

On the 2- Y -homogeneous condition of the incidence graphs of 2-designs

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Let Γ denote a bipartite graph with vertex set X and color partitions Y, Y' , and assume that every vertex in Y has eccentricity $D \geq 3$. For $z \in X$ and non-negative integer i , let $\Gamma_i(z)$ denote the set of vertices in X which are at distance i from z . Graph Γ is *2- Y -homogeneous* whenever for all i ($1 \leq i \leq D - 1$) and for all $x \in Y$, $y \in \Gamma_2(x)$ and $z \in \Gamma_i(x) \cap \Gamma_i(y)$, the number of common neighbours of x and y that are at distance $i - 1$ from z is independent of the choice of x, y and z .

In this talk, we discuss the 2- Y -homogeneous condition of the incidence graphs of 2-designs. We prove that quasi-symmetric 2-designs that are quasi-symmetric $3-(v, k, \lambda)$ designs with intersection numbers 0 and $y = \lambda + 1$ are the only 2-designs which have 2- Y -homogeneous distance-biregular incidence graphs. Moreover, every 2- Y -homogeneous distance-biregular graph with eccentricity $D = 3$ is the incidence graph of such a design.

Keywords: distance-biregular graph, incidence graph, 2-design