$3'$-GDDs with 3 Groups and Block Size 5

Zebene Girma Tefera, Addis Ababa University, Ethiopia

A t-(v, k, λ) design is a collection of k-subsets of a v-set $\mathbb{V}$, such that every t-subset of $\mathbb{V}$ is in exactly λ blocks.
A group divisible design, GDD$(n, m, k; \lambda_1, \lambda_2)$, is an ordered triple $(\mathbb{V}, \mathbb{G}, \mathbb{B})$ where $\mathbb{V}$ is a mn-set of symbols, $\mathbb{G}$ is a partition of $\mathbb{V}$ into m sets called groups of size n each, and $\mathbb{B}$ is a collection of k-subsets (called blocks) of $\mathbb{V}$, such that each pair of symbols from the same group occurs in exactly λ_1 blocks and each pair of symbols from different groups occurs in exactly λ_2 blocks.
GDDs and t-designs have been studied for their usefulness in statistics and for their universal application to constructions of new designs, and provide a challenging area in the construction of Combinatorial designs.
Definition of the group divisible designs can be generalized in several ways to define a t-GDD. One such generalization is the following. A 3'-GDD$(n, 3, 5, \mu_1, \mu_2)$, with 3 groups and block size 5 is defined by combining GDDs and t-designs; every triple occurs in exactly μ_1 blocks if it contains elements from at most 2 groups while it occurs in exactly μ_2 blocks if it has all three elements from different groups.
In this work, we formulate the necessary conditions for the existence of this GDD, obtain several new constructions and prove specific instances of non-existence.