

Extremal results on tournaments and 4-uniform hypergraphs

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One of the most important problems in Extremal Combinatorics is to determine the largest or the smallest possible number of copies of a given object in a finite combinatorial structure. In this talk, we address this problem in the case of tournaments and we give an application to 4-uniform hypergraphs. A *diamond* is a 4-vertex tournament which consists of a vertex dominating or dominated by a 3-cycle. Bondy [1] asked for the maximum number δ_n of diamonds in an n -tournament. This question has some relationship with the following particular case of a problem raised by Frankl and Füredi [2]: what is the maximum number of hyperedges of a 4-uniform hypergraph in which every 5 vertices span 0 or 2 hyperedges? We present a partial solution of these two problems for $n \equiv 0[4]$ and for $n \equiv 3[4]$. Our work improves some recent results obtained by Sameraro et al [3].

[1] J.A. Bondy. Private Communication to Boussaïri, 2008.

[2] P. Frankl, Z. Füredi, An exact result for 3-graphs, *Discrete Math.*50, 1984, 323 – 328.

[3] K. Gunderson and J. Semeraro, Tournaments, 4-uniform hypergraphs, and an exact extremal result, *Journal of Combinatorial Theory, Series B.*, 2017, pages 125.

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