

Consecutive patterns in Catalan Words and the Descent Distribution

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A Catalan word $w = w_1w_2 \cdots w_n$ is one over the set of positive integers satisfying $w_1 = 1$ and $1 \leq w_i \leq w_{i-1} + 1$ for $i = 2, \dots, n$. Catalan words of length n are enumerated by the Catalan number $C_n = \frac{1}{n+1} \binom{2n}{n}$. We compute the distribution of the descent statistic on the Catalan words avoiding a consecutive pattern of length at most three. Baril–Kirgizov–Vajnovszki began the study of patterns in Catalan words sequences, focusing on the enumeration of those that avoid classical patterns of length 3. We make use of the symbolic method to associate functional equations satisfied by the counting generating functions. As a consequence we enumerate the set of Catalan words that avoid consecutive patterns of length 3, and we also provide the total number of descents on this set.

Keywords: Catalan word, generating function, patterns.