Toughness and the Wiener Index of a Graph

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Let G be a finite simple graph with vertex set $V(G)$. The *Wiener index* of G is defined as $W(G) = \sum_{u,v \in V(G)} \mathrm{d}(u,v)$, where $\mathrm{d}(u,v)$ is the distance between u and v. This topological index was defined by Wiener in 1947 for studying the structural graphs of molecules. Since that time, many theoretical results have been derived. The results we present concern the *toughness* of a graph G, denoted $\tau(G)$. Let $\mathcal{S} = \{S \subseteq V(G) \mid S \neq \emptyset \text{ and } \omega(G-S) \geq 2\}$, where $\omega(G-S)$ is the number of components in the induced subgraph $G-S$. Then $\tau(G) := \min_{S \in \mathcal{S}} \frac{|S|}{\omega(G-S)}$. Lower bounds on the toughness have been used extensively to determine properties of graphs. Our main results provide upper bounds on the Wiener index of G that guarantee $\tau(G) \geq t$ for some specified $t \geq 1$.

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