Theta Graphs are Hall $t$-chromatic

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A graph satisfies Hall’s $t$-condition on $\kappa, G$ if for each subgraph $H$ of $G$

$$\sum_{\sigma=1}^{t} \alpha(H(\sigma, L)) = t\alpha(H) \geq \sum_{v \in V(H)} \kappa(v)$$

A graph $G$ is said to be Hall $t$-chromatic if the only color demands $\kappa$ for which there does not exist a proper $(t, \kappa)$ coloring of $G$ are those that fail Hall’s $t$-condition. A theta graph is a union of $m \geq 3$ internally disjoint paths, with the same end vertices. We show that every theta graph is Hall $t$-chromatic for all non-negative integers $t$.

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