Efficient Domination on Archimedean Lattices

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A set of vertices S is said to dominate a graph G=(V,E) if every vertex in V is either in set S or is adjacent to a vertex in set S. For a finite graph G, T is a minimal dominating set if it dominates G and has a cardinality less than or equal to every other dominating set of G. For finite graphs, the minimal domination ratio of G=\(|T|\)\(|V|\). We extend the notion of domination ratio to infinite graphs composed of replicable subgraphs. Archimedean lattices are vertex-transitive infinite graphs formed by regular polygons. Efficient domination is when each vertex is dominated by exactly one vertex in the dominating set, and when possible it always achieves the domination ratio for infinite vertex-transitive graphs of finite degree. We prove for all Archimedean lattices whether or not efficient domination is possible. We then calculate upper and lower bounds for domination ratios of Archimedean lattices. Furthermore, we have solved exact domination ratios for most Archimedean lattices.

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