From coloring the real numbers to constructing wave functions on graphs

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A convergence structure on a set X is a binary relation ($\mathbf{F}$ *converges to* x) (i.e. $\mathbf{F} \downarrow x$) between the filters on X and the points in X subject to the constraint that the set of filters $\{\mathbf{F} \mid \mathbf{F} \downarrow x\}$ is closed under reverse inclusion and the filter generated by $\{x\}$ converges to x, all $x \in X$. A *convergence space* is a set equipped with a convergence structure. A neighborhood of x is a member of every filter converging to x. Every topological space is a convergence space. If every point has a least neighborhood, then we have a directed graph. $f\colon (X, \downarrow_X) \longrightarrow (Y, \downarrow_Y)$ is continuous if the f-image of every $\mathbf{F}$ converging to x in X converges to $f(x)$ in Y. If Y is a directed graph then we say that a continuous function is a *coloring* of X. We can travel through the vertices of a directed graph continuously, and emphasize that we do not travel along edges. We are interested in measurable colorings of the reals and of the Euclidean plane, where the σ-algebra on Y is generated by the vertex neighborhoods in Y. A coloring turns Y into a measure space, induces a measure on the graph neighborhoods, and from there we obtain wave functions on Y partially constrained by the coloring.

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