**On Combinatorial Interpretations of some Elements of the Riordan Group**

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Let $g(z) = 1 + \sum_{k=1}^{\infty} g_k z^k$ and $f(z) = \sum_{k=1}^{\infty} f_k z^k$, where $f_1 \neq 0$. A Riordan array $R = (g(z), f(z))$ is an infinite lower triangular matrix whose column generating functions are $g(z)(f(z))^k$, where $k = 0, 1, 2, 3, \cdots$. Riordan arrays, equipped with Shapiro's multiplication rule $(g(z), f(z)) * (h(z), k(z)) = (g(z)h(f(z)), k(f(z)))$, form a group and this group provides an interesting algebraic framework to solve combinatorial problems. In this talk, we provide a combinatorial proof of an identity involving the Central Trinomial Numbers and the Bell subgroup element $(M(z), zM(z))$, where $M(z)$ is the generating function of the Motzkin numbers, and settle Shapiro's uniqueness question regarding this identity. We also provide a combinatorial interpretation for the Bell subgroup elements of the form $R = (g(z), zg(z))$, where $g(z)$ is a generating function satisfying a functional equation $g(z) = 1 + z * g(z)^k (k \geq 2)$, and use our combinatorial interpretation to obtain their inverses explicitly in terms of powers of $g(z)$.

Keywords: Riordan arrays; Bell subgroup; Central Trinomial and Motzkin numbers.