Rigidity Matroids: Combinatorial, Linear and Algebraic

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The rigidity theory of bar-and-joint frameworks has a well defined linear matroid structure (valid in all dimensions), a combinatorial structure (well understood only in dimension 2) and an algebraic structure, which has only recently started being investigated. I will survey the connecting aspects of these representations, focusing on graph theoretical inductive constructions of circuits and their algebraic interpretations in dimension 2.

We have recently introduced a novel operation on graphs, called a combinatorial resultant, which is an abstraction of the classical resultant in algebraic elimination theory. We showed that all combinatorial rigidity circuits in 2D can be constructed inductively from K4 graphs by applying this operation. Our construction has an algebraic counterpart leading to the calculation of a so-called circuit polynomial in the Cayley-Menger ideal. The algebraic aspects will be presented in a subsequent talk by Goran Malic. Here, I will focus on the combinatorial and algorithmic aspects, alongside with several emerging open questions.

Keywords: rigidity matroids, sparsity, inductive constructions, linear and algebraic representations