Rigidity and Matroid Theory for Circle Contact Structures in the Plane

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A *circle contact structure (CCS)* is a realization of a graph with vertices representing circles on the plane with edges representing tangential contacts. For a *generic* circle contact structure, there is a neighborhood of radius vectors in which each radius can be independently increased and decreased while preserving the contacts and remaining radii. We show that graphs with generic circle contact structures are the independent sets of a matroid over the edges of the complete graph K_n, whose circuits include K_4 and $K_{3,3}$. Interestingly the proof involves both the even cycle matroid and the $(2,3)$-sparsity (or 2-dimensional generic framework rigidity) matroid. In addition, we show that the (possibly nongeneric) bar-joint frameworks arising from circle contact structures are independent if and only if the structures are generic, and give common inductive constructions for bases. We discuss connections to a hyperconnectivity matroid that has several equivalent characterizations (arising from matrix completions, spline spaces, polynomial rigidity, bar-joint rigidity on the moment curve), and conjectured to be the unique free-est matroid (maximal matroid in the weak order posets) with K_4 and $K_{3,3}$ as circuits.

Keywords: circle contact structures, rigidity, matroids, weak order posets