

Beyond the Symmetries of the Base graphs of a Canonical Double Cover Irene Sciriha

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The factorization of a bipartite connected graph under direct product may not be unique but has a unique bipartite factor. The canonical double cover (CDC) of a base graph G is its direct product $G \times K_2$ with K_2 . Thus if a graph is a CDC, then its bipartite factor under direct product is necessarily K_2 . Non-isomorphic graphs with the same CDC have the same degree sequence and the same number of walks of arbitrary length from corresponding vertices. We define a walk-colouring of the vertices of a graph, which assigns the same colour to vertices having the same number of walks of any specific length starting from them. We also consider the inverse problem of determining the base graphs that produce isomorphic CDCs, equivalent of a 2-1 mapping of the vertices of a CDC to the vertices of a base graph. The CDC may have unexpected automorphisms which cannot be lifted from automorphisms of the base graph leading to non-isomorphic graphs having the same CDC. We show that the graphs with isomorphic CDCs are related by Ryser-switching of disjoint edges with the same end-vertex colouring.

Keywords: canonical double covering, Ryser switch, walk colouring.