Additive Coloring of Cycles

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An *additive coloring* of a graph G is a labeling of the vertices of G from $\{1, 2, \ldots, k\}$ so that any two adjacent vertices have distinct sums of labels on their neighbors. The *additive coloring number* of G, denoted $\chi_\Sigma(G)$, is the minimum positive integer k such that G has an additive coloring. In 2009 Czerwiński, Grytczuk, and Żelazny conjectured that $\chi_\Sigma(G) \leq \chi(G)$, where $\chi(G)$ is the chromatic number of G. The *additive choice number* of a graph G, denoted $\mathrm{ch}_\Sigma(G)$, is the minimum positive integer k such that whenever each vertex of G is given a list of at least k integers, then an additive coloring can be chosen from the lists. In 2016 Ahadi and Dehghan showed that χ_Σ and ch_Σ can be arbitrarily far apart. In this talk, we show that $\chi_\Sigma(C_n) = \mathrm{ch}_\Sigma(C_n) = \chi(C_n)$ for all cycles C_n. The proof for even cycles relies on counting placements of non-attacking rooks on a specific chessboard in application of the Combinatorial Nullstellensatz, and the proof for odd cycles relies on directed walks in an auxiliary graph.

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