

Hamiltonicity of 3-connected, planar, $K_{1,1,5}$ -minor-free graphs

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The Hamiltonicity of various classes of graphs has long been a topic of interest. Tutte proved in 1956 that all 4-connected planar graphs are Hamiltonian. Weakening the connectivity condition to 3-connectivity is not sufficient for Hamiltonicity, however; there exist infinite families of graphs that are 3-connected and planar, but not Hamiltonian.

We look at the restricted class of 3-connected planar graphs obtained by forbidding $K_{1,1,5}$ as a minor. Using certain properties of induced fans, we are able to prove that with exactly one exception, all 3-connected, planar, $K_{1,1,5}$ -minor-free graphs are Hamiltonian. The one non-Hamiltonian graph in our class is the well-known Herschel graph, a bipartite graph on 11 vertices.

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