 *For every two natural numbers q and $n, n \geq 3$, there is a natural number $C(n,q)$ satisfying the following: Let S be any set of points in the plane such that no three points are collinear. If $|S| \geq C(n,q)$, then there are n points of S which are the vertices of a convex n-gon for which the number of points of S in its interior is divisible by q.*

CONJECTURE 3 *If $a_1a_2a_3\ldots$ is an arbitary sequence of elements from Z_n, then for every ℓ there exists ℓ consecutive blocks of equal length such that in each block the sum of the elements equals their product.*

Conjecture 1 was proved by the authors in the case where $n-1$ is a prime. Conjecture 2 was proved by the authors jointly with W. Voxman for every n and q such that $n \geq q+2$. Conjecture 3 was proved by N. Alon in the case when n is a prime.

Key words: Ramsey theory.

133 **Sorting on a mesh-connected computer: lower bounds**

Yijie Han
Yoshihide Igarashi
Miroslaw Truszczynski
University of Kentucky, Lexington

Abstract

A mesh-connected array of processors is widely accepted as a realistic model of a parallel computer. Sorting algorithms for such computers have been studied extensively. In the talk we discuss a method of proving lower bounds for sorting algorithms. The method, called *joker zone* method, is powerfull enough to show optimality of some of the algorithms. The power of the method is discussed. It is tied to a certain combinatorial parameter of rectangular arrays, called *stretch*. By studying this parameter we can conclude, in particular, that in the case of square $n \times n$ arrays, no matter what *indexing function* (it assigns the final destination in the array of processors for the element of rank i) is used, sorting in less than $2.27n$ steps is impossible. On the other hand, we also show that there are indexing functions such that the best lower bound that the joker zone method implies is $2.46n$.

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On strong complete mappings.

Anthony B. Evans, Wright State University.

We ask ourselves which groups admit strong complete mappings, i.e. complete mappings that are also orthomorphisms. A permutation θ of the elements of a group is a complete mapping if the mapping $x \to x\theta(x)$ is a permutation and an orthomorphism if the mapping $x \to x^{-1}\theta(x)$ is a permutation.

We give a partial answer to this question and show that the existence of a strong complete mapping for the cyclic group of order n corresponds to the existence of a Knut Vic design (or pandiagonal latin square) of order n. We are then able to derive the known existence results for Knut Vic designs as special cases of our existence results for strong complete mappings.

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Integrity of Graphs and Their Complements

R. Laskar and S. Stueckle*
Clemson University

For a graph, G, the integrity of G, I(G), is defined by I(G)=min{ |S| + m(G-S)}, where the minimum is taken over all S contained in V(G) and m(G-S) is the order of the maximum component of G. Similarly the edge-integrity of G, I'(G), is defined by I'(G)=min{|E| + m(G-E)}, where the minimum is taken over all E contained in E(G). In this paper we investigate the relationships between I(G) and $I(\overline{G})$ and between I'(G) and $I'(\overline{G})$. In particular, Nordhaus-Gaddum type results are studied.

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Max-balancing weighted directed graphs

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A weighted directed graph $G = (V, A, g)$ is called *max-balanced* if for every cut W the maximum weight over arcs leaving W equals the maximum weight over arcs entering W. For a potential p (i.e., a function defined on the vertices) we define the *reweighted graph with respect to p* by $G^p = (V, A, g^p)$ where $g^p_a = p_u + g_a - p_v$ for $a = (u,v) \in A$. We show that for strongly-connected G there exists a unique (up to additive constant) potential p such that G^p is max-balanced. We describe an $O(|V|^2|A|)$ algorithm for computing the potential p that uses the maximum cycle mean algorithm. We describe extensions to arbitrary connected graphs and present several characterizations of max-balanced graphs using lexicographic order properties of the arc weight function. Time permitting, we describe a decomposition/contraction theorem for max-balanced graphs in terms of their cycle structure.

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Minimum Degree and Neighborhood Union Conditions for Hamiltonian Properties in Graphs

Terri E. Lindquester, Rhodes College

Let NC2 = min $|N(u) \cup N(v)|$ where the minimum is taken over all pairs of vertices u, v that are at distance two in a graph G. In this paper, we pair a minimum degree condition with NC2 to obtain results for hamiltonicity, hamiltonian-connectedness, and traceability. In particular, we prove that if G is a 2-connected graph of order p with $\delta \geq t$ such that NC2 $\geq$ p-t, then G is hamiltonian.

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Scheduling of Real-Time Communicating Tasks

Jie Wu (*) and Eduardo B. Fernandez ,Florida Atlantic University

In general, scheduling problems are NP-complete even under some simple assumptions about computation and communication costs. Therefore, most methods use heuristics to get a suboptimal solution by emphasizing some requirements or compromising some conflicting requirements. However, the most frequently used task models such as task precedence graphs with no communication or task interaction graphs with no precedences cannot totally reflect tasks in real time applications which are described by sets of cooperating tasks with precedences. In this paper, a communicating task model with precedence ordering is considered. Methods are proposed for the three major steps of task scheduling, namely, processor interconnection, grain size decision, and task allocation, by assuming minimization of communication delay as the main objective. These results can be useful to allow control systems to satisfy real-time deadlines. Given the increasing computational loads of systems such as robotic controllers, this optimization is becoming very necessary.

Keywords = Communicating tasks, multiprocessing, task scheduling.

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Simulations of Turing Machines by 2NPDA and Their Applications to Open Problems

Shigeki Iwata (Tokai Univ., Japan)*
Takumi Kasai (Univ. of Electro-Communications, Japan)

Let $NSPACE_2(S(n))$ ($ASPACE_2(S(n))$) denote the family of languages accepted by nondeterministic S(n) space bounded Turing machines with binary alphabet. Let 2NPDA ($2NPDA_{poly}$) denote the family of those accepted by two-way (polynomial time bounded, resp.) nondeterministic pushdown automata. Then for a constant c,

$$NSPACE_2(c \log n) \subseteq 2NPDA_{poly}, \text{ and}$$
$$ASPACE_2(c \log n) \subseteq 2NPDA$$

are obtained. Applications of these results to some open problems in formal language theory are considered.

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Unbeatable Sets in Graphs

E. O. Hare *
S. T. Hedetniemi
Clemson University

Let $G=(V,E)$ be a graph, $X \subseteq V(G)$ and $W = V(G) - X$. Define the boundary of X, $Bd(X) = X \cap N[W]$. Let $S \subseteq V(G)$. S is beatable if there exists $S' \subseteq V(G)$ such that (1) $|S'| = |S|$ and $N[S'] \supset N[S]$ or (2) $|S'| < |S|$ and $N[S'] \supseteq N[S]$; otherwise, S is unbeatable. We prove that for a tree T, if $S \subseteq V(T)$, $\langle N[S] \rangle$ connected, $\langle W \rangle$ connected and S unbeatable, then $\gamma(T) = \gamma(T-N[S]) + |S|$. We also discuss properties of beatable and unbeatable sets.

142

Pigeons, Cliques, and Equidistant Permutation Arrays

W. J. Myrvold, University of Victoria

Two permutations on n symbols are *equidistant* if they agree in exactly one position. An *equidistant permutation array (epa)* is a set of permutations which are pairwise equidistant. We describe an exhaustive computer search used to verify that a largest epa on eight symbols has sixteen permutations.

This exhaustive computer search required the development of a fast algorithm for finding a maximum clique in a graph. We describe a new approach which we call a *pigeonhole algorithm*. This algorithm was approximately 2000 times faster for the graphs in the exhaustive search than a recursive approach based on the observation that any maximum clique either contains a vertex v (examine the neighbourhood of v) or it does not contain v (examine $G-v$).

143

SOCIALLY BALANCED PLANNING AND COLORED COMBINATORIAL OPTIMIZATION PROBLEMS

Horst W. Hamacher, Center for Optimization and Combinatorics, University of Flórida, USA, and Department of Mathematics, Universität Kaiserslautern, FRG
and
Franz Rendl, Technische Universiät Graz, AUSTRIA

Starting with two examples we show that job planning subject to given minority quota can be modeled using combinatorial optimization problems with additional coloring constraints. For given number of coloring constraints we develop polynomial algorithms for colored assignment and colored matching intersection problems.

144

BIPARTITE GRAPHS AND DEGREE CONDITIONS: Some More Results

Badri N. Varma
University of Wisconsin-Fox Valley

Some more results are presented in search of an answer to the following:

Let G be a graph of order n and for some property P let S be a sufficient condition, involving degree of vertices and the order of G. In the case G is bipartite, for what property P does S remain sufficient when the value of n in S is replaced by n/2 + 1?

145

Numerical Transversality Conditions in Optimization Problem

John Gregory and Cantian Lin*, Southern Illinois University

Many problems in the calculus of variations and optimal control theory involve an extra end point condition called the transversality condition. For example, if the value of the optimal solution $x(t)$ is not specified at the right hand end point $t = b$, then auxiliary dependent variables are used in optimal control problems as the authors have done. In this paper, we give numerical transversality conditions, discuss their local error and the resulting global error of numerical solution and show how these transversality condition are used to obtain accurate and efficient numerical method for extremal problem.

146

Fractional Domination on Double Cone Graphs

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Let $G = (V,E)$ be a graph. A function $g:V \to [0,1]$ is dominating if $g(N[v]) \geq 1$ for every vertex v, where N[v] denotes the closed neighborhood of v. Let $\Gamma_f(G)$ and $\gamma_f(G)$ denote the upper and lower fractional domination numbers, i.e. $\Gamma_f(G)$ and $\gamma_f(G)$ is the maximum, respectively minimum, of the sum g(V), where g is a minimal dominating function. We examine Γ_f for double cone graphs and present results of the following type:

Theorem 1: Any graph T can be embedded in a double cone graph G such that for any Γ_f– function g on G we have $g(N[T]) = \gamma_f(T)$.

Theorem 2: Let $g:V \to [0,1]$ be a function on a graph G then G can be embedded in a graph H such that any γ_f– function on H is an extension of g.

Theorem 3: Let $g:V \to [0,1]$ be a function on a graph G then G can be embedded in a graph H such that there exists a Γ_f– function on H that is an extension of g.

147

A Parallel Algorithm for Solving Mazes

D. I. A. Cohen* CUNY and M. Blaze Princeton U.

A variation of the "Game of Life" can be adapted to determine whether a particular maze has a solution or not. The complexity of this method is superior to standard methods. Three forms of this algorithm will be presented.

148

THE Gg-NETWORK: A NEW LINEAR-COST COMPUTER NETWORK

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In a natural way, a computer network can be represented as a graph whose nodes represent the computers and whose edges represent the links between the computers. The G-network, a computer network topology proposed by Rice, Guha, Brigham and Dutton, posses many desirable characteristics such as fault-tolerance, efficient routing, small number of links etc. In this paper, we generalize the G-network and name it Gg-network. A method for generating the topology of a Gg-network from the topology of a "core" graph is described. We also study it as a fault-tolerant computer network. In particular, we study its diameter, fault-tolerance, number of links and connectivity relations. It is shown that Gg-network has small number of edges as G-network. Two classes of Gg-networks whose core graphs are complete graphs and hypercubes are also studied. The G-networks are a special class of Gg-networks, since the G-networks are obtained by taking complete graphs as core graphs for the construction of the Gg-networks.

2-Hamiltonian Connectively of the 4-Connected Halin Graphs

149

* C.A. Barefoot and Farhad Shahrokhi
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Let end(T) denote the set of endvertices of the tree T. A 4-connected Halin Graph is a graph $H=T_1VT_2VC$ of vertex connectively 4, where T_1 and T_2 are identical plane trees with end(T_1) = end (T_2) and C is the cycle connecting the endvertics of T_1, in the cycle order determined by the embedding of T_1. It will be shown that H is 2-Hamiltonian connected. This result is shown constructively and the resulting algorithm can determine a Hamiltonian (u,v) path in H or H-x in linear time, where $x \neq u$ and $x \neq v$.

150

Path formalism to solve ordinary and partial finite difference equations.

Nguyen Ky Toan , Universite du Quebec a Trois-Rivieres

In two papers in J. Math. Physics 20 (1979) and 21 (1980) Nguyen Ky Toan and Adel Antippa has present a discrete path formalism to solve system of linear equations. In this paper we apply this formalism to solve finite difference equations represented by a signal flow graph (linear lattice of vertices). On the other hand, for a partial finite difference equation with two variables, a two dimensional lattice of vertices is needed. The method also treats homogeneous and inhomogeneous in the same way. The inhomogeneous terms combines with the boundary conditions to form the sources of signal flow graph. Finally, the method permits an arbitrary specification of boundary conditions provided they are compatible with the equation.

Keywords : signal flow graph, difference equations, path.

WEDNESDAY, FEBRUARY 22, 1989 4:20 p.m.

151

A Fractional Version of Domatic Number

Douglas F. Rall, Furman University

A (fractional) dominating function of a graph G=(V,E) is a function f from V into the closed unit interval [0,1] so that for each vertex w in V, $f(N[w]) \geq 1$. This generalizes the usual definition of a dominating set which in this context is a subset of V whose characteristic function is a dominating function. The domatic number of G is the maximum number of dominating sets of G into which V can be partitioned. In this paper we extend this definition in a natural way to fractional dominating functions and investigate the resulting fractional domatic number.

152

AN APPROXIMATE ALGORITHM FOR MINIMUM EDGE DELETION BIPARTITE SUBGRAPH PROBLEM

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In the area of graph algorithms the optimization problem of obtaining a minimum set of edges removal of which leaves a graph bipartite is known to belong to the NPC class. Recently, polynomial time algorithms for proper circular-arc, circular-arc, permutation, and split graphs have been reported. At the same time a polynomial time algorithm for obtaining maximum cut, a related problem, for planar graphs is known. However, any polynomial time bounded algorithm is not yet known for general graphs. This has motivated the present work in which we suggest a simple algorithm which gives a near optimal solution guaranteeing that the error is within a constant bound.

153

Elementary Bounds for the Probability of Winning a Series Competition

A. Meir and J.W. Moon*, University of Alberta

Suppose two teams A and B compete in a series of separate games; the first team to win m games wins the series. We assume that A and B win any given game with probabilities p and q, respectively, when $p + q = 1$. Let $W(m,p)$ denote the probability that team A wins the series and let $E(m,p)$ denote the expected duration of the series. Several authors have given various expressions for $W(m,p)$ and $E(m,p)$ in terms of sums involving binomial coefficients or, equivalently, the incomplete beta function. Our main objects are to point out an identity relating $W(m,p)$ and $E(m,p)$ and to derive upper and lower bounds for $W(m,p)$ and $E(m,p)$ that yield reasonably satisfactory numerical results when, say, $m \geq 10$.

154

Supereulerian graphs of minimum degree at least 4

Paul A. Catlin, Wayne State University, Detroit 48202

We discuss sufficient conditions for a graph to be supereulerian, i.e., to have a spanning closed trail. For example, let G be a 2-edge-connected graph of order n. Suppose that for any bond $E \subseteq E(G)$ with $|E| \leq 3$, both components of $G - E$ have order at least $n/5$. Then either G has a spanning closed trail, or G has five disjoint connected subgraphs, all of order $n/5$, such that when all of them are contracted, G is contracted to $K_{2,3}$. Prior results of X. T. Cai, P. A. Catlin, F. Jaeger and X. W. Li follow.

155

Networks for Greatest Common Divisor Computations

C.N. Purdy* and G.B. Purdy, University of Cincinnati

We consider the problem of computing the greatest common divisor (gcd) of two n-bit integers on a network G of processors P(1),P(2),...P(N), where each P(i) can perform functions in a fixed set F. We are interested in computing bounds on the time T needed to compute the gcd, the area A required to realize G, and the number of processors N included in G, as functions of G, F, and n. We discuss upper and lower bounds on A and T when G represents a VLSI chip and F is restricted to a set of bit-level operations. In this situation the graph G will be drawn in the plane and there will be an upper bound, independent of n and N, on the number of edges of G which can meet at a vertex or cross over one another. We prove lower bounds on A and T under various assumptions about the time required for a signal to propagate along an edge as a function of the length of the edge. When the time is linear in the length of the edge, we show that a gcd algorithm of Brent and Kung is best possible. We also discuss the efficiency of several competing gcd algorithms for various choices of G and F.

156

Stable Edges and Cohesion Galaxies[†]

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Key words: cohesion, cutvertex, stable edges

The cohesion of a vertex x, denoted $\mu(x)$, is the minimum number of edges whose removal results in a subgraph for which x is a cutvertex. An edge e is said to be stable if its deletion changes the cohesion of no vertex in the graph and a characterization of stable edges is given. The main results here deal with finding graphs with a large proportion of stable edges. The proof techniques are constructive and involve a special vertex set which we call a cohesion galaxy.

[†]Research supported by the U.S. Office of Naval Research.

Optimal Scheduling of Subtasks Under Compatibility and Precedence Constraints

157

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In this paper, we consider the problem of optimally scheduling a large task composed of several subtasks, each with a specified minimum time requirement r_x. Some pairs of subtasks are compatible and may be performed simultaneously, while other pairs are incompatible and must be scheduled at non-overlapping intervals. In addition, the formulation accommodates precedence constraints between incompatible pairs of subtasks. We consider two problems: (a) scheduling the overall task in the minimum possible time; and (b) finding a schedule which maximizes the total sum of times allocated to all subtasks given a fixed total time for the whole task. We find efficient algorithms to solve these two problems when the compatibility graph is an interval graph and the specified precedence relation can be extended to a complete transitive orientation of the incompatibility graph.

Keywords: Optimal scheduling, subtasks, compatibility graphs, precedence relation, transitive orientation, interval graphs.

158

CYCLIC DECOMPOSITION OF MONOMIAL PERMUTATIONS

Oscar Moreno and Ivelisse Rubio*, University of Puerto Rico

We will consider the monomial permutation $x \mapsto x^i$ acting in the finite field F_q. The problem is : for what i the permutation decomposes into cycles of uniform length with (0, 1, -1) as fixed points.

The objective of this paper is to give the necessary and sufficient conditions in order to obtain permutations that decomposes into cycles of uniform length. As a consequence of this also we will prove the validity of some formulas for counting the i's that produce these permutations.

159

Generalized Degrees, Connectivity and Hamiltonian Properties in Graphs

R.J. Faudree, Memphis State Univ., R.J. Gould*, Emory Univ., M.S. Jacobson, Univ. of Louisville, L.M. Lesniak, Drew Univ.

We consider a form of generalized degree for sets of vertices based on neighborhood unions. Bounding this generalized degree from below, we obtain results about connectivity and highly Hamiltonian properties in graphs. In particular, we determine bounds implying when a graph is Hamiltonian-connected and when it is pancyclic.

160

An upper bound of the diameter of certain distance regular graphs

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For a distance regular graph with valency k and diameter d, if $c_1=\dots=c_{s-1}=1$, $c_s\geq 2$ and $a_{s-1}<c_s$ $(s\geq 2)$, then we can show $c_{n-(s-1)}\leq c_n$ for any n with $s<n\leq d$ by counting the number of paths between two vertices. From this results, we can conclude that the diameter of a distance regular graph satisfying the above condition is bounded above by $(k-c_s+1)(s-1)+2$.

WEDNESDAY, FEBRUARY 22, 1989 5:00 p.m.

162

Reliability Covering Problems

M. O. Ball, University of Maryland
J. S. Provan, University of North Carolina
D. R. Shier*, College of William and Mary

Suppose we are given a set of objects S = {1, 2, ..., m}, together with a collection $\mathcal{R} = \{R_1, R_2, ..., R_n\}$ of subsets of S. It is supposed that each subset R_k has a known probability p_k of being available. We are interested in calculating the probability p(S, $\mathcal{R}$) that each object in S is covered by some available set, assuming that the subsets R_k fail (are unavailable) randomly and independently. The calculation of p(S, $\mathcal{R}$) is shown to be NP-hard, even for the special case when the subsets R_k correspond to paths in a tree and the vertices of the tree are to be covered. Polynomially solvable special cases (on a tree) are presented.

161

A Constructive Characterization of (α, c)-Critical Graphs

Jennifer Zito, University of Pennsylvania.

A graph is said to be (α, c)-*critical* when the removal of any edge will increase the independence number, α, and/or the number of connected components, c. We provide a constructive characterization of (α, c)-critical graphs. The construction uses α-critical graphs and trees with unique maximum independent sets as the building blocks. It is also shown that trees with unique maximum independent sets can be constructed from trees which have all endpoints in the same bipartite class.

165

A Characterization of Separating Pairs and Triplets in a Graph

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Vijaya Ramachandran
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University of Texas

We obtain tight upper bounds for the maximum number of separating pairs and triplets in an undirected biconnected and triconnected n-node graph, respectively. The bound is $\frac{n(n-3)}{2}$ for the maximum number of separating pairs in a biconnected graph, and the bound is $\frac{(n-1)(n-4)}{2}$ for $n \neq 6$ and 6 for $n = 6$ for the maximum number of separating triplets in a triconnected graph. We present worst-case graphs that exactly achieve these upper bounds. The graph for a biconnected case is a cycle and for a triconnected case is a wheel for $n \neq 6$, and is a tri-regular triconnected graph for $n = 6$. Finally, we give an $O(n)$ characterization for the separating pairs in a biconnected graph.

163

G-balanced and weakly G-balanced sets

Paul Terwilliger, University of Wisconsin-Madison.

Let W be a finite dimensional Euclidean space, and let X be a finite set of unit vectors spanning W. Let G be a group of orthogonal linear transformations on W that permute the elements of X, and for each pair of elements x, y in X let G_{xy} (resp. G_{xy^-}) denote the set of elements in G that fix x and y (resp. interchange x and y). Then X is said to be G-*balanced* (resp. *weakly* G-*balanced*) if (i) any two elements in X are interchanged by some element in G, and (ii) for any x, y in X, the subspace of W fixed by everything in G_{xy} is just Span{x, y} (resp. (ii) for any x, y in X, the subspace of W consisting of those vectors z sent to -z by everything in G_{xy^-} is just Span{x - y}). We know of several infinite families of G-balanced and weakly G-balanced sets (in each case G is a classical group), and conjecture these are the only examples if the permutation rank of G on X is sufficienty large. We give some remarkable combinatorial properties of G-balanced and weakly G-balanced sets, which show they are related to the Q-polynomial association schemes.

Key Words: Generously transitive permutation group, Association scheme.

164

Hamilton-type Properties of Permutation Graphs

Barry Piazza, U. of Southern Mississippi

A graph G is hamiltonian connected if for any x and y in G , there exists an x-y hamiltonian path. A bipartite graph G with bipartition (X,Y) is called (i) equitable Hamilton laceable if $|X| = |Y|$ and for any x in X and any y in Y , there exist an x-y hamiltonian path; or (ii) nearly equitable Hamilton laceable if $|X| = |Y|+1$ and for any x and w in X , there exists an x-w hamiltonian path. For a graph G with vertices 1,2,...,n and a permutation α in S_n , the permutation graph P(G,α) is formed by taking two copies of G , G_1 and G_2, together with n edges which join vertex i in G_1 and α(i) in G_2 , for $1 \leq i \leq n$. The author proves the following:

T1: If G is hamilton connected, P(G,α) is hamiltonian connected for all permutations α .

T2: If G is equitable Hamilton laceable, P(G,α) is equitable Hamilton laceable if and only if (i) $\alpha(X) = X$ or (ii) $\alpha(X) = Y$.

T3: If G is nearly equitable Hamilton laceable, P(G,α) is equitable Hamilton laceable if and only if $\alpha(X) = X$.

Keywords: Hamilton laceable, hamiltonian connected, permutation graph

166 **THE CONSTRUCTION OF K-CONNECTED D-CRITICAL GRAPHS OF GIVEN ORDER & SIZE**

L. Caccetta* & W.F. Smyth#

Curtin University of Technology McMaster University

We call a simple undirected graph G of finite diameter $D \geq 2$ D-critical if the addition of any edge to G reduces the diameter. We consider the class $\mathcal{G} = \mathcal{G}(n,m,D,K)$ of K-connected D-critical graphs of order n and size m. For given positive integers n, m, D, and K, we show how to determine whether or not $\mathcal{G}(n,m,D,K)$ is empty, and, if not, how to construct graphs $G \in \mathcal{G}(n,m,D,K)$.

167 **Continuous-time Traversal in Dynamically Random Digraphs**

Dan Pritikin, Miami University, Oxford, OH

Consider a shuttle system providing nonscheduled transportation along the arcs of a digraph as follows. To each arc xy correspond parameters λ and ω. A stream (Poisson) of shuttles depart from x to y along xy, on average one shuttle departing every λ minutes. Once embarked, shuttles along xy spend (on average) ω minutes of travel time before arriving at y. The main problem solved: Given a starting vertex A and destination vertex B, efficiently compute a travel strategy for minimizing the expected time of arrival at B. Note that typically an optimal strategy does not merely specify a particular AB-path to follow. Instead, a strategy is a rule for deciding whether to board a shuttle when one happens to depart from the traveller's current location.

169 **A Menger-Type Characterization of Local-Connectivity**

Nell Kimmins Rayburn, Austin Peay State University

A (k+1)-connected graph is locally k-connected if the (vertex-deleted) neighborhood of each vertex is k-connected. G. Chartrand and R.E. Pippert introduced this concept in a 1974 paper. Since then a number of papers have appeared dealing with characteristics of locally k-connected graphs. We present a necessary and sufficient condition for a graph to be locally k-connected. Our condition gives a local analogue of Menger's Theorem for (k+1)-connected graphs.

168 **Broadcasting: Between Whispering and Shouting**

Arthur M. Farley and Andrzej Proskurowski*

Computer and Information Science, University of Oregon

An important communication process is that of *broadcasting*, ie., dissemination throughout a network of a message originating at a single site. We assume a *store-and-forward* network architecture in which broadcasting requires coordinated communication between adjacent sites. We model this site-to-site communication in two stages. In the first (*switching*) stage, the message is directed to the appropriate output channel (communication link). In the second (*sending*) stage, the message is transmitted over that link. We investigate the time behavior of the broadcasting process as a function of the switching and sending time parameters.

170 **Node-Packing Problems with Integer Rounding Properties**

S. K. Tipnis*, Illinois State University and L. E. Trotter, Jr., Cornell University

We consider an integer programming formulation of the node-packing problem, $\max\{1 \cdot x:\ Ax \leq w,\ x \geq 0,\ x \text{ integral}\}$, and its linear programming relaxation, $\max\{1 \cdot x:\ Ax \leq w,\ x \geq 0\}$, where A is the edge-node incidence matrix of a graph G and w is a nonnegative integral vector. We give an excluded subgraph characterization quantifying the difference between the values of these two programs. One consequence of this characterization is an explicit description for the "integer rounding" case. Specifically, we characterize those graphs G with the property that for every subgraph of G and for any choice of w, the optimum objective function values of these two problems differ by less than unity.

Key Words: Node-Packing, Integer Rounding, α-critical graphs

THURSDAY, FEBRUARY 23, 1989 8:30 a.m.

176

P_4-separable graphs: a tree representation and some consequences

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We introduce and characterize a class of graphs featuring a special P_4-structure. It turns out that the graphs in our class are uniquely tree representable in a way which generalizes previously known results about P_4-reducible graphs and cographs. Further, the tree representation can be used in the design of efficient algorithms for solving graph isomorphism, minimum fill-in and dominating set, and the four classic optimization problems.

KEYWORDS: Graph Theory, Efficient Algorithms, P_4-Structure

A Threshold for Hypercube Bandwidth in Random Graphs

177

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M. K. Goldberg, Rensselaer Polytechnic Institute
Zevi Miller*, Miami University

Let G be a random graph on $N=2^n$ points with edge probability $p=p(n)$, and $f:G \rightarrow Q(n)$ a bijection between the vertex sets of G and the n-dimensional hypercube Q(n). Let B(G) be the minimum, over all such bijections f, of the maximum, over all edges xy in G, of the distance in Q(n) between f(x) and f(y). For each integer $k=o(n^{\frac{1}{2}})$, we find a probability threshold t(n) such that if $p \geq t(n)$ then almost every graph G satisfies $B(G) \geq n-k$ while if $p < t(n)$ then almost every graph G satisfies $B(G) \geq n-k$.

Cycles through 11 vertices in 3-connected cubic graphs

178

R.E.L. Aldred*
Bau Sheng
D.A. Holton

We determine the circumstances under which a set of 11 vertices in a 3-connected cubic graph lies on a cycle. In addition, we consider the number of such cycles that exist and characterize those graphs in which a set of 9 vertices lies on exactly two cycles.

179

"On harmonlously colored edge critical graphs"

Georges, J.P. Trinity College

Let C be a vertex coloring of the graph G. For every edge {a,b} in G, there corresponds a pair of colors {c(a), c(b)}. A graph is said to be harmoniously colored if adjacent vertices receive different colors and all edges receive different color pairs. This paper examines the harmonious chromatic number and the harmonious edge criticality of graphs.

181

A General Parsing Algorithm, *(based on algorithms due to Younger and Knuth)*

FRANK HADLOCK, *Computer Science Dept., Tennessee Technological U.*

The general parsing algorithm due to Younger is applicable to any context free language but requires that the grammar be in Chomsky normal form. Although it is $\Theta(n^3)$, it has the advantage that it does not require the grammar to be unambiguous. If a string has multiple derivations, they are all captured in the tabular structure employed by Younger's algorithm. In the algorithm presented in this paper, the $LR(0)$ *items* employed by Knuth's LR(k) parsing algorithm are used as entries in this tabular structure, thus obviating the need to employ Chomsky normal form. Besides being general, it is shown that the algorithm is compatible with error recovery techniques based on the insertion/deletion of a single *character* or *phrase*.

Keywords: grammar, parse, language, ambiguous, error recovery

182

Automorphisms of powers of the n-cube

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The kth power of a graph Γ is by definition the graph $\Gamma^{(k)}$ whose vertices are the vertices of Γ, two being adjacent if and only if they are at distance $\leq k$ in Γ. In this talk we are interested in determining the automorphism group of powers of the n-cube $\Gamma(n)$. Denote by $G^{(k)}(n)$ the automorphism group of $\Gamma^{(k)}(n)$, for $1 \leq k \leq n$. The problem of computing the size of $G^{(k)}(n)$ arises in estimating the probability that a random graph on 2^n points contains $\Gamma^{(k)}(n)$. Estimating this probability is a step in finding a probability threshold for the analogue of bandwidth in hypercubes.

It is known that $G^{(1)}(n)$, the automorphism group of the cube, is isomorphic to the semi-direct product $\mathbb{Z}_2^n \ltimes S_n$, of order $2^n \cdot n!$, and $G^{(n)}(n)$ is isomorphic to S_{2^n}, of order $(2^n)!$ What is the situation for $2 \leq k \leq n-1$? We have the following.

Theorem: (*i*) For $n \geq 2$, $G^{(n-1)}(n)$ is isomorphic to $\mathbb{Z}_2^{2^{n-1}} \ltimes S_{2n-1}$, of order $2^{2^{n-1}}(2^{n-1})!$.

(*ii*) for $n \geq 4$, $G^{(2)}(n)$ is isomorphic to $\mathbb{Z}_2^n \ltimes S_{n+1}$, of order $2^n(n+1)!$.

(*iii*) For $2 \leq k < n-1$, $G^{(k)}(n) = \begin{cases} G^{(2)}(n), & \text{if } k \text{ is even} \\ G^{(1)}(n), & \text{if } k \text{ is odd} \end{cases}$ (except possibly for finitely many n).

183

CYCLE COVERINGS OF GRAPHS

by

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Cun-Quan Zhang*
Department of Mathematics
West Virginia University

Symour and Szekeres conjectured that any 2-connected graph has a family of cycles such that each edge of the graph is contained in exactly two cycles of that family.

With the aid of four coloring theorem, Symour proved that if each edge of a 2-connected *planer* graph is asigned a weight 1 or 2 such that the total weight of each edge-cut is even, then there is a family of cycles of the graph such that for each edge e of the graph, the number of the cycles of the family containing the edge e equals the weight of the edge e. In this paper, we generalized the theorem by Symour without the assumption of the four coloring theorem.

If each edge of a 2-connected cubic graph without a subgraph isomorphic to a subdivision of the Peterson Graph is signed a weight 1 or 2 such that the total weight of each edge-cut is even, then there is a family of cycles of the graph such that for each edge e of the graph, the number of the cycles of the family containing the edge e equals the weight of the edge e.

Furthermore, a direct corollary of that theorem is that *the Symour-Szekeres conjecture is true for any 2-connected cubic graph without a subgraph isomorphic to a subdivision of the Peterson Graph.* Note that the minimum counterexample to the Symour-Szekeres conjecture is a cubic graph.

184

THE EDGE SUBCHROMATIC NUMBER OF A GRAPH AND ITS COMPLEMENT

Gayla S. Domke*, Georgia State University, Atlanta, GA and
Renu C. Laskar, Clemson University, Clemson, SC

The subchromatic number, denoted $X_s(G)$, of a graph $G = (V, E)$ is the smallest order n of a partition $\{V_1, V_2, \ldots, V_n\}$ such that the subgraph induced by the vertices of V_i forms a disjoint union of complete subgraphs. The edge subchromatic number of G, denoted $X_s'(G)$, is the smallest order n of a partition $\{E_1, E_2, \ldots, E_n\}$ of E such that the subgraph induced by the edges of E_i forms a disjoint union of complete subgraphs. This talk will include Nordhaus-Gaddum type result for the edge subchromatic number of a graph and its complement.

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The scordatura Problem

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The term scordatura refers to the tuning of stringed instruments that deviates from the normal tuning. For example, the sixteen Sonatas on the Mystery of Mary's Life by Heinrich Ignaz Franz von Biber (1644-1704), considered the high-point of violin scordatura literature, involve fifteen different tunings. To reduce time-consuming and distracting retunings, it is customary to use several instruments. The scordatura problem then is to minimize the retuning required during the performance, for a prescribed sequence of tunings and a given number of instruments.

We give an efficient solution of the scordatura problem, assuming that the underlying distance function $\delta(t_1,t_2)$ between two tunings t_1 and t_2 is given by $\delta(t_1,t_2)=0$ if t_1 and t_2 are identical and $\delta(t_1,t_2)=1$ otherwise. We also discuss other distance functions and their implications on solutions.

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The Isomorphism Problem for Varieties of Lattices

Ross Willard, University of Waterloo

L. Babai, P. Klingsberg and E. Luks have shown that the isomorphism and automorphism problems for colored graphs become solvable in polynomial time when restricted to instances $\langle V,E,\text{coloring}\rangle$ whose color classes have size $O(\log n)$, where $n = |V|$. We show that in certain circumstances the same conclusion can be obtained when the color classes have size $O((\log n)^k)$ for fixed $k \geq 2$. Some deep facts about lattices then allow us to show that for each finitely generated variety of lattices $\mathcal{V}$, the isomorphism problem for the finite members of $\mathcal{V}$ is in P. (Equivalently, if K is a finite set of finite lattices and K^* is the class of all finite lattices which can be embedded into direct products of members of K, then the isomorphism problem for K^* is in P.) This generalizes the result of Babai, Klingsberg and Luks for distributive lattices.

KEY WÔRDS: colored, graph, isomorphism, automorphism, lattice, variety

188

Small cycles in Steinhaus graphs

Wayne M. Dymacek, Washington and Lee University

In this paper we prove that every Steinhaus graph that is not a tree has girth three or four. Furthermore, a Steinhaus graph is bipartite if and only if it is a tree or has girth four. Finally, let G be any Steinhaus graph on n vertices except for those generated by $0,1,2,3,2^{n-1}-2$, and their partners. Let P be the vertices of G of degree one. Then $G\backslash P$ has a spanning eulerian subgraph.

Key words: girth, spanning eulerian subgraph, Steinhaus graph

189

Monochromatic Coverings in Colored Complete Graphs.

P. ERDÖS, R. FAUDREE, R. GOULD, A. GYÁRFÁS, C. ROUSSEAU, R. SCHELP*

Hungarian Academy of Sciences, Memphis State University, Emory University

Let G be a graph, $A, B \subseteq V(G)$. The set A is said to cover or dominate the set B if $y \in B - A$ implies there exists an $x \in A$ such that $xy \in E(G)$. Let the complete graph K_n be edge colored with k colors and let t be a fixed positive integer. The following question is considered. What is the largest subset B of $V(K_n)$ that is monochromatically covered by some t element subset A of $V(K_n)$?

THURSDAY, FEBRUARY 23, 1989 11:50 a.m.

191

Short Encodings of Evolving Combinatorial Structures

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A derivation in a transformational system such as a graph grammar is redundant in the sense that the exact order of the transformations does not affect the final outcome; all that matters is that each transformation, when applied, is applied to the correct substructure. By taking advantage of this redundancy, we are able to develop an efficient encoding scheme for such derivations. This encoding scheme has a number of diverse applications. It can be used in efficient enumeration of combinatorial objects or for compact representation of program and data structure transformations. It can also be used to derive lower bands on lengths of derivations. We show for example, that $\Omega(n \log n)$ applications of the associative and commutative laws are required to transform an n-variable expression over a binary associative, commutative operation into any equivalent expression. Similarly, we show that $\Omega(n \log n)$ "diagonal flips" are required to transform one n-vertex numbered triangulated planar graph into any other. Both of these lower bounds are tight.

192

Stanley Decompositions of the Bracket Ring

B. Sturmfels, RISC-Linz, Austria, and N. White*, Univ. of Florida

We give an explicit Stanley decomposition of the bracket ring, that is, the commutative ring generated by the d x d minors of a generic n x d matrix. A Stanley decomposition is a direct sum decomposition each summand of which is a bracket monomial times a subring generated freely by brackets. The decomposition is obtained by a shelling of the simplicial complex consisting of the standard tableaux. This construction associates to each summand a standard (n-d) x d tableau, hence the number of summands is easily predictable by the well-known hook-length formula. The construction has important applications to normal form theory for nilpotent vector fields. Key words:. Stanley decomposition, bracket ring, shelling, standard tableau, distributive lattice.

193

Cyclomatic Numbers of Connected Induced Subgraphs

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abstract

In this paper, we only deal with simple, finite graphs. Basic consepts can be found in [2].The cyclomatic number of a graph H, denoted by cy(H), is $|E(H)| - |V(H)| + 1$. Let A be an independent set of vertices of a graph G. Let C(A) be the collection of all connected induced subgraphs of G which contain A. Define

$$\omega(A) = \min \{cy(H) : H \in C(A)\}.$$

In [1], B. Alspach and H. Oral asked the following question: what can be said about $\omega(A)$ for various classes of graphs? Of particular interest are the cases when A is a maximal independent set of vertices in G or A is a color class in a proper vertex coloring of G with number of colors equal to the chromatic number of G. In this paper, we obtain:

Theorem 1. $\omega(A) \leq \binom{|A|}{2} - |A| + 1$, if $|A| \geq 2$.

For triangle-free graphs, we have:

Theorem 2. $\omega(A) \leq |A|^2/4 - |A| + 1$, if $|A| \geq 2$.

The upper bounds in Theorems 1 and 2 are best possible. Also the edge version of Alspach-Oral problem is considered and similar results are given. It is interesting to obtain the upper bound for some other classes of graphs, for example planar graphs, 3-connected graphs, etc.

[1]. B. Alspach and H. Oral, Research Problem 95, 71, (1988), p.185.
[2]. J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Elsvier, New York, 1976.

194

Application of Forbidden Difference Graphs to T-Colorings

Barry Tesman, Rutgers University

Let $T = \{0\} \cup S$ where S is a subset of the positive integers N. A T-coloring of a graph G is a vertex coloring $F : V(G) \to N \cup \{0\}$ such that $[x, y] \in E(G) \Rightarrow |F(x) - F(y)| \notin T$. The T-span of G, $sp_T(G)$, is the minimum n such that there exists a T-coloring $F : V(G) \to \{0, 1, \ldots, n\}$. For a given set T, we define the graph $\mathcal{T}$, the forbidden difference graph of T, by $V(\mathcal{T}) = \{0, 1, 2, \ldots\}$ and $\{i, j\} \in E(\mathcal{T})$ if $|i - j| \in T$. In this talk, we study forbidden difference graphs and their application to T-colorings. In particular, we improve the Cozzens-Roberts bound for $sp_T(G)$.

196

LOAD BALANCING GRAPHS

Naveed Sherwani*, Alfred Boals, Western Michigan University
Hesham Ali, University of Nebraska-Omaha

In this paper we introduce the concept of load balancing graphs. These graphs arise in the design of fault tolerant networks since they allow node balancing in case of node failures.
We define a graph to be n-load balancing if for each vertex v there exists at least n other vertices with the property that the union of their neighborhoods contain the neighborhood of v. Several well known classes of LB(2) graphs are characterized. We investigate the relationship of regularity, connectivity, and diameter in these graphs.

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On the Covering Graph of a Poset of Rank k.

Dr. James A. Wiseman - Rochester Institute of Technology

The maximal number of edges possible in the covering graph of a poset of rank k is determined. Issues related to this question are analyzed (such as specified rank type) and some theorems and conjectures are stated.

Key Words: Covering Graph, Poset, Rank.

198

On Long Cycles in Tough Regular Graphs

Douglas Bauer*, Stevens Institute of Technology, Hoboken, NJ 07030
H.J. Broersma and H.J. Veldman, University of Twente, Enschede, The Netherlands

We consider the problem of finding the minimum order of a 1-tough k-regular nonhamiltonian graph.

Key words: 1-tough, hamiltonian cycle.

199

Graceful subgraphs of complete Cayley graphs

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Let $n = 2k+1$. We prove that the undirected Cayley graph C_n of Z_n with generating set $\{1,2,..,k\}$ contains a subgraph K_4, graceful in the sense that its edges have different colors, as a partial answer to an Erdös-Pyber-Tuza question.
Initially, inspired by Heffner's embedding of C_7 into a torus subdivided by its 14 graceful K_3's, we get what follows: we omit a corner unit cube in a cubic 3x3x3-lattice and get, in each of the other 26 unit cubes, a tetrahedron with alternate vertices, so that face identifications (related to the tessellation of R^3 by regular truncated octahedra) on the union of the resulting 26 tetrahedra realizes C_{13}, once vertices are labelled in adequate arithmetic progressions mod 13 in each admissible face diagonal direction. These tetrahedra give 26 similarly colored K_4's among the 52 graceful K_4's of C_{13}. The other 26 have similar properties.
The graph whose vertices are the graceful tori of C_n and whose edges are their bicolor intersection graphs admits an action of a subgroup of index 2 in the group of units mod n, making it representable by a canonically labeled F-diagram.

201

ASSOCIATION SCHEMES ON TRIPLES AND PARTIAL 3-DESIGNS

D. M. Mesner and P. Bhattacharya, U. of Neb., Lincoln

We define an association scheme on triples (AST) as a partition of the set of ordered triples from a v-set into ternary relations $R_0, \ldots, R_m$ subject to certain regularity conditions involving quadruples and parameters p^h_{ijk}. We have found lots of AST's although, like classical association schemes, they can be hard to construct. We have an algebraic formulation using 3-dimensional (0,1)-matrices and a ternary product. The algebra has many uses but is non-associative and has not led us to regular representations or eigenvalue properties. We say that a 2-design is partially balanced for triples subject to some AST if for each relation R_i in the AST there is a number λ_i such that {x,y,z} occurs in exactly λ_i blocks whenever $(x,y,z) \in R_i$. Some 2-designs have this property and others do not, and this provides an interesting way to classify 2-designs.

202

Clique partitions of split graphs

W D Wallis*, Southern Illinois University, and Julin Wu, Qingdao University

Split graphs are those graphs formed by taking a complete graph and an empty graph disjoint from it and a subset of the edges joining the two. These graphs have proven important in studying clique partition problems, both in examples and in general theory. We shall prove that deciding the clique partition number is NP-complete when restricted to the class of split graphs.

Keywords: clique partition, complexity, split graph, NP.

203

SOME EXTREMAL SET THEORY RESULTS

Richard Anstee, University of British Columbia

Two simple proof techniques are discussed. A straightforward induction yields results about forbidden families of configurations including some new best possible extremal results. An elementary amortized complexity argument yields bounds for a case where the sets and the forbidden configurations are ordered.

204

TREE CODES ARISING FROM A CLASSICAL PERMUTATION CODE

Germain Kreweras, Institut de Statistique, Universite Paris
Paul Moszkowski,* Syracuse University, Syracuse, New York

The classical permutation bijection exchanging the number of records and the number of cycles can be generalized to Cayley's trees. This bijection yields generating functions of Cayley's trees with respect to degrees and increases.

Key Words: Cayley tree, degree sequence, increasing edge, generating function, record.

205

A New Construction of S(4,7,23)

A. Baartmans, Oakland University
W. Wallis, Southern Illinois University
J. Yucas*, Southern Illinois University

In this paper we give a construction of the unique Steiner System S(4,7,23). This construction employees subsets of PG(3,2), subsets of the Steiner System S(3,4,8) and concatenations of both.

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"Checking unicursality of a mixed graph by minimum cuts: a base for an optimal algorithm for the mixed chinese postman problem".

Yves Nobert*, Jean-Claude Picard, Patrick Samson (University of Quebec at Montreal).

The chinese postman problem is well solved when the original graph contains only arcs or only edges. The mixed chinese postman problem is NP-complete and very few papers are devoted to this case. In this paper we present a very efficient way to check the unicursality of a mixed graph which enables us to calculate a sharp lower bound on the optimal value of the mixed chinese postman problem using the simplexe method. An algorithm is also proposed using Gomory cuts and blosom inequalities to achieve optimality.

207

Pair Labellings with Given Distance

Zoltán Füredi, Jerrold R. Griggs*, and Daniel J. Kleitman
Institute for Mathematics and its Applications, University of Minnesota

Given a graph G and $d \in Z^+$, the pair-labelling number, $r(G,d)$, is defined to be the minimum n such that each vertex in G can be assigned a pair of numbers from $\{1,\ldots,n\}$ in such a way that any two numbers used at adjacent vertices differ by at least d. We answer a question of Roberts by determining all possible values of $r(G,d)$ given the chromatic number of G. The answer follows by determining the chromatic number of the graph that has pairs of integers as vertices and edges joining pairs that are distance at least d apart. For general $t \in Z^+$, the analogous questions for t-sets instead of pairs are considered. A solution for general t is conjectured which, for $d = 1$, reduces to Lovász's theorem on Kneser graphs.

208

THE ORDER DIMENSION OF A RANDOM PARTIALLY ORDERED SET

P. Erdős, H. A. Kierstead, and W. T. Trotter*
Arizona State University

The dimension of a poset on n points can be as large as $n/2$, and it is easy to show that a random poset has dimension at least $n/c \log n$. We show that a random poset has dimension at most $n/2 - \varepsilon_1 n/\log n$ and at least $\varepsilon_2 n$. We believe the lower bound can be strengthened to $n/2 - \varepsilon_3 n/\log n$. Such a bound would imply a super linear lower bound on the problem of bounding the dimension of a poset in terms of the maximum degree.

209

Some Small Simple t–designs

Yeow Meng Chee *, Charles J. Colbourn and Donald L. Kreher
University of Waterloo and Rochester Institute of Technology

Using the basis reduction algorithm of Kreher and Radziszowski, we construct a number of new simple t–designs for t=2,3,4 and 5. We then outline a method of employing basis reduction to produce large sets of disjoint designs. Among the large sets we have found so far, of particular interest is the large set of 3–(23,4,4) designs.

210

Geometric figures, Instabilities and the Traveling Salesman Problem

Randy Garrett, Science Applications International Corporation

A graphical presentation of work–in–progress research illustrates how instabilities develop when solving a two–dimensional Euclidean Traveling Salesman Problem. A new type of geometric figure shows the type and location of representative instabilities which develop from a given set of boundary conditions. The cause of these instabilities as well as the implications for heuristic solutions are discussed.

211

ASCENDING SUBGRAPH DECOMPOSITION FOR TREES

R.J. FAUDREE*, MEMPHIS STATE UNIVERSITY
R.J. GOULD, EMORY UNIVERSITY

It has been conjectured that if a graph G has $\binom{n+1}{2}$ edges, then the edge set of G can be partitioned into n graphs $G_1, G_2, \ldots, G_n$ such that G_i has i edges $(1 \leq i \leq n)$, and G_i is isomorphic to a subgraph of G_{i+1} $(1 \leq i < n)$. Such a graph G is said to have an *ascending subgraph decomposition (ASD)*. It will be shown that any forest with $\binom{n+1}{2}$ edges has an ASD.

212

Graceful Labelings of Cycles and Prisms with Pendent Points

Daniel Ropp, Department of Mathematics, Washington University

We prove that for any positive integers n and T with $T \leq n$ there is an n–cycle with pendent points attached at T vertices that is graceful.

We also show that the graphs obtained by attaching one pendent point to each vertex of a prism or by attaching one pendent point to each vertex of one cycle of a prism are graceful.

213

Some perpendicular arrays for arbitrarily large t

Earl S. Kramer*, Spyros S. Magliveras, Tran van Trung, Qiu-rong Wu
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We show that perpendicular arrays exist for arbitrarily large t and with $\lambda=1$. In particular, if d divides (t+1) then there is a $PA_1(t,t+1,t+((t+1)/d))$. If $v \equiv 1$ or 2(mod 3) then there is a $PA_\lambda(3,4,v)$ for any λ. If 3 divides λ then there is a $PA_\lambda(3,4,v)$ for any v. If $n \geq 2$ there is a $PA_1(4,5,2^n+1)$. Using recursive constructions we exhibit several infinite families of perpendicular arrays with $t \geq 3$ and relatively small λ. We finally discuss methods of constructing perpendicular arrays based on automorphism groups. These methods allow the construction of PA's with $(k-t) > 1$.

214

Theoretical Aspects of Global Routing

M. Sarrafzadeh (Northwestern Univ.) and D. Zhou* (Univ. of IL-Urbana)

In two-dimensional array global routing of two-terminal nets there are a set $\eta = \{N_1, \ldots, N_n\}$ of multiterminal nets in an $m \times m$ grid, being a square tessellation of the plane. Each two-terminal net N is specified by a pair $[(x_1, y_1), (x_2, y_2)]$, where (x_i, y_i), $i = 1, 2$, are the tiles containing terminals of N. In a global routing, for each net, a sequence of tiles through which it passes, is specified. Span s of a two-terminal net in a two-dimensional array is the distance, in L_∞ norm (i.e., maximum distance in x- and y-direction), between its two terminals. Span s^* of an instance of global routing in a two-dimensional array is the largest span a net has; placement algorithms produce (or at least, try to produce) layouts with "small" s^*. We propose an algorithm that solves an arbitrary instance of global routing in an $m \times m$ array with span s^* using channel width not greater than $min(s^*, m/2)$; we establish tightness of this result. Also, we propose an algorithm achieving channel width $O(\omega_{opt} log(s^*/\omega_{opt}))$, where ω_{opt} is the optimal width. Finally, we propose a global router, suitable for "x-layouts", that produces optimal channel width in the x-direction and channel width not greater than m in the y-direction. Proposed results are extended to global routings involving multiterminal nets.

Key words: Global routing, Gate array, Standard cell, Short nets, Channel routing.

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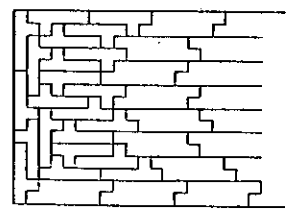
TILING IMPLICATIONS

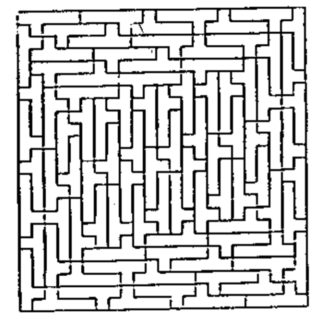
Aaron Meyerowitz Florida Atlantic University

In [1], S. Golomb noted that perhaps every polyomino which tiles some semi-infinite half strip also tiles some rectangle (perhaps of different width). He mentioned the y-hexomino as a possible counter-example. 20 years later K. Dahlke has shown that it is not [2]. We present some related results and open questions . Among them: restricting to translations of one orientation of a single tile (no reflections or rotations), tiling a half strip implies ability to tile a rectangle (of the same width); tiling a quarter plane implies ability to tile a bent strip. Similar results hold in higher dimensions. Dimension 1 is non-trivial too. The analogous implcation is valid for sets of size 3 [3].

[1] S. Golomb Tiling with Polynomials JCT 1 280–296 (1966)
[2] I. Peterson Pieces of a polyomino puzzle Science News 132 (1986)
[3] A. Meyerowitz Tilings in $\mathbb{Z}$ with triples JCT(A) 48 229–235 (1988)



(from [1])



(from [2])

216

Regular Spider Graphs are Edge-Graceful

Don Small, Colby College, Waterville, ME 04901

This paper verifies Lee's conjecture ('87), in the affirmative, that all regular spider graphs are edge-graceful. A spider graph is a connected graph with a "center" node of degree > 2 and all other nodes of degree < 2. A spider graph is called regular when the distance of the center node to every end-node is the same. A graph (V, E) is edge-graceful if there exists a bijection $f: E \to \{1, 2, \ldots, |E|\}$ such that the induced mapping $f^+: V \to \{0, 1, \ldots, |V|-1\}$ defined by $f^+(v) = \sum f(u,v) \bmod(|V|)$ for all $v \in V$ and $\{u,v\} \in E$ is a bijection. A labeling algorithm and discussion of its development will be presented.

Key Words: edge-graceful, degree, spider.

217

A Survey of New Simple t-Designs

Donald L. Kreher, Rochester Institute of Technology

The concept of using basis reduction for finding $t-(v,k,\lambda)$ designs without repeated blocks was introduced in my joint presentation with S.P. Radziszowski at the Seventeenth Southeastern International Conference on Combinatorics, Graph Theory and Computing. This tool and other algorithms were packaged into a system of programs I have called the design theory toolchest. It was distributed to several researchers different institutions. At this meeting I report on the many new open parameter situations that were settled using this toolchest.

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On the Power of 45-degree Lines

Charles Chiang and Majid Sarrafzadeh*, Northwestern University

A *knoch-knee* layout is a collection of vetrex disjoint paths in a square grid graph. Mapping of edges into one of ν conducting layers, each of which is a graph isomorphic to the grid graph, is called a ν-layer wiring; in a wiring, if a grid point is used by a path from layer L_i to layer L_j then no layer in between can be used by other paths. Problem of 3-layer wiring a knock-knee layout is known to be NP-complete. We propose an algorithm for wiring a given knock-knee layout W in three layers by replacing knock-knees with 45° wires, that is, mapping the layout into a 45° grid graph, being dual of an octo-square tessellation of the plane. A technique for efficiently "stretching" W, employing 45° wires, to ensure its two-layer wirability is also proposed. All algorithms run in time O(b) and use O(b) vias (i.e., communication paths in the third dimension), where b is the total number of bends.

Key words: *Grid graph, edge disjoint paths, octo-square tessellation, VLSI*

219

A Generalization of Pitteway's Algorithm

J.A. Hoskins and W.D. Hoskins, University of Manitoba*

The Dirichlet tesselation is used very frequently in the biological sciences to model competitive effects between individuals (either plant or animal) in efforts to predict, control or explain the organization of these individuals at observed spatial sites. Various attempts have been made to modify the basic weakness of the Dirichlet tesselation in this context viz. its lack of ability to incorporate the relative size, health, vigor or other competitive advantage as modifiers on the geometry of the tesselation. However, it appears that the algorithm and tesselation generalization to be described has not been used before since it relies on a new geometry theorem, and possesses the advantage that it reduces to the standard tiling for equally weighted vertices. The new tesselation is related to previous work concerning nucleation sites in crystallizing metals but, the latter work is concerned with a region centered about a fixed initial nucleation site whereas for animals there is no need to operate with this initial restriction and as a consequence the structure of the new weighted distance tiles is very different.

220

EXPONENTIAL LOWER BOUNDS FOR THE NUMBER OF GRACEFUL NUMBERINGS OF SNAKES

J. Abrham*, University of Toronto, and
A. Kotzig, Université de Montréal

It is shown that the number of graceful numberings of the snake with n edges grows at least exponentially with n. In particular exponential lower bounds are obtained for two special classes of graceful numberings of this snake.

Key words: Graceful numberings, snakes.

221

A Lower Bound for (t,k,L,v)-Designs

G. Rubin-Thomas, Benedict College

Consider a set of v objects, called varieties, and any k-set or a L-set containing these varieties without repetition. A (t,k,L,v)-design is a collection of k-sets such that any L-set intersects at least one of these k-sets in at least t varieties. Let B(t,k,L,v) denote the cardinality of a minimal (t,k,L,v)-design. Let

$$\eta(t,k,L,v) = \left\lceil \frac{1}{\binom{k}{t}} \left\lceil \frac{v}{v-t} \left\lceil \frac{v-1}{v-t-1} \left\lceil \cdots \left\lceil \frac{L+1}{L-t+1} \right\rceil \right\rceil \cdots \right\rceil\right\rceil\right\rceil$$

where $\lceil x \rceil$ denotes the least integer greater than or equal to x.

We show the following:

(1) $\eta(t,k,t,v) = \left\lceil \binom{v}{t} / \binom{k}{t} \right\rceil$

so that, for a Steiner system, $B(t,k,t,v) = \eta(t,k,t,v)$;

(2) $B(2,2,L,v) = \eta(2,2,L,v)$; and

(3) $B(t,k,L,v) \geq \eta(t,k,L,v)$.

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A Polynomial Algorithm for Chain Packings

Garth Isaak, RUTCOR, Rutgers University

Define a chain packing in a graph $G = (V, E)$ to be a collection P of edge disjoint chains such that no two chains share a common endpoint and such that in the partial graph $H = (V, E(P))$ formed by the edges of P, the degree of each vertex is less than or equal to a given integer constraint $b(v)$. DeWerra has examined odd chain packings (each chain in P has odd length) and deWerra–Pulleyblank reduces computation of a maximum cardinality odd chain packing to computation of a maximum capacitated b-matching. DeWerra–Roberts shows that the augmenting chain theorem for matchings extends to chain packings and also that a maximum cardinality chain packing can always be obtained by short chains (each chain in P has length one or two). In this paper, the augmenting chain theorem is used to give a polynomial algorithm for maximum cardinality chain packings. A modified version produces short chain packings at each stage of the algorithm.

223

Digraphs with Constant Walk Numbers

Hari Iyer, Bennet Manvel*, and Richard Osborne
Colorado State University

An investigation of covering spaces of digraphs gave rise to the question: Which digraphs have exactly k walks of length n, for every n? Clearly a digraph with this constant walk property must have at least one directed cycle, but no two directed cycles joined by a directed walk. Any digraph with such a cycle structure will have walk numbers which are eventually periodic. For such digraphs, we present a reduction process which generates two sequences $a = (a_0, a_1, a_2, \ldots, a_{m-1})$ and $b = (b_0, b_1, b_2, \ldots, b_{m-1})$. The digraph has walk numbers which are eventually constant if and only if the sum

$$\sum_{j=0}^{m-1} a_j b_{(j+k)}$$

is independent of k, where the subscripts are taken modulo m. We characterize the pairs of sequences $(a), (b)$ with this constant cyclic sum property.

224

On edge-gracefulness of the kth power graphs.

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A (p,q)-(multi) graph G is said to be edge-graceful if there exists a bijection f: E --> {1,...,q} such that the induced mapping f^+ --> {0,1,..,p-1} defined by $f^+(u) = \Sigma\ \{f(u,v): uv \in E\} (\mathrm{mod}\ |V|)$ is a bijection.

We consider graph G such that its kth power G^k is edge graceful. For k = 2 and 3 the problem is completely solved for path P_n and cycle C_n.

225

DESIGN ISOMORPHISMS

*Neal Brand, University of North Texas and Somporn Sutiniuntopas, Ramkamhaeng University, Bangkok.

Let B and B' be 3–(v,4,2) designs with v–set an abelian group, G, such that translations by elements of the group give design automorphisms. From the designs one can construct difference sets and topological spaces associated to the designs and the difference sets. We exploit the topology to show that under certain geometric conditions, an isomorphism from B to B' must be a translation in G composed with a group automorphism of G. These conditions can often be used to compute the automorphism group of a design.

KEY WORDS

combinatorial design, design isomorphism, difference set, covering projection

226

On the bisection width of partial $k-trees$

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A $k-tree$ is a graph that can be reduced to a k clique by a pefect elimination scheme. A partial $k-tree$ is a partial graph of a $k-tree$. Sufficiently large partial $k-trees$ can be disconnected by the removal of atmost k vertices. We use this bounded decomposability property of partial $k-trees$ to develop an $O(n^2)$ algorithm for the bisection width problem on these graphs. The bisection width problem is equivalent to finding an optimal edge separator in a graph. The bisection width problem is NP-complete for general graphs and polynomial time algorithms are only known for co-graphs and some sub-classes of partial $k-trees$ such as trees and $k-$ *outerplanar* graphs. Our interest in the bisection width problem stems from its fundamental importance in graph embedding and VLSI layout problems. Partial $k-trees$ arise naturally in hierarchical design techniques for VLSI layout problems.

Keywords:- Graph bisection width, Graph Embedding, Algorithms, Complexity and VLSI.

227

ON THE EIGENVALUES OF CERTAIN WEIGHTED DIRECTED GRAPHS †

Madhav P. Desai and Vasant B. Rao

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Urbana, Illinois 61801.

We study a weighted directed graph $G = (V, E)$ with edge weights chosen so that the adjacency matrix $A(G)$ of G is the transition matrix of a reversible Markov chain. We derive an upper bound on the second largest eigenvalue of $A(G)$. This bound is a function of the *expansion properties* of G, and on the *skewness* of the left eigenvector of $A(G)$ corresponding to its largest eigenvalue. We show that the upper bound is tight when considered as a function of the skewness alone. Knowledge of the second largest eigenvalue enables us to study the speed of convergence (in particular, the *finite-time behavior*) of a reversible Markov chain, which can be used to model probabilistic algorithms such as Simulated Annealing (SA).

Keywords: Expander graphs, eigenvalues, Markov chains and simulated annealing.

228

On edge-graceful (multi)graphs which are decomposable into Hamiltonian cycles.

Y.S.Ho and Sin-Min Lee*
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Dept. Maths and Computer Science,San Jose State University, San Jose,CA95192.

A (p,q)-(multi) graph G is said to be edge-graceful if there exists a bijection f: E --> {1,...,q} such that the induced mapping f^+ --> {0,1,..,p-1} defined by $f^+(u) = \Sigma\{f(u,v): uv \in E\} (\text{mod } |V|)$ is a bijection.

The second author conjectured that every connected (p,kp)-multigraph G which are decomposable into k Hamiltonian cycles is edge-graceful. if and only if p is odd. Some partial results are given to support this conjecture.

229

The Design of Template Immune Networks: Path and Star Immunity

Arthur M. Farley* and Andrzej Proskurowski
Computer and Information Science, University of Oregon

A network is specified by a topology and a protocol. A network's topology defines its connection structure, while the protocol defines its operational behavior. A template is a connected graph. A network is template immune with respect to a set of templates T if its topology remains connected under removal of any imbedding of an element of T and its protocol guarantees that all remaining sites can still communicate. Isolated template immunity with respect to a set of templates T maintains immunity under multiple imbeddings of elements of T, where each template imbedding contains no vertices that are neighbors of another imbedding. We discuss the design of networks that are isolated template immune with respect to simple path (P_n) and star ($K_{1,n}$) templates.

230

Improved binary and ternary code coverings by simulated annealing.

L. T. Wille, Florida Atlantic University

New upper bounds have been obtained for K(n,R), the minimal number of codewords in any binary code of length n and covering radius R (=1,2) and for ternary code coverings of covering radius 1 (the football pool problem). These results were found by a computer algorithm based on the simulated annealing technique. The implementation of this algorithm to the problem at hand will be discussed, with particular attention to the cooling rate dependence of the results.

Key words: code coverings, football pool problem, simulated annealing algorithm.

231

Results and questions about the limit points of graph eigenvalues

Michael Doob, University of Manitoba

The problem of finding the values of the limit points of graph eigenvalues, after being dormant for ten years,has seen a new burst of activity. About a year ago James Shearer showed that every real number r satisfying $r > \tau^{3/2}$ is the limit point of the the largest eigenvalues of a sequence trees (τ is the golden mean). This result has than the largest by the author. In this case it possible to use a sequence of trees also. The limit point of least eigenvalues can be determined for all real values except for those r in the tiny gap $-2.06 < -\tau^{3/2} < r < -2$. Some new constructions will be given and some new questions raised concerning these limit points.

Key Words: Graphs, Eigenvalues, Limit points.

232

On k-gracefulness of the sequential join of null graphs.

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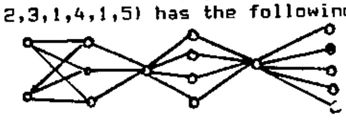
Dept. of Computer Science, Stevens Institute of Technology, Hoboken, NJ07030.

For integers $k \geq 1$, a (p,q) graph G is said to be <u>k-graceful</u> if there exists an injection $f: V \rightarrow \{0,1,\ldots,k+q-1\}$ such that the induced mapping $f^*: E \rightarrow \{k,k+1,\ldots,k+q-1\}$ defined by $f^*(uv)=|f(u)-f(v)|$ is a bijection. G is said to be arbitrarily graceful if it is k-graceful for all $k \geq 1$.

We show that for any sequence $\{a_1,\ldots,a_n\}$ of integers where $a_i \geq 1$. The sequential sum $SP(a_1,\ldots,a_n)$ of the null graphs $\{N(a_1),\ldots,N(a_n)\}$ is arbitrarily graceful.

The graph SP(2,3,1,4,1,5) has the following configuration:



233

Computation of Galois Groups
John B. McKay, Concordia University

A problem in Galois theory is the determinations of the Galois group from a polynomial f(x). The Galois group is the group of automorphisms of the splitting field of f, and is realisable as a group of permutations on the roots. It is easy to compute lower bounds on G but hard to compute upper bounds. Surprisingly, the existence of certain designs attached to G provides a mechanism for obtaining upper bounds. A p–adic method has been developed by Darmon (Harvard) and Ford (Concordia) which is used to establish the Mathieu groups M_{11} and M_{12} as Galois groups of polynomials over the rationals. It is hard to distinguish between non–isomorphic groups having the 'same' character tables and the 'same' class power maps. The eigen–values of corresponding respresentations on corresponding elements are equal. I expect the groups have the same Polya combinatorics. For small degree (n up to 7) an algorithm has been implemented by my student Ron Sommeling on Maple and is available as part of Maple. Research is needed in understanding how to use ramified primes.

234

On q-Derangement Numbers

MICHELLE WACHS, UNIVERSITY OF FLORIDA

We derive a q-analogue of the classical formula for the number of derangements of an n element set. Our derivation is entirely analogous to the classical derivation, but relies on a descent set preserving bijection between the set of permutations with a given derangement part and the set of shuffles of two permutations.

235

A lower bound for the weighted balancing problem

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A *signed graph* is a graph $G = (V, E)$, with a positive or negative sign associated to each of its edges $e \in E$. A signed graph is called *balanced*, if there is no negative cycle in it, i.e., a cycle with an odd number of negative edges. The *weighted balancing problem* is to find a subset $F \subseteq E$ of the edges such that the deletion of these edges from E makes G balanced, and $\omega(G) = \sum_{e \in F} \alpha_e$ is minimal, where α_e are given positive reals associated to the edges.

In this paper we present a network model and an $O(|V|^3)$ algorithm to obtain a lower bound for $\omega(G)$. We also present a decomposition of the graph, resulting in an improved bound in $O(|V|^4)$ time, and implying that some of the edges cannot belong to F at the optimum.

236

Multidimensional Matching for Chordal Graphs is Polynomial

Elias Dahlhaus, University of Bonn

By the multidimensional matching problem we mean the following: Given a natural number k and a graph $G = (V, E)$, such that $\|V = n = km$. The problem is the construction of m pairwise disjoint subsets of V, which form a complete subgraph of G and have size k. The following is known (see for example M. Garey/D.S. Johnson, "Computers and Intractability", San Francisco), that multidimensional matching is NP-complete, even for fixed $k = 3$.

Define a graph to be chordal if it has no induced cycle of length > 3. By a result of Dirac we know that the number of maximal cliques of a chordal graph is bounded by the number of vertices. Using bipartite matching we can prove the following:

Multidimensional matching restricted to chordal graphs can be executed in polynomial time.

237

COMPUTATIONS WITH COHERENT CONFIGURATIONS (9:00 am)

D. G. Higman, University of Michigan, Ann Arbor

This talk will be about software under development for computations with cc's and associated designs and chamber systems; including computation of coherent closures, homogenous components, parameters, character-multiplicity tables, fusion, tensor products, chamber systems, and cc's of small type.

238

Is Self-Duality Involutory?

Joseph McCanna, University of New Mexico

Given a self-dual polyhedron (or graph) H, the duality-mappings are those taking vertices to faces & faces to vertices, reversing inclusion. Let $k = k(H)$ be the smallest positive integer for which a duality-mapping φ can be found with $\varphi^k = \text{id}$, the identity map. Even though it is true that "the dual of the dual is the original", there nevertheless exist polyhedra for which $k(H) \neq 2$. We show that the values acheived by k are precisely the powers of two.

239

The connected components of some geometric graphs

Klaus G. Fischer, George Mason University

Let N be a positive square free integer and consider the geometric graph whose vertices are the points of $\mathbb{Q}(\sqrt{N})^2$ with an edge between vertices if their Euclidean distance equals 1. As a consequence of the existence of additive colorings on these graphs, it is shown that these have a countable infinite number of connected components and a description of these is given. For example, if p is a prime for which $p = 3 \bmod 4$, then $1/p$ is in the component containing the origin iff $-N$ is a quadratic residue of p.

Key words: geometric graphs, connected components

240 A Heuristic Algorithm for the Computation of The Edge-Integrity of Halin Graphs

Marc J. Lipman* and Robert L. Sedlmeyer
Indiana University - Purdue University at Fort Wayne

Given a graph G and a set S of its edges, we let $m(G - S)$ denote the largest order of a component of $G - S$. The edge-integrity of G is the minimum taken over all such sets S of the sum $m(G - S) + |S|$. The computation of the edge-integrity of trees and cacti can be done in small polynomial time, while the problem of computing the edge-integrity of an arbitrary graph is known to be NP-Complete. A halin graph is the union of a plane tree with no vertices of degree two and the cycle through the tree's end-vertices in the order determined by the plane drawing of the tree. In this talk we present a heuristic, an A^*-based algorithm employing the heuristic, and an implementation of the algorithm, for computing the edge-integrity of halin graphs. We present an empirical investigation of the performance of the algorithm.

Key words: edge-integrity, halin graphs, A^*, heuristic

241

CONDENSATION PROGRAMS AND THEIR APPLICATION TO THE DECOMPOSITION OF MODULAR REPRESENTATIONS. (10:30am)

A.J.E. Ryba, University of Michigan, Ann Arbor

Condensation programs allow us to use Parker's Meat-Axe to study modules which would be much too large to investigate with the Meat-Axe alone.
In this talk, I shall show how a condensation program replaces a large matrix representation of a group algebra by a related but much smaller representation for a Hecke algebra. I shall describe a recent, and efficient implementation of such a program and discuss an example of its use.

242

On the Number of Lattice Points Inside a Convex Lattice n-gon

Stanley Rabinowitz, Westford Massachusetts

A lattice point in the plane is a point with integer coordinates. A lattice polygon is a polygon whose vertices are all lattice points. In 1980, Arkinstall (Bull. Austral. Math. Soc. 22(1980)259-274) showed that a convex lattice pentagon must contain an interior lattice point. In this paper, additional results of this type are shown. For example, it is shown that a convex lattice heptagon must contain at least 4 interior lattice points and a convex lattice nonagon must contain at least 7 interior lattice points. The combinatorial nature of the set of lattice points in the interior of a lattice n-gon is investigated. For example, if K is a convex lattice polygon, we define the interior hull, H, to be the boundary of the convex hull of the lattice points in the interior of K. It is shown that $v(K) \leq 2v(H)$ For $v(K) \geq 7$ where v(K) denotes the number of vertices of K. The function $g(v)=\min\{g(K) \mid v(K)=v\}$ is investigated where g(K) denotes the number of lattice points in the interior of K and the min is taken over all convex lattice polygons, K. For example, it is shown that $3\left\lfloor \frac{v-1}{2} \right\rfloor - 5 \leq g(v) \leq \binom{\lceil v/2 \rceil}{3}$.

A computer search leads to several conjectures concerning g(v).

243 REVERSE BINARY DIGRAPHS

Sajal K. Das, University of North Texas, Denton
*Narsingh Deo and Sushil Prasad, University of Central Florida, Orlando

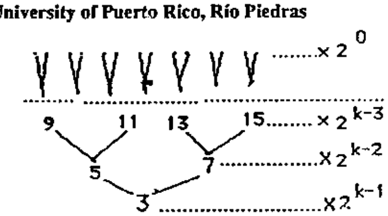
We define a family of digraphs as follows. Given a positive integer n, an $n \times n$ binary matrix $RA(n)$ is constructed such that the first $\lfloor \log i \rfloor + 1$ elements of the i^{th} row consist of the reverse bit-string of integer i and the remaining $n - (\lfloor \log i \rfloor + 1)$ elements are 0's, for $2 \leq i \leq n$. The first element in the first row of the matrix is 1 and the rest are all 0's. The *reverse binary digraph* $RBDG(n)$ of order n has $RA(n)$ as its adjacency matrix. We study various properties of these digraphs including connectivity, degree-sequence, transitive closure, coloring, and eigenvalues. For example, the digraph $RBDG(n)$, $n \geq 3$, is weakly connected with no cycles of length greater than one and it has $O(n \log n)$ edges. Exact expressions for the total number of edges, the in-degree of any vertex v_i in $RBDG(n)$, and the length of the longest path are derived. It is also found that digraph $RBDG(n)$ is nonplanar for $n \geq 15$. Since the reverse binary digraphs have many other parameters analytically defined, they can be used as possible test inputs for digraph algorithms.

244

A Natural Tree

Pablo M. Salzberg, Department of Mathematics, University of Puerto Rico, Río Piedras

The natural numbers in the interval (2^k, 2^{k+1}) can be represented by the binary tree shown in the figure. By overlapping these trees we obtain a kind of binary tree representation for the natural numbers, where the basic operations have a simple geometrical interpretation. In this framework, the Collatz "3x+1 problem" translates into an interesting jumping problem. Similar representations can also be obtained by considering the trees associated with the intervals (p^k, p^{k+1}), for any prime number p.



245

"COMPLETION METHODS IN COMPUTATIONAL GROUP THEORY"

Charles C. Sims, Rutgers University

Versions of the Knuth–Bendix procedure for strings and the Groebner basis algorithm are beginning to play a significant role in computational group theory. This talk will describe some computer experimentation with these techniques. Applications to the study of Burnside groups and the computation of certain solvable quotients of finitely presented groups will be discussed. Keywords: computational group theory, Knuth–Bendix procedure, Groebner basis algorithm, finitely presented groups

246

K-arcs in projective planes and in sets of M.O.L.S.

R. Killgrove*, R. Sternfeld, Indiana State University
C. Gordon, California State University, Los Angeles

Keywords: projective planes, orthogonal Latin squares

We suggest 6-arc completions to planes using the Bruck paper "Enriched Cliques with Six Vertices" Annals of Discrete Math. 18(1983) 133-168 as the starting point, or, the alternative, show, if true, failure to have complete k-arcs other than ovals and leave oval completions to others.

We suggest 6-arc completions of 3 M.O.L.S. of order 10. We concentrate on the case started earlier by Killgrove and Milne "Two elementary principles for combinatorial computations" Second Annual Houston Conference on Circuits, Systems and Computers 1970. We now have reduced the number of Latin squares of order 6 used to 79. Our preliminary computations suggest that one cannot even construct 3 Latin rectangles with six rows maintaining the necessary conditions.

247

Numbers of nonisomorphic drawings for small graphs

Heiko Harborth, Techn. University Braunschweig, West Germany

Realizations of graphs in the plane are called drawings if two lines have at most one point in common, either an endpoint or a simple crossing. Two drawings of a graph are isomorphic if there is a one-to-one correspondence which preserves the incidences of vertices, crossings, parts of edges, and regions. The numbers C(G) of nonisomorphic drawings of graphs G are determined for all G with up to five vertices. As a result for a general class of graphs we have $C(K_{1,n}+v)=\frac{1}{2}(1+3^{n-1})$. - This is common work with Hans-Dietrich O.F. Gronau, Greifswald, GDR.

248

A NEW BANDWIDTH REDUCTION ALGORITHM FOR TREES

Chandra GowriSankaran*, Dawson College, Montreal.
Zevi Miller, Miami University, Oxford, Ohio.
Jaroslav Opatrny, Concordia University, Montreal.

The most successful bandwidth reduction algorithms for graphs are level structure algorithms. This paper studies a new bandwidth reduction algorithm for trees, algorithm LST, which defines recursively a level structure for trees. Some theoretical properties and the time complexity of algorithm LST are discussed. Furthermore, we present the results of empirical studies, in particular comparing LST with the most successful bandwidth reduction algorithm to date, the GPS algorithm. It is shown that in almost all examples of trees studied, algorithm LST produced level structures of smaller widths than did GPS algorithm.

FRIDAY, FEBRUARY 24, 1989 11:10 a.m.

250

Fano free Coxeter free projective planes of order 11

R. Killgrove, R. Sternfeld*, Indiana State University
C. Gordon, California State University, Los Angeles

Keywords: finite projective plane

Preliminary report on work to show that the only Fano free, Coxeter free projective plane of order 11 is the known one. All order 10 latin squares that can be partitioned into 25 order 2 squares are found. Completion of each square to a plane of order 11 is attempted. A plane is Fano free iff it contains no Fano configuration. A plane is Coxeter free iff it contains no Coxeter configuration. A Coxeter configuration is two quadrangles sharing the same diagonal points and exactly one vertex.

251.

Vertex Partitions of Graphs and Induced Subgraphs

Hugh Hind, University of Southern Mississippi

Let H be graph. A graph G is called *H-free* if G does not contain H as a vertex-induced subgraph.

Given a graph H, it may be asked: Can we find a postive integer k (dependent only on H) such that for every graph G there exists a partition of the vertex set of G into k subsets where each subset in the partition induces a subgraph of G that is H-free?

It is shown that such a k cannot exist. More specifically, it is shown that for any finite graph H and n sufficiently large, there exists an $\epsilon > 0$ (where ϵ depends only on $|H|$) and a graph G on n vertices such that each vertex subset of G of size at least $n^{1-\epsilon}$ induces a subgraph of G containing an induced copy of H.

KEYWORDS: vertex partition, forbidden subgraph

252

MULTIPLICATIVITY OF ORIENTED CYCLES

Pavol HELL and Huishan ZHOU*, Simon Fraser University

A graph G is multiplicative if the class of all graphs X that do not admit a G-colouring (i.e., a homomorphism X-G) is closed under taking the (categorical) product. A longstanding conjecture of Hedetniemi states that all complete graphs are multiplicative. All directed and undirected paths and cycles are known to be multiplicative, with the exception of directed cycles of non-prime-power length. We present a new, purley combinatorial,proof of the multiplicativity of directed cycles of prime-power length, based on the method of Sauer and El-Zahar. The previous proof of this result (by Häggkvist, Hell,Miller,and Neumann-Lara) used the Lefschetz duality theorem of homology theory. We also give a complete classification of multiplicativity of all oriented paths (and almost all oriented cycles.) If time permits other versions of multiplicativity will be mentioned.

Keywords: Graph products,graph homomorphisms, generalized colourings, multiplicativity, Hedetniemi's product conjecture.

253

Polynomial Algorithms to Count Linear Extensions in Certain Posets

George Steiner, McMaster Univ. Hamilton, Canada

Counting the linear extensions of a partially ordered set is of considerable interest in computer science, operations research and other fields. The problem has been conjectured to be #p–complete for general posets. Atkinson and Chang [1] have presented solutions requiring $o(n^{k^2})$ time and $o(n^{k^2-k})$ space for posets of width k. In this paper we discuss an improved algorithm with $o(n^{k+1})$ time and $o(n^k)$ space complexity for this case, and other polynomial algorithms for classes of certain highly decom– posable posets. For these partial orders, we also present polynomial time solutions to count linear extensions with additional properties, e.g. having a fixed point or a fixed comparability relation.

[1]: M.D. Atkinson and H.W. Chang: Extensions of Partial Orders of Bounded Width, Congressus Numer. 52(1986), pp. 21–35.

254

Representative arcs in prime planes

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California State University
Los Angeles, CA 90032

Ray Killgrove
Department of Mathematics and Computer Science
Indiana State University
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Consider the projective plane over a field of prime order. Two arcs in the plane are *equivalent* if there is a collineation which maps one onto the other. We are interested in selecting an arc from each equivalence class. We restrict attention to arcs containing the points with homogeneous coordinates (1, 0, 0), (0, 1, 0), (0, 0, 1), and (1, 1, 1). Such an N-arc can be represented by an $(N-4)\times 2$ matrix whose rows are the affine coordinates of the other points. Two such representations are *equivalent* if their corresponding arcs are. Let a linear ordering of the field elements be given. Extend it lexicographically to the affine coordinate pairs and then to arc representations. An *N-key* is the least member of an equivalence class of N-arc representations. A representation A of an $(N+1)$-arc *extends* a representation B of an N-arc if the first $N-4$ rows of A agree with the $N-4$ rows of B. An easy but useful result is that every $(N+1)$-key *extends* an N-key. A more interesting result is that there are $N-1$ (nonlinear) matrix transformations $R_1, \ldots, R_{N-4}, C_1, C_2, J$ with the property that the orbit of an arc representation A under them is exactly the equivalence class of A. The *normal form* of a representation A is the least representation which can be obtained from A by row and column permutations. The normal form of A is equivalent to A.. Let C_0 and R_0 be the identity transformations and consider the set $\{R_iC_jA:\ i=0,\ldots,N-4\ \&\ j=0,1,2\}$. A third result is that the normal forms of these $3N-9$ representations are exactly of the normal forms of the representations in the orbit of A under C_1, $C_2, R_1, \ldots, R_{N-4}$. These three results have been used to obtain all N-keys in projective planes of various prime orders.

255

An Algorithm for k-Coloring a Graph Bounding the Monochromatic Index

Kenneth A. Berman* & Jerome L. Paul, University of Cincinnati

The *index* $\lambda(G)$ of a graph G is the eigenvalue of G of maximum absolute value. Given a vertex k-coloring C of G (i.e. a partition of vertices of G into k classes), the *monochromatic index* of C is $\lambda(G_C)$, where G_C is the subgraph of G induced by the set of all monochromatic edges. In this talk we discuss an efficient algorithm for obtaining a k-coloring C such that $\lambda(G_C) \le \lambda(G)/k$. The latter inequality was used to solve the paving problem in operator theory for non-negative matrices in [Berman, Halpern, Kaftal, Weiss, "Matrix Norm Inequalities and the Relative Dixmer Property", Integral Equations and Operator Theory 11, (1988), 28-48.]. Setting $k = [\lambda(G)] + 1$ the inequality also yields Wilf's Theorem which states that the the vertex chromatic number of G is bounded above by $\lambda(G) + 1$.

Keywords: graph algorithm, index, vertex coloring, chromatic number

256

Polynomials for Directed Graphs

Gary Gordon* and Lorenzo Traldi, Lafayette College

The Tutte polynomial for an ordinary graph can be generalized to rooted or unrooted directed graphs in many ways. We discuss several distinct (although related) generalizations. Many evaluations of these two variable polynomials have graph theoretic interpretations, such as the number of spanning arborescences (i.e., directed spanning trees) and the number of feasible sets (i.e., rooted sub-arborescences). When D is a rooted arborescence, one of these polynomials completely determines D. We discuss the proof of this fact and the generalization of this polynomial to greedoids.

Key words: Arborescence, Greedoid, Tutte polynomial

257

The Ultimate Algorithm for Counting Subsequences (12:00 N)

Ewa Kubicka and Allen J. Schwenk*, Western Michigan University

A sequence of length n has $\binom{n}{k}$ subsequences of length k. For binary sequences, each subsequence is one of 2^k possibilities. A subsequence counting algorithm determines how often each of the possibilities occurs. Beginning with a naive algorithm, we present four or five improvements based on binary search trees and deBruijn sequences that dramatically improve the speed of our algorithm.

The algorithm relates to the reconstruction problem for sequences. If I have a hidden sequence of length n and I tell you how often each possible subsequence of length k appears, can you determine the hidden sequence? For each k we can prove the existence of a minimum length n such that sequences of length n or more are not reconstructible. The difficult problem is determining the minimum nonreconstructible n for each value of k. We have found the minimum n for each $k \leq 6$. We suspect that n grows exponentially in k.

Keywords: binary search tree, reconstruction, deBruijn sequences

258

ON POLYADIC CODES

Vanessa R. Job, Goucher College

The polyadic codes are a generalization of the duadic codes and the triadic codes, which were generalizations of quadratic residue codes to non-prime lengths. For a particular value of m, a polyadic code is called an m-adic code. Existence criteria are given for m-adic codes of arbitrary length. The concept of an m-splitting is introduced and a one-to-one correspondence between m-splittings of $\mathbb{Z}_n$ over $GF(q)$ and families of m-adic codes of length n over $GF(q)$ is shown. A sub-family of the m-adic codes, called the m-adic codes of minimal degree, is defined and necessary and sufficient conditions for the existence of these codes are given. Lower bounds are given for the minimum of the weights of the odd-like vectors in m-adic codes of minimal degree. Also, the existence of a sub-class of m-adic codes with low minimum weight is shown and a method for constructing idempotents of binary duadic codes with low minimum weight is demonstrated.

Many binary polyadic codes have minimum weights which are the best possible or the best possible known for their lengths and dimensions. Tables of idempotents and minimum weights of all of polyadic codes of lengths less than or equal 100 and of selected additional polyadic codes of minimal degree are included.

259

VERTEX-EDGE ADJACENCY MATRIX OF A GRAPH AND AN ALGORITHM TO GENERATE ALL CLIQUES

Sajal K. Das *, University of North Texas, Denton, TX
Phalguni Mukherjee and Indranil Sengupta, University of Calcutta, India

A new matrix, called *vertex-edge adjacency matrix* (VEAM), is defined as another useful data structure for representing undirected graphs. This matrix is obtained by multiplying the adjacency matrix of a graph with its incidence matrix. As a by-product, we derive matrices, the elements of which are interpreted as 1−*connectors* (or γ-connectors) between two specified edges in the graph. We also study several graphical configurations corresponding to these connectors. Using a reduced version of VEAM as a data structure, a simple algorithm is presented to generate all cliques in a given graph, where a clique is a maximal complete subgraph. Although our algorithm requires exponential time in the worst-case, it has the advantage that non-maximal complete subgraphs and/or duplicate cliques are never generated and hence the average-case performance is better.

Key words: Vertex-edge adjacency matrix, 1-connector, clique, algorithm.

260

Introduction to the Conception of Impredictability

Gao Hongxun

Department of Methematics Nankai University , Tianjin , P. R. China

By a general review of several measurements of complexity for a given sequence , it appears an intrinsic deficiency in these measurements . Thus , a new conception , the activity or dynamic analysis of measurement of impredictability is given . Then , a mathematical model of the measurement of impredictability is derived . This activity analysis is tightly related to some of the classifications of the regarded sets of sequences . Some theoretical considerations under the sets of de Bruijn sequences(M-sequences) , for several case , are considered .

261

SEMIDUAL DECOMPOSITION OF A MOP GRAPH (12:30 p.m.)

Larry I. Basenspiler, University of South Alabama

An algorithm and a set of routines constructing a semidual tree from a maximal outerplanar (MOP) graph are presented. The algorithm runs in time linear in the size of input and uses only an insignificant amount of space (beyond that used to store initial graph and output semidual). The running package is presented in full, and the procedures which are of a theoretical interest and practical use are discussed in detail.

262

GRAPHS, LINEAR CODES AND MATCHINGS

Horacio Tapia-Recillas

A class of (error detecting-correcting) linear codes associated in a natural way to a (finite) graph is described and some of the invariants of the code (length of the words, dimension and minimum distance) are given in terms of the number of vertices, edges and cycles of the graph. A characterization of these codes is given in terms of perfect matchings when the graph is a tree. We relate some of these results with Cohen-Macaulay graphs.

264

Applications of Random Graphs to Circuit Design & Analysis

E Scheinerman, Johns Hopkins U.

Algorithms are traditionally analyzed from a *worst-case* perspective. However, *average-case* analysis often explains why theoretically bad algorithms (e.g. Simplex Method) perform well. Furthermore, random algorithms are often preferable to their deterministic counterparts.

We will discuss applications of random graph theory to the analysis and the design of computer hardware. In particular, we will discuss expected performance of and random designs for concentrators. Similar discussion of self-diagnosing systems and broadcasting will be presented as time permits.

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"FINITE BOL QUASIFIELDS ARE NEARFIELDS"

Jill Hanson, University of Aukland and Michael Kallaher
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A long standing problem in finite algebras and finite translation planes has been the determination of all finite Bol quasifields. Let Q be a finite Bol quasifield of order q where q is the dth power of the prime p. Work of Kallaher and Ostrom has shown that Q is a nearfield except possibly when p=5,7,11,19,23,29,59 and d=2 or p=3 and d=4,6. In this paper, using Cayeley, we prove: THEOREM: Q is a nearfield unless p=3 and d=4,6.

266 GROWTH OF MONOIDS AND FREE ALGEBRAS

John Pedersen, Dept. of Mathematics. U. South Florida, Tampa

When a variety or algebra has a canonical presentation (confluent and terminating), computing its growth series becomes a problem in enumerative combinatorics. The coefficients of the series are just the number of strings or trees of a given size not containing substrings or subtrees of certain kinds. We show how standard generating function techniques can be used to calculate such growth series.

267 Rodica Simion, George Washington University, Washington, D.C.

On the structure of the lattice of non-crossing partitions.

A partition of $[n] := \{1, 2, \ldots, n\}$, with *blocks* $B_1, B_2, \ldots, B_k$ is called *non-crossing* if whenever a, b, c, d satisfy $1 \le a < b < c < d \le n$, $a, c \in B_i$, and $b, d \in B_j$, then $i = j$. Kreweras showed that the non-crossing partitions of $[n]$ form a lattice, $NC(n)$, under the refinement ordering, and determined the Möbius function of $NC(n)$. Later, Edelman studied the Zeta polynomial and established formulae for the Whitney numbers of $NC(n)$. It is apparent from his formulae that $NC(n)$ is rank symmetric and rank unimodal. We extend these results by showing that $NC(n)$ is self-dual and admits a symmetric chain decomposition.

Key words: set partition, lattice, chain decomposition

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THE COMPUTATIONAL COMPLEXITY OF FINDING FULL SUB GENERALIZED n-GONS OF A GENERALIZED n-GON

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Is it reasonable to expect that we can find new examples of generalized n-gon for n=4,6,8 by computer search? The answer to this question is yes. In this paper, I prove:

Theorem: Given a generalized n-gon, it is possible in polynomial time to determine if it has or does not have a full sub generalized n-gon.

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Sharply 2-homogeneous sets of permutations

Gordon F. Royle, University of Waterloo

Sharply 2-homogeneous sets of permutations appear in the literature under the alternative names of half-planes and perpendicular arrays. It is known that a sharply 2-homogeneous set of permutations exists for every prime-power degree. We give a simple construction for this result based on splitting a sharply 2-transitive set of permutations into two parts.

On the Max Cut Problem in Interval Graphs

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Abstract

For a given graph $G = (V, E)$, the max cut problem is to find the proper partition of V into two disjoint sets V_1 and V_2 such that the number of edges from E that have one endpoint in V_1 and one endpoint in V_2 is maximum. The max cut problem is proved to be NP-complete for general graphs by Karp. In this paper, we investigate the max cut problem in interval graphs and we study the equivalent augmentation problem in interval orders. We also show that the problem is polynomial for special classes of interval graphs.

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Mahonian Statistics on Labeled Forests

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MICHELLE WACHS, UNIVERSITY OF FLORIDA

A permutation statistic is called *Mahonian* if it has the same distribution on the symmetric group S_n as the inversion statistic. The major index and the inversion index are the fundametal examples of Mahonian statistics. In a recent paper, Björner and Wachs generalized the major index to labeled forests and show that the major index has the same distribution as the inversion index on labeled forests of fixed shape. In this paper we give a direct combinatorial proof of this result by constructing an explicit bijection on labeled forests which takes the major index to the inversion index. For the symmetric group this bijection reduces to a new bijection on S_n taking the major index to the inversion index which is similar to a bijection of Foata. We also generalize the Mahonian statistics of Rawlings and Kadell to labeled forests and show that they have the same distribution as the inversion index as well.

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Cycle Bases from Orderings and Coverings

David Hartvigsen*, Russel Mardon, Eitan Zemel: Northwestern University

We use notions of "ordering" and "covering" to define several classes (some new, some old) of cycle bases for graphs. Each class is characterized stucturally and by its relationship to the other classes. For each class we also characterize (both constructively and by excluded subgraphs) those graphs for which every cycle basis is in the class. In addition, we characterize those graphs with a cycle basis that covers every edge two or more times; we show that for every odd k there exists a graph with a cycle basis that covers every edge exactly k times; and we describe a connection between cycle bases and ear decompositions.

Key words: cycle basis, fundamental, ear decomposition, greedoid