On the Edge-Balance Index Sets of Distance Two of Wheels


Let $G$ be a simple graph with vertex set $V(G)$ and edge set $E(G)$, and let $\mathbb{Z}_2 = \{0, 1\}$. Any edge labeling $f$ induces a partial vertex labeling $f^+ : V(G) \to \mathbb{Z}_2$ assigning 0 or 1 to $f^+(v)$, $v$ being an element of $V(G)$, depending on whether there are more 0-edges or 1-edges within the distance 2 to $v$, and no label is given to $f^+(v)$ otherwise. For each $i \in \mathbb{Z}_2$, let $v_f(i) = |\{v \in V(G) : f^+(v) = i\}|$ and let $e_f(i) = |\{e \in E(G) : f(e) = i\}|$. An edge-labeling $f$ of $G$ is said to be edge-friendly if $|e_f(0) - e_f(1)| \leq 1$. The distance-2 edge-balance index set of the graph $G$ is defined as $\text{EBI}_2(G) = \{|v_f(0) - v_f(1)| : f$ is edge-friendly.$\}$. In this paper, exact values of the distance-2 edge-balance index sets of wheel graphs are presented.

Keywords: vertex labeling, edge labeling, friendly labeling, cordiality, edge-balance index set, distance-2, wheels