Additive Coloring of Cycles

Axel Brandt*, Savannah Williams, Davidson College

An additive coloring of a graph $G$ is a labeling of the vertices of $G$ from $\{1, 2, \ldots, k\}$ so that any two adjacent vertices have distinct sums of labels on their neighbors. The additive coloring number of $G$, denoted $\chi_{\Sigma}(G)$, is the minimum positive integer $k$ such that $G$ has an additive coloring. In 2009 Czerwiński, Grytczuk, and Żelazny conjectured that $\chi_{\Sigma}(G) \leq \chi(G)$, where $\chi(G)$ is the chromatic number of $G$. The additive choice number of a graph $G$, denoted $\text{ch}_{\Sigma}(G)$, is the minimum positive integer $k$ such that whenever each vertex of $G$ is given a list of at least $k$ integers, then an additive coloring can be chosen from the lists. In 2016 Ahadi and Dehghan showed that $\chi_{\Sigma}$ and $\text{ch}_{\Sigma}$ can be arbitrarily far apart. In this talk, we show that $\chi_{\Sigma}(C_n) = \text{ch}_{\Sigma}(C_n) = \chi(C_n)$ for all cycles $C_n$. The proof for even cycles relies on counting placements of non-attacking rooks on a specific chessboard in application of the Combinatorial Nullstellensatz, and the proof for odd cycles relies on directed walks in an auxiliary graph.

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