An improved upper bound for the site percolation threshold of the square lattice.

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Percolation models are infinite random graph models which have applications to phase transitions and critical phenomena. In the site percolation model, each vertex in an infinite graph G is retained independently with probability p and deleted otherwise, while an edge is retained only if both of its endpoints are retained. The percolation threshold is the critical probability $p_c(G)$ such that if $p > p_c(G)$ there is positive probability that the random subgraph of retained elements has an infinite connected component, while the probability that all of its components are finite is one if $p < p_c(G)$.

There are a few lattice graphs for which the site percolation threshold is exactly known, but rigorous bounds for unsolved lattices have been very inaccurate. We present an upper bound for the site percolation threshold of the square lattice, which improves the best previous result, obtained in 1995. A two-stage application of the substitution is used, comparing the square lattice with a specially designed self-matching lattice. The substitution method achieves significant computational reductions from the use of symmmetry, non-crossing partitions, and conversion to a network flow problem.

Keywords: percolation, threshold, self-matching lattice, set partition, non-crossing partition, network flow.