

A Combinatorial Model for Merging in Traffic: Lattice Path Approach

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A two lane road approaches a stoplight. The left lane merges into the right just past the intersection. Vehicles approach the intersection one at a time, with some drivers always choosing the right lane, while others will always choose the shorter lane, or the right lane if they are equal in length. We identify an arrival sequence of vehicles as a binary string, where the 0s represent drivers always choosing the right lane, and the 1s represent drivers choosing the shorter lane.

A merging path is a lattice path formed by an arrival sequence where right steps represent 1s and up steps represent all 0s, and 1s choosing the right lane when the lanes are even. We give closed formulas for all merging paths reaching the point (m, n) with exactly k 0s in the arrival sequence, and the expected length of the right lane for all arrival sequences with exactly k 0s. Proofs involve an adaptation of Andre's Reflection Principle. Other interesting identities also emerge, such as a connection with the Ballot numbers.

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